

## Hot Spot Temperature Analysis in 3 Phase Transformers Using FEM Method

Diako Azizi<sup>1</sup>, Ahmad Gholami<sup>2</sup>, Diar Azizi<sup>3</sup>

<sup>1</sup>(Department of Biomedical Engineering, Science and Research Branch, Islamic Azad University, Tehran, Iran)

<sup>2</sup>(Department of Electrical Engineering, Iran University of Science and Technology, Tehran, Iran)

<sup>3</sup>(Department of Mechanical Engineering, Science and Research Branch, Islamic Azad University, Tehran, Iran)

### ABSTRACT

Transformers represent the largest portion of capital investment in transmission and distribution substations. In addition, transformer outages have a considerable economic impact on the operation of an electrical network. Hot-spot temperature (HST) value is a significant factor that directly impacts on the operation of transformers. For this purpose, energy (thermal) and Maxwell equation are solved with together until temperature is obtained. For numerical solution of above equation, finite element method (FEM) is used. The selected model for simulation is 3 phases, 10 MVA transformers.

*Keywords*-Energy, hot spot, magnetic, temperature, transformer

### I. INTRODUCTION

Transformers are static pieces of apparatus with two or more windings, which by electromagnetic induction; transform a system of alternating voltage and current into another system of voltage and current usually of different values and at same frequency for the purpose of transmitting electrical power. Since transformers belong to the most valuable assets in electrical networks, it is suitable to pay higher attention to these operating resources. An outage impacts the stability of the network and the associated financial penalties for the power utilities can be increased. In a transformer operation, a part of the electrical energy is converted into the heat. Although this part is quite small comparing to total electric power transferred through a transformer, it causes significant temperature rise of transformer constructive parts, which represents the limiting criteria for possible power transfer through a transformer. That is why the precise calculation of temperatures in critical points is of practical interest. Thermal impact leads not only to long-term oil/paper insulation degradation; it is also a limiting factor for the transformer operation [1]. Therefore the knowledge of the temperature, especially the hot-spot temperature, is of high interest.

A hot-spot temperature calculation procedure is given in the International Standards [2]-[4]. In [5], [6] the algorithm for calculating the hot-spot temperature of a directly loaded transformer, using data obtained in a short circuit heating test, is given. These papers propose improvements in the modeling of thermal processes inside the transformer tank.

Calculation methods have to be based on an energy balance equation. Some attempts of heat transfer theory results application to the heat transfer from winding to oil are exposed in [7]. The usage of average heat transfer coefficient is typical in a transformer designing process to calculate needed number (area) of cooling surfaces [8]-[13]. In this paper, a procedure for obtaining the temperature distribution in the transformer is proposed. For numerical simulation of mentioned equation, it has been used finite element method.

### II. MATHEMATICAL FORMULATION FOR HEAT CONDUCTION EQUATION [14]

The structure of a transformer winding is complex and does not conform to any known geometry in the strict sense. Under fairly general conditions, the transformer windings can be assumed cylindrical in formation; hence, a layer or a disc winding is a finite annular cylinder [10]. The thermal and physical properties of the system would be equivalent to a composite system of insulation and conductor. It has been assumed that heat is generated throughout the body at a constant rate, and oil in the vertical and horizontal ducts take away the heat through the process of convection. However, in an actual transformer winding, the conductor is the only heat source. Later in this section, formulations are given for calculating different thermal and physical properties of the system. It has been assumed that temperature is independent of space variable, due to the fact that winding structure is symmetrical. The temperature at any point on the periphery of circle for a specific value of  $r$  and  $z$  is deemed a constant (i.e., presence of spacers has been ignored, thus reducing three dimensional problem to a two-dimensional) one with  $r$  and  $z$  as space variables. Dielectric loss in insulation is assumed to be small compared to copper losses in the conductor. The surface of disc or layer has been assumed smooth.

The generalized system of non-homogeneous heat conduction equation with non-homogeneous boundary condition, in Cartesian coordinate system is written thus [10,15]:

$$k \cdot \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + Q = 0 \quad (1)$$

In the region  $a < x < b$  and  $0 < y < l$ .  
 At the inner cylindrical surface:

$$-k_1 \frac{\partial T}{\partial x} + h_1 T = f_1(y) \quad (2)$$

At the outer cylindrical surface:

$$k_2 \frac{\partial T}{\partial x} + h_2 T = f_2(y) \quad (3)$$

At the bottom flat surface:

$$-k_3 \frac{\partial T}{\partial y} + h_3 T = f_3(x) \quad (4)$$

At the top flat surface:

$$k_4 \frac{\partial T}{\partial y} + h_4 T = f_4(x) \quad (5)$$

Equations (1) through (5) represent the general heat conduction equation with convection at all four boundary surfaces. In the above equations, temperature T is a function of space variables x and y. The term Q is the heat source function and has been modified here to take care of variation of resistivity of copper with temperature. The heat source term Q can be of the form:

$$Q = Q_0 \cdot [1 + \alpha_c \cdot (T - T_{amb})] \quad (6)$$

Where,  $\alpha_c$  is the temperature coefficient of electrical resistance of copper wire. With this representation, the function Q becomes temperature dependent, distributed heat source. Boundary functions  $f_1(y)$  and  $f_2(y)$ , derived from Newton's law of cooling are of the form (7) and (8):

$$f_1(y) = h_1 \cdot (T_b + m_1 y) \quad (7)$$

$$f_2(y) = h_2 (T_b + m_2 y) \quad (8)$$

The term  $T_b$  is the temperature at the bottom of the disc or layer, as applicable. Terms  $m_1$  and  $m_2$  are the temperature gradient along the winding height (for layer) or along disc thickness for a disc. Similarly, functions  $f_3$  and  $f_4$  representing temperature profiles across bottom and top surfaces, having the form same as shown in (7) and (8), where temperature gradient term has been taken as zero:

$$f_3(x) = h_3 T_b \quad (9)$$

$$f_4(x) = h_4 T_{top} \quad (10)$$

In general, thermal conductivities are shown different, and are indicated accordingly. But in an actual case, the thermal conductivities in radial directions ( $k_1$  and  $k_2$ ) are equal and conductivities in axial direction will also be the same ( $k_3$  and  $k_4$ ). Thermal conductivity has been treated as a vector quantity, having components in both radial and axial direction.

Resultant thermal conductivity of the system can be estimated as:

$$K = \sqrt{k_r^2 + k_z^2} \quad (11)$$

Where:

$$k_r = \frac{\log \frac{r_n}{r_1}}{\left( \frac{\log \frac{r_2}{r_1}}{k_1} + \frac{\log \frac{r_3}{r_2}}{k_2} + \dots + \frac{\log \frac{r_n}{r_{n-1}}}{k_n} \right)} \quad (12)$$

$$k_z = \frac{k_{cu} k_{in} (t_{cu} + t_{in})}{(t_{in} k_{cu} + t_{cu} k_{in})} \quad (13)$$

Term K represents resultant thermal conductivity of insulation and conductor system. Heat transfer coefficients  $h_1$  to  $h_4$  ( $h_{tc}$ ), are different across all the four surfaces. To determine boundary functions  $f_1$  to  $f_4$ , it is necessary to calculate heat transfer coefficients across the four surfaces. Difficulty has been encountered in calculation heat transfer coefficient. It is reported elsewhere [10], that it depends on as many as 13 factors (e.g., winding size, type, duct dimensions, oil velocity, type of oil circulation, heat flux distribution, oil thermal properties, etc.). In this work, corrections have been given for temperature dependence of the thermal and physical properties of oil, such as viscosity, specific heat, volumetric expansion and thermal conductivity. It was found that there is negligible effect of specific heat, coefficient of volumetric expansion and conductivity in the present working range of loading. Following are some of the heat transfer relations and relevant formulae in natural cooling (ON) mode. These formulae have been used to calculate the  $h_{tc}$  [16]. Local Nusselt number for laminar flow over vertical plates has been shown below:

$$Nu = 0.6 Ra_{hf}^{0.2} \quad (14)$$

$$Ra_{hf} = Gr_{hf} Pr \quad (15)$$

Where  $Ra_{hf}$  and  $Gr_{hf}$  are the local Rayleigh and Grashof number based on heat flux ( $q_w$ ) at characteristic dimension ( $\delta$ ).  $Pr$  is the Prandtl number of transformer oil. Expression of Rayleigh number based on constant heat flux is:

$$Ra_{hf} = \frac{g \beta C_p \rho^2 q_w \delta^4}{k_{oil}^2 \mu} \quad (16)$$

Mean Nusselt number in this case can be computed as:

$$Nu_m = 1.25 [Nu]_{\delta=1}$$

However, correction to formula (14) has to be given for cylindrical curvature. The correction factor in this case is of the following form ( $30 < Pr < 50$ ):

$$f(\xi) = 1 + 0.12 \xi \quad (17)$$

Where:

$$\xi = \frac{2\sqrt{2}}{Gr^{0.25}} \times \frac{\delta}{r} \quad (18)$$

$$Nu_m = 1.75 \left[ G_z + 0.012 \left( G_z Gr^{1/3} \right)^{4/3} \right]^{1/3} \times \left( \frac{\mu_b}{\mu_w} \right)^{0.125} \quad (25)$$

Here  $G_r$  is Grashof number based on temperature difference. Heat transfer coefficient can be computed as:

$$h = \frac{Nu_{k_{oil}}}{\delta} \quad (19)$$

$$G_z = RePr \frac{D}{l} \quad (27)$$

Where,  $h$  represents local coefficient. Mean coefficient ( $h_m$ ) can be calculated from mean Nusselt number, as in (15). After knowing  $h_m$  for a particular surface, temperature difference between the winding surface and oil can be found out, dividing the  $h_m$  by the heat flux through the surface. Mean Nusselt number of top surface of annular cylindrical winding for laminar and turbulent regime, will normally be of the form:

$$Nu_m = 0.54Ra^{0.25} = 0.61Ra_{hf}^{0.2} \quad (20)$$

$$Nu_m = 0.15Ra^{1/3} = 0.24Ra_{hf}^{1/4} \quad (21)$$

$$\delta = \frac{b-a}{2} \quad (22)$$

Where  $a$  and  $b$  are the inner and outer radius of annular disc or layer. Nusselt number of bottom surface of annular cylindrical winding for laminar and turbulent regime is in (23):

$$Nu_m = 0.27Ra^{0.25} = 0.35Ra_{hf}^{0.2} \quad (23)$$

The axial oil temperature gradient in presence of cooling by a constant heat flux can be found out by using (24), due to [17] thus:

$$\frac{\partial T}{\partial y} = 4.25 \times 10^{-2} \frac{q_w}{k_{oil}} \left( \frac{1}{\pi D_m} \right)^{4/9} Ra_{hf}^{-1/9} \quad (24)$$

Where  $l$  is the winding height,  $D_m$  is the mean diameter of annular disc or layer of windings. Determination of boundary conditions in forced convection (OF mode) too requires calculating  $h_{tc}$ . The expression when the oil velocity is lower values, the mean Nusselt number based on temperature difference is of the form of (25), corresponding mean Nusselt number based on constant heat flux is of the form of (26):

Where  $G_z$  called Graetz number. The terms  $\mu_b$  and  $\mu_w$  are viscosity of oil computed at oil bulk mean temperature of oil and at winding wall temperature, respectively.  $Re$  is the Reynolds number and  $Pr$  is the prandtl number. The relative importance of natural and forced cooling is indicated by the factor  $f_r = Gr/Re^2$ . If  $f_r \geq 1$  then both of the cooling modes have to be considered. At a lower value of this factor, natural cooling can be ignored. The oil viscosity is an important property, which depends on temperature. The formula to take the oil temperature variation into consideration is given in (28) below from [9, 10]:

$$\mu = \alpha \cdot \exp\left(\frac{\gamma}{T_m}\right) \quad (28)$$

Where:

$$\alpha = 0.0000013573 \text{ (kg.m}^{-1}\text{.s}^{-1}\text{)} \quad (29)$$

$$\gamma = 2797.3 \text{ (K)} \quad (30)$$

The viscosity was calculated at the mean oil and wall temperature. Initially, the winding surface temperature is not known, so a starting guess for winding surface temperature has to be made, after calculating the value of  $h$ , the temperature difference ( $T_w - T_{oil}$ ) is to agree with the assumed value. To calculate winding wall temperature at different surfaces, only bottom oil temperature is necessary. In this paper, the bottom oil rise over ambient temperature has been calculated as:

$$\theta_u = \theta_{fl} \cdot \left( \frac{l^2 R + 1}{R + 1} \right)^n \quad (31)$$

Where  $\mu_u$  is the bottom oil temperature rise over ambient Temperature  $\mu_{fl}$  is the full load bottom oil temperature rise over ambient temperature obtained from an off-line test;  $R$  is the ratio of load loss at rated load to no-load loss. The variable  $l$  is the ratio of the specified load to rated load:

$$I_r = \frac{I}{I_{rated}} \quad (32)$$

The exponent  $n$  depends upon the cooling state. The loading guide recommends the use of  $n=0.8$  for natural convection and  $n=0.9-1.0$  for forced cooling.

Assumptions have been made in the calculation of  $h_{tc}$  in the ducts provided in the disc-type windings under oil-forced (OF) modes of heat transfer (DOF and NDOF). The cooling in OF mode is due to mixed mode (natural and forced) convection. While the oil-flow velocity in both vertical and horizontal ducts has been assumed to be equal, the mode is DOF.

If the flow velocity in horizontal ducts is assumed negligible compared to velocity in the vertical ducts, the mode is NDOF. In case of DOF mode, the  $h_{tc}$  was assumed to be a function of both heat flux through the surface and the oil-flow velocity. The mechanism of heat transfer in this mode of cooling is same in both axial and radial direction of the disc. In case of NDOF mode, the  $h_{tc}$  in the vertical duct has been estimated by the same formula as for DOF. But the convection of heat in the horizontal ducts has been assumed to be a purely natural type.

### III. SIMULATION AND RESULTS

The FEM of the transformer is performed in order to verify the effectiveness of the theoretical equations used in the design process and validate the design parameters.

The FEM in the case study involves these stages: magnetic and thermal field distribution within the transformer.

#### III.A. Design parameters of the transformer

To investigate the proposed approach, a three-phase transformer was designed. Solenoid-type windings are used for the conceptual design. The primary and secondary windings have 16 layers and each layer 45 turns. The main parameters of the transformer are shown in table 1.

TABLE I  
 DESIGN PARAMETERS OF TYPICAL TRANSFORMER WITH AUXILIARY WINDINGS.

Rating:	Voltage:	Current:	Frequency:	Phase:
10MVA	63/20 kV	90/290A	50 Hz	three-phase

#### III.B. Simulations of magnetic field

In this section, the simulation results obtained using COMSOL Multiphysics software is discussed. The color plotting of the magnetic field distribution in the case study obtained as a result of the simulation process is explained. The color plots of the magnetic field distribution obtained as a result of 2D simulations is shown in figure 1.

Referring to the numerical flux density values shown in the figure 1, the maximum value of the leakage magnetic field is 0.015 T.

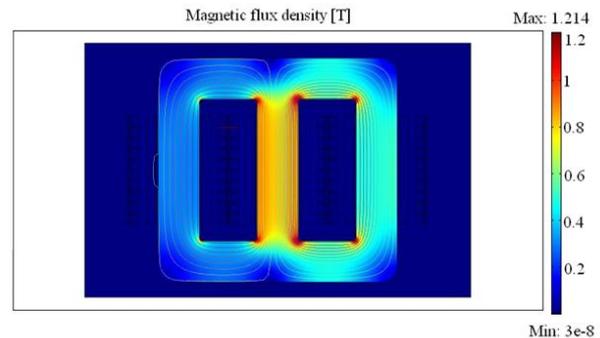


Fig. 1. Magnetic field distribution in typical transformer

#### III.C. Simulation of thermal field

The color plots of the thermal field distribution obtained as a result of 2D simulations is shown in figure 2.

It can be seen that, in figure 2, the maximum value of the temperature is 136°C..

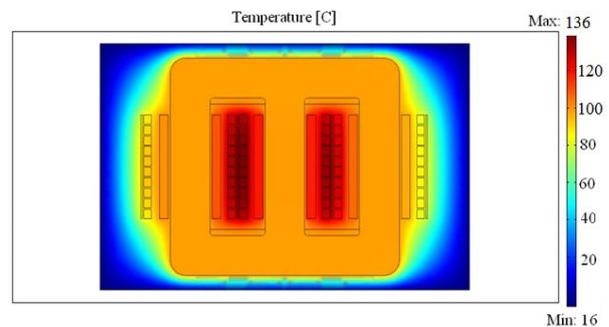


Fig. 2. Thermal field distribution in typical transformer

### IV. CONCLUSION

In this paper, an attempt has been made to suggest a method to improve the accuracy of prediction of the temperature of the hottest spot in power transformer by solving the heat transfer partial differential equation (PDE) numerically.

The purely numerical approach for evaluating hot spot and its location followed in this paper seems to correspond reasonably well with the results of calculations and actual tests and on site measurements [9, 10]. The authors wish to point out that the IEEE loading guide and other similar documents offer relations for the calculation of the HST based on p.u. load. The formulations tend to ignore the possibilities of two transformers that are rating identical but have a different winding structure and varying heat loss/ unit volume. The method suggested by the authors gives due representation for this omission and, hence, is believed to give more accurate estimates. The thermal model presented here can predict the hot-spot location, with a reasonable degree of accuracy.

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## Authors' information



**1-Diako Azizi** was born in 1985. He has received B.Sc. degree in Electrical Engineering from Tabriz University, Tabriz, Iran in 2007. And he received the Master degree in Electrical Power Engineering from the University of Science and Technology, Tehran, Iran in 2009. He is presently pursuing the Ph.D. degree in Electrical Power Engineering, Iran University of Science and Technology. His research interests are aging of insulations in electrical machines.



**2-Ahmad Gholami** has received his B.Sc. Degree in electrical engineering from IUST, Tehran, Iran, in 1975, the M.Sc. and Ph.D. Degrees in electrical engineering from UMIST, Manchester, England, in 1986 and 1989 respectively. He is currently an associate professor in the Electrical Engineering Department of Iran University of Science and Technology. His main research activities are high voltage engineering, electrical insulation, insulation coordination, transmission lines and substations planning.



**3-Diar Azizi** was born in 1982 in Iran. He has received B.Sc. degree in Mechanical Engineering from the Iran University of Science and Technology, Tehran, Iran in 2005. He is presently pursuing the M.sc degree in Mechanical Engineering in University Science and Research branch, Islamic Azad University, Tehran, Iran. His research interests are Heat Transfer, Fluid Mechanic, and Thermodynamic.