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Continuous And Irresolute Functions Via Star Generalised Closed Sets

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ABSTRACT: *In this paper, we introduce a new class of continuous functions called semi*δ-continuous function and semi* δ-irresolute functions in topological spaces by utilizing semi* δ-open sets and to investigate their properties.*

Keywords and phrases: *semi* δ-open set, semi* δ-closed set, semi* δ-continuous, semi* δ-irresolute function.Mathematics Subject Classifications: 54C05*

I. INTRDUCTION

In 1963, Levine[3] introduced the concept of semi-continuity in topological spaces. In 1968 N.V.Velicko introduced the concept of δ-open sets in topological spaces. T.Noiri[8] introduced the concept of δ-continuous functions in 1980.In 2014 Raja Mohammad Latif[15] investigate the properties of δ-continuous functions and δ-irresolute functions. S.Pasunkili Pandian[10] defined semi*pre- continuous and semi*pre-irresolute functions and investigated their properties. S.Pious Missier and A.Robert[12,16] introduced the concept of semi*-continuous and semi*α-continuous functions in 2014.Quite recently ,the authors[13] introduced some new concepts, namely semi*δ-open sets, semi*δ- closed sets. The aim of this paper is to introduce new class of functions called semi*δ-continuous, semi*δ-irresolute functions and investigated their properties.

II. PRELIMINARIES

Throughout this paper (X, τ), (Y, σ) and (Z, η) represent non-empty topological spaces on which no separation axioms are assumed unless otherwise mentioned.

Definition 2.1: A subset A of a space X is
(i) generalized closed (briefly g-closed) [4] if Cl(A)⊆U whenever A⊆U and U is open in X.
(ii)generalized open (briefly g-open) [4] if X\A is g-closed in X.

Definition 2.2: If A is a subset of X,
(i) the generalized closure of A is defined as the intersection of all g-closed sets in X containing A and is denoted by Cl*(A).
(ii) the generalized interior of A is defined as the union of all g-open subsets of A and is denoted by Int*(A).

Definition 2.3: Let (X, τ)be a topological space. A subset A of the space X is said to be
1. Semi-open [3] if A⊆Cl(Int(A)) and semi*-open[12] if A⊆Cl*(Int(A)).
2. Preopen [5] if A⊆Int(Cl(A)) and pre*open[14] if A⊆Int*(Cl(A)).
3. Semi-preopen [7] if A⊆Cl(Int(Cl(A))) and semi*-preopen[10] if A⊆Cl*(pInt(A))).
4. α-open [6] if A⊆Int(Cl(Int(A))) and α*-open [11] if A⊆Int*(Cl(Int*(A))).
5. Regular-open[13] if A=Int(Cl(A)) and δ-open[8] if A= δInt(A)
6. semi α-open[9] if A⊆Cl(αInt(A)) and semi* α-open[16] if A⊆Cl*(αInt(A))).
7. δ-semi-open [2] if A⊆Cl(δInt(A)) and semi*δ-open**[**13] A⊆Cl*(δInt(A)).
The complements of the above mentioned sets are called their respective closed sets.

Definition 2.4: A function *f : (X, τ)→ (Y, σ)* is said to be
1. g-continuous [1] if f^{-1}(V) is g-open in (X, τ) for every open set V in (Y, σ).
2. Semi-continuous [3] if f^{-1}(V) is semi-open in (X, τ) for every open set V in (Y, σ).

3. Semi*-continuous [12] if f^{-1}(V) is semi*-open in (X, τ) for every open set V in (Y, σ).
4. Pre-continuous [5] if f^{-1}(V) is pre-open in (X, τ) for every open set V in (Y, σ).
5. Pre*-continuous [14] if f^{-1}(V) is pre*-open in (X, τ) for every open set V in (Y, σ).
6. α- continuous [6] if f^{-1}(V) is α-open in (X, τ) for every open set V in (Y, σ).
7. α*- continuous [11] if f^{-1}(V) is α*-open in (X, τ) for every open set V in (Y, σ).
8. Semi-pre-continuous[7] if f^{-1}(V) is semi-preopen in (X, τ) for every open set V in (Y,σ).
9.Semi*pre-continuous[14] if f^{-1}(V) is semi*-preopen in(X,τ) for every open set V in (Y,σ).
10. Semi α-continuous [9] if f^{-1}(V) is semi-α-open in (X, τ) for every open set V in (Y, σ).
11.Semi*α-continuous [16] if f^{-1}(V) is semi*-α-open in (X, τ) for every open set V in (Y,σ).
12. δ-continuous [8] if f^{-1}(V) is δ-open in (X, τ) for every open set V in (Y, σ).
13. δ-semi-continuous [2] if f^{-1}(V) is δ-semi-open in (X, τ) for every open set V in (Y, σ).

Definition 2.5: A topological space (X, τ) is said to be $T_{\frac{1}{2}}$ if every g-closed set in X is closed.

Theorem 2.6: [13] Every δ-open set is semi*δ-open.
Theorem 2.7: [13] In any topological space,
(i) Every semi*δ-open set is δ-semi-open.
(ii) Every semi*δ-open set is semi - open.
(iii) Every semi*δ-open set is semi* - open.
(iv) Every semi*δ-open set is semi*-preopen.
(v) Every semi*δ-open set is semi-preopen.
(vi) Every semi*δ-open set is semi*α-open
(vii) Every semi*δ-open set is semiα-open.

Remark 2.8: [17] Similar results for semi*δ-closed sets are also true.
Theorem 2.9: [13] For a subset *A* of a topological space (*X*, τ) the following statements are equivalent:
(i) A is semi*δ-open.
(ii) A⊆Cl*(δInt(A)).
(iii) Cl*(δInt(A))=Cl*(A).

Theorem 2.10: [17] For a subset *A* of a topological space (*X*, τ), the following statements are equivalent:
(i) A is semi*δ-closed.
(ii) Int*(δCl(A))⊆A.
(iii) Int*(δCl(A))=Int*(A).

III. SEMI*δ -CONTINUOUS FUNCTIONS

Definition 3.1: A function f: (X, τ) → (Y, σ) is said to be **semi* □-continuous** if f^{-1}(V) is semi*δ-open in (X, τ) for every open set V in (Y, σ).
Theorem 3.2: Every δ-continuous is semi*δ-continuous.
Proof: Let f: (X, τ) → (Y, σ) be δ- continuous. Let V be open in (Y, σ). Then f^{-1}(V) is δ-open in (X, τ). Hence by Theorem 2.6, f^{-1}(V) is semi*δ-open in (X, τ). Therefore f is semi*δ-continuous.
The converse of the above theorem need not be true as it is seen from the following example.
Example 3.3: Let X = {a, b, c, d}, Y={a, b, c, d}, τ = {ϕ, {a},{b}, {a, b}, {a, b, c}, X} and σ= {ϕ, {a}, {a, b}, Y}. Define f: (X, τ) → (Y, σ) by f(a) = c; f(b) = a; f(c) = d; f(d) = b. Then f is semi*δ-continuous but not δ-continuous. Since for the open set V = {a, b} in (Y, σ), f^{-1}(V) = {b, d} is semi*δ-open but not δ-open in (X, τ).
Theorem 3.4: Every semi*δ-continuous is δ-semi-continuous.
Proof: By Theorem 2.7(i), every semi*δ-open set is δ-semi-open, the proof follows. The converse of the above theorem need not be true as it is seen from the following example.
Example 3.5: Let X= {a, b, c, d}, Y= {a, b, c, d}, τ = {ϕ, {a}, {b}, {a, b}, {a, b, c}, X} and σ= {ϕ,{a},Y}. Define f: (X, τ) → (Y, σ) by f(a) = b; f(b) = f(c) = a; f(d) = c. Then f is δ-semi-continuous but not semi*δ-continuous. Since for the open set V = {a} in(Y, σ), f^{-1}(V) = {b, c} is δ-semi-open but not semi*δ-open in (X, τ).

Theorem 3.6: Every semi*δ-continuous is semi-continuous.
Proof: By Theorem 2.7(ii), every semi*δ-open set is semi-open, the proof follows. The converse of the above theorem need not be true as it is seen from the following example.
Example 3.7: Let X= {a, b, c, d}, Y= {a, b, c, d}, τ = {ϕ, {a}, {a, b, c}, X} and σ= {ϕ, {a, b, c},Y}. Define f: (X, τ) → (Y, σ) by f(a) = b; f(b) = a; f(c) = d; f(d) = c. Then f is semi-continuous but not semi*δ-continuous. Since for the open set V = {a, b, c} in(Y, σ), f^{-1}(V) = { a, b, d} is semi-open but not semi*δ-open in (X, τ).

Theorem 3.8: Every semi*δ-continuous is semi*-continuous.
Proof: By Theorem 2.7(iii), every semi*δ-open set is semi*-open, the proof follows. The converse of the above theorem need not be true as it is seen from the following example.
Example 3.9: Let X= {a, b, c, d}, Y= {a, b, c, d}, τ = {ϕ, {a}, {a, b}, {a, b, c}, X} and σ= {ϕ, {a, b, c}, Y}. Define f: (X, τ) → (Y, σ) by f(a) = b; f(b) = a; f(c) = d; f(d) = c. Then f is semi*-continuous but not semi*δ-continuous. Since for the open set V = {a, b, c} in(Y, σ), f^{-1}(V) = {a, b, d} is semi*-open but not semi*δ-open in (X, τ).

Theorem 3.10: Every semi*δ-continuous is semi*pre-continuous.
Proof: By Theorem 2.7(iv), every semi*δ-open set is semi*pre-open, the proof follows. The converse of the above theorem need not be true as it is seen from the following example.
Example 3.11: Let X= {a, b, c, d}, Y= {a, b, c, d}, τ = {ϕ, {a}, {b, c, d}, X} and σ= {ϕ, {a, b}, {a, b, c}, Y}. Define f: (X, τ) → (Y, σ) by f(a) = b; f(b) = d; f(c) = a; f(d) = c. Then f is semi*pre-continuous but not semi*δ-continuous. Since for the open set V = {a, b} in(Y, σ), f^{-1}(V) = { a, c} is semi*pre-open but not semi*δ-open in (X, τ).
Theorem 3.12: Every semi*δ-continuous is semi-pre-continuous.
Proof: By Theorem 2.7(v), every semi*δ-open set is semi-pre-open, the proof follows. The converse of the above theorem need not be true as it is seen from the following example.
Example 3.13: Let X= {a, b, c, d}, Y= {a, b, c}, τ = {ϕ, {a, b}, {a, b, c}, X} and σ= {ϕ, {a}, {b}, {a, b}, Y}. Define f: (X, τ) → (Y, σ) by f(a) = b; f(b) = f(c) = a; f(d) = c. Then f is semi-pre-continuous but not semi*δ-continuous. Since for the open set V = {a} in(Y, σ), f^{-1}(V) = {b, c} is semi-pre-open but not semi*δ-open in (X, τ).
Theorem 3.14: Every semi*δ-continuous is semi*α-continuous.
Proof: By Theorem 2.7(vi), every semi*δ-open set is semi*α-open, the proof follows. The converse of the above theorem need not be true as it is seen from the following example.
Example 3.15: Let X= {a, b, c, d}, Y= {a, b, c}, τ = {ϕ, {a}, {a, b}, {a, b, c}, X} and σ= {ϕ, {b}, {a, b}, Y}. Define f: (X, τ) → (Y, σ) by f(a) = f(c) = b; f(b) = a; f(d) = c. Then f is semi*α-continuous but not semi*δ-continuous. Since for the open set V = {b} in(Y, σ), f^{-1}(V) = {a, c} is semi*α-open but not semi*δ-open in (X, τ).

Theorem 3.16: Every semi*δ-continuous is semiα-continuous.
Proof: By Theorem 2.7(vii), every semi*δ-open set is semiα-open, the proof follows. The converse of the above theorem need not be true as it is seen from the following example.
Example 3.17: Let X= {a, b, c}, Y= {a, b, c}, τ = {ϕ, {a, b}, X} and σ= {ϕ, {c}, Y}. Define f: (X, τ) → (Y, σ) by f(a) = f(b) = c; f(c) = a. Then f is semiα-continuous but not semi*δ-continuous. Since for the open set V = {c} in(Y, σ), f^{-1}(V) = {a, b} is semiα-open but not semi*δ-open in (X, τ).
Remark 3.18: The concept of semi*δ-continuity and continuity are independent as shown in the following example.
Example 3.19:
1. Let X = {a, b, c}, Y ={a, b, c}, τ= {ϕ,{a},{b},{a, b},X} and σ= {ϕ,{a},Y}. Define f: (X, τ)→ (Y, σ) by f(a) = b; f(b) = f(c) = a. Then f is semi*δ-continuous but not continuous. Observe that for the open set V = {a} in (Y, σ), f^{-1}(V) = {b, c} is semi*δ-open but not open in (X, τ).
2. Let X = {a, b, c}, Y={a, b, c}, τ = {ϕ, {a, b}, X} and σ= {ϕ, {a, b},Y}. Define f: (X, τ) → (Y, σ) by f(a) = b; f(b) = a; f(c) = c . Then f is continuous but not semi*δ-continuous. It is clear that for the open set V = {a, b} in (Y, σ), f^{-1}(V) = {a, b} is open but not semi*δ-open in (X, τ).
Remark 3.20: The concept of semi*δ-continuity and g-continuity are independent as shown in the following example.
Example 3.21:
1. Let X = {a, b, c, d}, Y ={a, b, c, d}, τ= {ϕ,{a},{b},{c},{a, b}, {a, c}, {b, c}, {a, b, c},X} and σ= { ϕ,{a},{b},{a, b}, {a, b, c},Y}. Define f: (X, τ)→ (Y, σ) by f(a) = b; f(b) = a; f(c) = d; f(d) = c. Then f is semi*δ-continuous but not g-continuous. Observe that for the open set V = {a, b, c} in (Y, σ), f^{-1}(V) = {a, b, d} is semi*δ-open but not g-open in (X, τ).
2. Let X = {a, b, c, d}, Y={a, b, c, d}, τ = {ϕ, {a},{a, b, c}, X} and σ= { ϕ, {a},{a, b, c}, Y}. Define f: (X, τ) → (Y, σ) by f(a) = f(d) = b; f(b) = c; f(c) = a . Then f is g-continuous but not semi*δ-continuous. It is clear that for the open set V = {a} in (Y, σ), f^{-1}(V) = {c} is g-open but not semi*δ-open in (X, τ).
Remark 3.22: The concept of semi*δ-continuity and α-continuity are independent as shown in the following example.
Example 3.23:

1. Let X = {a, b, c, d}, Y ={a, b, c, d}, τ= { ϕ,{a},{b},{c},{a, b}, {a, c}, {b, c}, {a, b, c},X} and σ= { ϕ, {a}, {a, b}, {a, b, c},Y}. Define f: (X, τ)→ (Y, σ) by f(a) = b; f(b) = a; f(c) = d; f(d) = c. Then f is semi*δ-continuous but not α-continuous. Observe that for the open set V = {a, b, c} in (Y, σ), f^{-1}(V) = {a, b, d} is semi*δ-open but not α-open in (X, τ).
2. Let X = {a, b, c, d}, Y ={a, b, c, d}, τ= {ϕ, {a, b, c}, X} and σ= { ϕ, {a, b}, {a, b, c},Y}. Define f: (X, τ)→ (Y, σ) by f(a) = f(c) = b; f(b) = a; f(d) = c. Then f is α-continuous but not semi*δ-continuous. It is clear that for the open set V = {a, b} in (Y, σ), f^{-1}(V) = {a, b, c} is α-open but not semi*δ-open in (X, τ).

Remark 3.24: The concept of semi*δ-continuity and pre-continuity are independent as shown in the following example.

Example 3.25:

1. Let X = {a, b, c}, Y ={a, b, c}, τ= { ϕ,{a},{b},{a, b}, X} and σ= { ϕ, {a}, {a, b}, Y}. Define f: (X, τ)→ (Y, σ) by f(a) = c; f(b) = a; f(c) = b. Then f is semi*δ-continuous but not pre-continuous. Observe that for the open set V = {a, b} in (Y, σ), f^{-1}(V) = {b, c} is semi*δ-open but not pre-open in (X, τ).
2. Let X = {a, b, c, d}, Y ={a, b, c, d}, τ= {ϕ, {a}, X} and σ= { ϕ, {a, b, c},Y}. Define f: (X, τ)→ (Y, σ) by f(a) = c; f(b) = d; f(c) = b; f(d) = a. Then f is pre-continuous but not semi*δ-continuous. It is clear that for the open set V = {a, b, c} in (Y, σ), f^{-1}(V) = {a, c, d} is pre-open but not semi*δ-open in (X, τ).

Remark 3.26: The concept of semi*δ-continuity and α*-continuity are independent as shown in the following example.

Example 3.27:

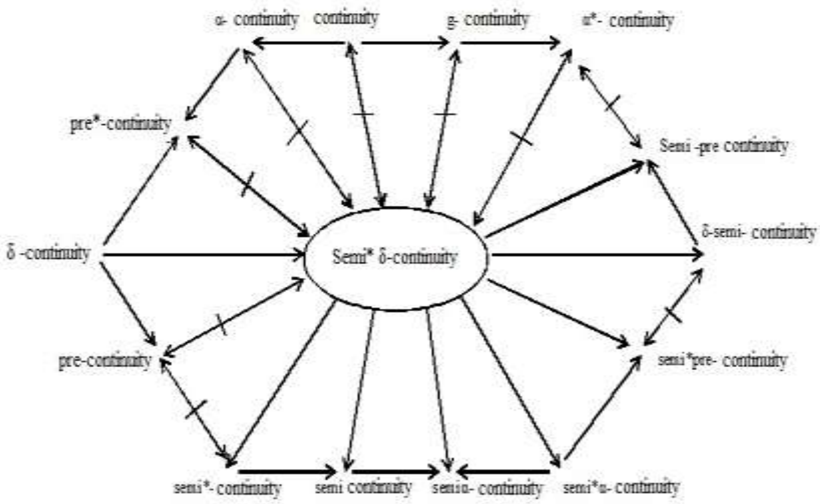
1. Let X = {a, b, c, d}, Y ={a, b, c, d}, τ= { ϕ,{a},{b},{c},{a, b}, {a, c}, {b, c}, {a, b, c},X} and σ= { ϕ, {a}, {a, b}, {a, b, c},Y}. Define f: (X, τ)→ (Y, σ) by f(a) = c; f(b) = a; f(c) = d; f(d) = b. Then f is semi*δ-continuous but not α*-continuous. Observe that for the open set V = {a, b} in (Y, σ), f^{-1}(V) = { b, d} is semi*δ-open but not α*-open in (X, τ).
2. Let X = {a, b, c, d}, Y ={a, b, c, d}, τ= {ϕ,{a, b}{a, b, c}, X} and σ= { ϕ, {a}, {b, c, d},Y}. Define f: (X, τ)→ (Y, σ) by f(a) = c; f(b) = d; f(c) = a; f(d) = b. Then f is α*-continuous but not semi*δ-continuous. It is clear that for the open set V = {b, c, d} in (Y, σ), f^{-1}(V) = {a, b, d} is α*-open but not semi*δ-open in (X, τ).

Remark 3.28: The concept of semi*δ-continuity and pre*-continuity are independent as shown in the following example.

Example 3.29:

1. Let X = {a, b, c}, Y ={a, b, c}, τ= { ϕ,{a},{b},{a, b}, X} and σ= { ϕ, {a, b}, Y}. Define f: (X, τ)→ (Y, σ) by f(a) = b; f(b) = c; f(c) = a. Then f is semi*δ-continuous but not pre*-continuous. Observe that for the open set V = {a, b} in (Y, σ), f^{-1}(V) = {a, c} is semi*δ-open but not pre*-open in (X, τ).
2. Let X = {a, b, c, d}, Y ={a, b, c, d}, τ= {ϕ, {a}, {a, b, c}, X} and σ= { ϕ, {a}, {b},{a, b},{a, b, c},Y}. Define f: (X, τ)→ (Y, σ) by f(a) = b; f(b) = d; f(c) = a; f(d) = c. Then f is pre*-continuous but not semi*δ-continuous. It is clear that for the open set V = {a ,b ,c} in (Y, σ), f^{-1}(V) = {a, c, d} is pre*-open but not semi*δ-open in (X, τ).

From the above the discussions we have the following diagram:



Theorem 3.30: A function f : (X, τ)→ (Y, σ) is semi*δ-continuous if and only if f^{-1}(U) is semi*δ-closed in (X, τ) for every closed set U in (Y, σ).

Proof. Let f : (X, τ)→ (Y, σ) be semi*δ-continuous and U be a closed set in (Y, σ). Then V=*Y*\U is open in *Y*. Then f^{-1}(V) is semi*δ-open in X. Therefore
f^{-1}(U) =f^{-1}(*Y**V*)=*X*\f^{-1}(*V*) is semi*δ-closed.
Converse: Let V be an open set in Y. Then U=Y\V is closed in Y. By assumption, f^{-1}(U) is semi*δ-closed in X. Hence f^{-1}(V)= f^{-1}(*Y*\U)=*X*\f^{-1}(U) is semi*δ-open in *X*.Therefore f is semi*δ-continuous.

Remark 3.31: The composition of two semi*δ-continuous functions need not be semi*δ-continuous and this can be shown by the following example.
Example 3.32: Let X= Y= Z= {a, b, c, d}, τ = {ϕ, {a}, {b}, {a, b},{a, b, c}, X}, σ ={ϕ, {a}, {b}, {a, b},{b, c}, {a, b, c}, Y} and η ={ϕ, {a}, {a, b, c}, Z}. Define f: (X, τ)→(Y, σ) by f(a)=b; f(b)=a; f(c)=d; f(d)=c and define g: (Y, σ)→(Z, η) by g(a)= g(d)=a ; g(b)=c and g(c)=b.Then f and g are semi*δ-continuous but g ∘ f is not semi*δ-continuous. Since {a} is open in (Z, η) but $(g \circ f)^{-1}$({a}) = f^{-1} (g^{-1} ({a})) = f^{-1} ({a, d}) ={b, c} which is not semi*δ-open in (X, τ).
Theorem 3.33: Let (X, τ) and (Z, η) be topological spaces and (Y, σ) be a space in which every semi*δ-open set is open, then the composition g ∘ f : (X, τ) →(Z, η) of semi*δ-continuous functions f : (X, τ) →(Y, σ) and g : (Y, σ) →(Z, η) is semi*δ-continuous.
Proof: Let G be any open set of (Z, η). Since g is semi*δ-continuous g^{-1} (G) is semi*δ-open in (Y, σ). Then by assumption, g^{-1}(G) is open in (Y, σ).Hence f^{-1}(g^{-1}(G)) = $(g \circ f)^{-1}$(G) is semi*δ-open in (X, τ). Thus g ∘ f is semi*δ-continuous.
Theorem 3.34: Let (X, τ) and (Z, η) be topological spaces and (Y, σ) be $T_{1/2}$-space then the composition g ∘ f : (X, τ) →(Z, η) of semi*δ-continuous f : (X, τ) →(Y, σ) and g-continuous function g : (Y, σ) →(Z, η) is semi*δ-continuous.
Proof: Let G be any closed set of (Z, η). Since g is g-continuous g^{-1} (G) is g-closed in (Y, σ). Also since (Y, σ) is $T_{1/2}$-space , g^{-1}(G) is closed in (Y, σ).
Hence f^{-1}(g^{-1}(G)) = $(g \circ f)^{-1}$(G) is semi*δ-closed in (X, τ). Thus g ∘ f is semi*δ-continuous.
Theorem 3.35: If f : (X, τ) → (Y, σ) is semi*δ-continuous and g : (Y, σ) →(Z, η) is continuous. Then their composition g ∘ f : (X, τ) →(Z, η) is semi*δ-continuous.
Proof: Let G be open in (Z, η). Thus g^{-1}(G) is open in (Y, σ). Since f is semi*δ-continuous f^{-1}(g^{-1}(G)) = $(g \circ f)^{-1}$(G) is semi*δ-open in (X, τ). Thus g ∘ f is semi*δ-continuous.
Theorem 3.36: Let f : (X, τ) → (Y, σ) be a function. Then the following statements are equivalent:
(i) The function f is semi*δ-continuous.
(ii) f^{-1}(F) is semi*δ-closed in X for every closed set F in Y.
(iii)f(s*δCl(A))⊆Cl(f(A)) for every subset A of X.
(iv)s*δCl(f^{-1}(B))⊆ f^{-1}(Cl(B)) for every subset B of Y.
(v)f^{-1}(Int(B)) ⊆s*δInt(f^{-1}(B)) for every subset B of Y.
(vi)Int*(αCl(f^{-1}(F)))=Int*(f^{-1}(F)) for every closed set F in Y.
(vii)Cl*(αInt(f^{-1}(V)))=Cl*(f^{-1}(V)) for every open set V in Y.
Proof: (i) ⇔(ii): This follows from theorem 3.30.
(ii)⇒(iii): Let A⊆X. Let F be a closed set containing f(A). Then by (ii), f^{-1}(F) is a semi*δ-closed set containing A. This implies that s*δCl(A)⊆f^{-1}(F) and hence f(s*δCl(A))⊆F.
Therefore f(s*δCl(A))⊆ Cl(f(A)).
(iii)⇒(iv): Let B⊆Y and let A= f^{-1}(B). By assumption, f(s*δCl(A))⊆Cl(f(A))⊆Cl(B). This implies that s*δCl(A)⊆f^{-1}(Cl(B)). Therefore s*δCl(f^{-1}(B))⊆ f^{-1}(Cl(B)).
(iv)⇔(v): The equivalence of (iv) and (v) can be proved by taking the complements.
(vi)⇔(ii): Follows from Theorem 2.10.
(vii)⇔(i): Follows from Theorem 2.9.

IV. SEMI* -IRRESOLUTE FUNCTIONS

Definition 4.1: A function f :X⟶Y is said to be **semi* -irresolute** if f^{-1}(V) is semi*δ-open in X for every semi*δ-open set V in Y.
Theorem 4.2: If f : (X, τ) → (Y, σ) is semi*δ-irresolute and g: (Y, σ)→ (Z, η) is semi*δ-continuous then g ∘ f : (X, τ) →(Z, η) is semi*δ-continuous.
Proof: Let G be any open set of (Z, η). Since g is semi*δ-continuous g^{-1}(G) is semi*δ- open in (Y,σ).Since f is semi*δ-irresolute f^{-1}(g^{-1}(G)) = $(g\circ f)^{-1}$(G) is semi*δ-open in (X, τ) .Thus (g∘ f) is semi*δ-continuous.
Theorem 4.3: If f : (X, τ) → (Y, σ) is semi*δ-irresolute and g: (Y, σ)→ (Z, η) is semi*δ-irresolute then g ∘ f : (X, τ) →(Z, η) is semi*δ-irresolute.

Proof: Let G be semi*δ-open in (Z, η). Since g is semi*δ-irresolute, $g^{-1}(G)$ is semi*δ-open in (Y, σ). Since f is semi*δ-irresolute $f^{-1}(g^{-1}(G)) = (g\circ f)^{-1}(G)$ is semi*δ-open in (X, τ). Thus (g∘ f) is semi*δ-irresolute.

Theorem 4.4: Let X be a topological space and Y be a space in which every semi*δ-open set is open. If f : (X, τ) → (Y, σ) is a semi*δ-continuous map, then f is semi*δ-irresolute.

Proof: Let G be semi*δ-open set in (Y, σ). Then by assumption, G is open in (Y, σ). Since f is semi*δ-continuous, $f^{-1}(G)$ is semi*δ-open in (X, τ). Thus f is semi*δ-irresolute.

Theorem 4.5: Let f :X⟶Y be a function. Then the following are equivalent:

(i) f is semi*δ-irresolute.

(ii) $f^{-1}(F)$ is semi*δ-closed in X for every semi*δ-closed set F in Y.

(iii)f(s*δCl(A))⊆s*δCl(f(A)) for every subset A of X.

(iv)s*δCl($f^{-1}(B)$)⊆f^{-1}(s*δCl(B)) for every subset B of Y.

(v)f^{-1}(s*δInt(B)) ⊆s*δInt($f^{-1}(B)$) for every subset B of Y.

(vi) Int*(δCl($f^{-1}(F)$))=Int*($f^{-1}(F)$) for every semi*δ-closed set F in Y.

(vii) Cl*(δInt($f^{-1}(V)$))=Cl*($f^{-1}(V)$) for every semi*δ-open set V in Y.

Proof:

(i)⇒(ii): Let F be a semi*δ-closed set in Y. Then V=Y\F is semi*δ-open in Y. Then $f^{-1}(V)$ is semi*δ-open in X. There-fore $f^{-1}(F)=f^{-1}(Y\setminus V)=X\setminus f^{-1}(V)$ is semi*δ-closed.

(ii)⇒(i): Let V be a semi*δ-open set in Y. Then F=Y\V is semi*δ-closed. By (iii), $f^{-1}(F)$ is semi*δ-closed. Hence

$f^{-1}(V)= f^{-1}(Y\setminus F)=X\setminus f^{-1}(F)$ is semi*δ-open in X.

(ii)⇒(iii): Let A⊆X. Let F be a semi*δ-closed set containing f(A). Then by (ii), $f^{-1}(F)$ is a semi*δ-closed set containing A. This implies that s*δCl(A)⊆$f^{-1}(F)$ and hence f(s*δCl(A))⊆F. Therefore f(s*δCl(A))⊆s*δCl(f(A)).

(iii)⇒(iv): Let B⊆Y and let A= $f^{-1}(B)$. By assumption, f(s*δCl(A))⊆s*δCl(f(A))⊆s*δCl(B). This implies that s*δCl(A)⊆f^{-1}(s*δCl(B)). Hence

s*δCl($f^{-1}(B)$)⊆ f^{-1}(s*δCl(B)).

(iv) ⇔ (v): The equivalence of (iv) and (v) can be proved by taking the complements.

(vi) ⇔ (ii): Follows from Theorem 2.10.

(vii) ⇔ (i): Follows from Theorem 2.9.

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