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International
Journal
Of Modern Engineering Research (IJMER)

On Alpha Generalized Star Preclosed Sets in Topological Spaces

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ABSTRACT: *In this paper, a new class of sets in topological spaces namely, alpha generalized star preclosed (briefly αg*p-closed) set is introduced. This class falls strictly in between the classes of preclosed and g*p-closed sets. Also we studied the characteristics and the relationship of this class of sets with the already existing class of closed sets. Further αg*p-open set is imtroduced which is the complement of αg*p-closed set and its properties are investigated.*

Keywords*: αg*p-closed set, αg*p-open set and αg*p-nbhd.*

I. INTRODUCTION

In 1970, Levine [6] introduced the concept of generalized closed set and discussed their properties, closed and open maps, compactness, normal and separation axioms. A.S.Mashor, M.E.Abd El-Monsef and E1-Deeb.S.N., [11] introduced preopen sets in topological spaces and investigated their properties. Later in 1998 H.Maki, T.Noiri [10] introduced a new type of generalized closed sets in topological spaces called gp-closed sets. The study on generalization of closed sets has lead to significant contribution to the theory of separation axioms, generalization of continuous and irresolute functions. H.Maki et.al. [8][9] generalized α-open sets in two ways and introduced generalized α-closed(briefly gα-closed) sets and α-generalized closed(briefly αg-closed) sets in 1993 and 1994 respectively.

M.K.R.S.Veera kumar [20] introduced the concepts of generalized star preclosed sets and generalized star preopen sets in a topological space. In this paper we introduce and study a new type of closed set namely 'αg*p-closed sets' in topological spaces.The aim of this paper is to study of αg*p-closed sets thereby contributing new innovations and concepts in the field of topology through analytical as well as research works. The notion of αg*p-closed sets and its different characterizations are given in this paper.

Throughout this paper (X,τ) and (Y,σ) represents topological spaces on which no separation axioms are assumed, unless otherwise mentioned. For a subset A of X, the closure of A and interior of A will be denoted by cl(A) and int(A) respectively.The union of all preopen sets of X contained in A is called pre-interior of A and it is denoted by pint (A). The intersection of all preclosed sets of X containing A is called pre-closure of A and it is denoted by pcl (A).

II. PRELIMINARIES

Definition 2.1.
A subset A of a topological space (X, τ) is called
(i) preopen [11] if A ⊆ int (cl (A)) and preclosed if cl (int(A)) ⊆ A.
(ii) semi-open [7] if A ⊆ cl (int (A)) and semi-closed if int (cl (A)) ⊆ A.
(iii) α-open [15] if A ⊆ int (cl (int (A))) and α-closed if cl(int(cl(A))) ⊆ A .
(iv) semi-preopen [1] (β-open) if A ⊆ cl(int(cl(A))) and semi-preclosed (β-closed set) if int (cl (int (A))) ⊆A .
(v) regular open [19] if A = int (cl(A)) and regular closed if A = cl (int (A)) .

Definition 2.2.
A subset A of a topological space (X, τ) is called
(i) generalized closed (briefly g-closed) [6] if cl(A) ⊆ U whenever A ⊆ U and U is open in X.
(ii) semi-generalized closed (briefly sg-closed) [3] if scl(A) ⊆ U whenever A ⊆ U and U is semi-open in X.
(iii) generalized semi-closed (briefly gs-closed) [2] if scl(A) ⊆ U whenever A ⊆ U and U is open in X.
(iv) generalized α-closed (briefly gα-closed) [9] if α cl(A) ⊆ U whenever A ⊆ U and U is α-open in X.

(v) generalized closed (briefly □ g-closed) [8] if □ cl(A) □ □ U whenever A □ U and U is open in X.
(vi) generalized preclosed (briefly gp-closed) [10] if pcl(A) ⊆ U whenever A □ U and U is open in X .
(vii) generalized semi-preclosed (briefly gsp-closed) [4] if spcl(A) □ U whenever A □U and U is open in X .
(viii) generalized pre regular closed (briefly gpr-closed)[5] if pcl(A) □ U whenever A □ U and U is regular open in X .
(ix) regular generalized closed (briefly rg-closed) [16] if cl (A) ⊆ U whenever A □ U and U is regular open in X.
(x) weakly generalized closed (briefly wg-closed)[13] if cl(int(A))⊆U whenever A⊆U and U is open in X.
(xi) regular weakly generalized closed (briefly rwg-closed)[13] if cl(int(A))⊆U whenever A⊆U and U is regular open in X.
(xii) strongly generalized closed (briefly g*-closed) [22] if cl(A) □ U whenever A□ U and U is g-open in X.
(xiii) mildly generealized closed (briefly mildly g-closed) [17] if cl(int(A))□ □U whenever A□ □U and U is g-open in X.
(xiv) generalized star preclosed (briefly g*p-closed set) [20] if pcl (A) □ U whenever A □ □ U and U is g-open in X.
(xv) pre-semi closed set [21] if spcl (A) □ U whenever A □ □ U and U is □ g-open in X.
The complements of the above mentioned closed sets are their respective open sets.

Definition 2.3.[14]
Let (X, τ) be a topological space , A □ X and x∈ X then x is said to be a pre-limit point of A iff every preopen set containing x contains a point of A different from x.

Definition 2.4.[14]
Let (X, τ) be a topological space and A □ X. The set of all pre-limit points of A is said to be the prederived set of A and is denoted by $D_p[A]$.

Definition 2.5.
Let (X, τ) be a topological space and let A,B be two non-void subsets of X . Then A and B are said to be pre-separated if A∩pcl(B)=pcl(A)∩B= ϕ

III. ALPHA GENERALIZED STAR PRECLOSED SETS

In this section we introduce alpha generalized star preclosed set and investigate some of their properties.
Definition 3.1. A subset A of a topological space (X, τ) is called alpha generalized star preclosed set (briefly □g*p-closed) if pcl(A) ⊆ U whenever A ⊆ U and U is □g-open in X.

Theorem 3.2. Every preclosed set is □g*p-closed.
Proof. Let A be any preclosed set in X . Let U be any □g-open set containing A. Since A is a preclosed set , we have pcl(A) = A.Therefore pcl(A) ⊆ U. Hence A is □g*p-closed in X.
The converse of above theorem need not be true as seen from the following example.

Example 3.3. Let X = {a,b,c,d} be given the topology τ = {X, ϕ ,{a},{b},{a,b},{a,c},{a,b,c}}.
Then {a,d} is a □g*p-closed set but not preclosed in X .

Theorem 3.4. Every □-closed set is □g*p-closed.
Proof. The proof follows from the definitions and the fact that every □-closed set is preclosed.
The converse of above theorem need not be true as seen from the following example.

Example 3.5. Let X = {a,b,c,d} be given the topology τ = {X, ϕ ,{a,b}}.
Then {a} is a □g*p-closed set but not □-closed in X.

Theorem 3.6. Every closed set is □g*p-closed.
Proof. The proof follows from the definitions and the fact that every closed set is preclosed.
The converse of above theorem need not be true as seen from the following example.

Example 3.7. Let X = {a,b,c,d} be given the topology τ = {X, ϕ ,{a},{b},{a,b}}.
Then {c} is a □g*p-closed set but not closed in X.
Corollary 3.8. Every regular closed set is □g*p-closed.
The converse of above corollary need not be true as seen from the following example.

Example 3.9. Let X = {a,b,c,d} be given the topology τ = {X, ϕ,{a},{b,c},{a,b,c}}.
Then {a,b,d} is a αg*p-closed set but not regular closed in X.

Remark. αg*p-closed sets are independent of semi-closed sets and semi-preclosed sets as seen from the following example

Example 3.10. Let X = {a,b,c,d} , τ = {X,ϕ,{a},{b},{a,b},{a,c},{a,b,c}}.
In this topological space (X, τ), a subset {a,c} is semi-closed and semi-preclosed but not αg*p-closed and {a,d} is a αg*p-closed set but not semi-closed , semi-preclosed.

Theorem 3.11. Every αg*p-closed set is gp-closed**.**
Proof. Let A be any αg*p-closed set in X . Let U be open set containing A.
Since every open set is αg-open, we have pcl(A) ⊆ U. Hence A is gp-closed.
The converse of above theorem need not be true as seen from the following example.

Example 3.12. Let X = {a,b,c,d} be given the topology τ = {X,ϕ,{a},{b},{a,b},{a,b,c}}.
Then {b,d} is a gp-closed set but not αg*p-closed in X.

Theorem 3.13. Every αg*p-closed set is a gpr-closed set.
Proof. The proof follows from the definitions and the fact that every regular open set is αg-open.
The converse of above theorem need not be true as seen from the following example.

Example 3.14. Let X = {a,b,c,d} be given the topology τ = {X,ϕ,{a},{b},{a,b},{a,b,c}}.
Then {a} is a gpr-closed set but not αg*p-closed in X.

Theorem 3.15. Every αg*p-closed set is gsp-closed**.**
Proof. The proof follows from the definitions and the fact that every open set is αg-open.
The converse of above theorem need not be true as seen from the following example.

Example 3.16. Let X = {a,b,c,d} be given the topology τ = {X,ϕ,{a,b}}.
Then {a,b,c} is a gsp-closed set but not αg*p-closed in X.

Theorem 3.17. Every αg*p-closed set is g*p-closed**.**
Proof. Let A be any αg*p-closed set in X. Let U be any g-open set containing A.
Since every g-open set is αg-open , we have pcl(A) ⊆ U. Hence A is g*p-closed.
The converse of above theorem need not be true as seen from the following example.

Example 3.18. Let X = {a,b,c,d} be given the topology τ = {X,ϕ,{a},{b},{a,b},{a,b,c}}.
Then {a,b,d} is a g*p-closed set but not αg*p-closed in X.

Theorem 3.21. Every αg*p-closed set is mildly generalized closed**.**
Proof. Let A be any αg*p-closed set in X . Let U be any g-open set containing A. Since every g-open set is αg-open and cl(int(A) ⊆ pcl(A) , we have cl(int(A)) ⊆ U. Hence A is mildly generalized closed.
The converse of above theorem need not be true as seen from the following example.

Example 3.22. Let X = {a,b,c,d} be given the topology τ = {X,ϕ,{a},{b},{a,b},{b,c},{a,b,c}}.
Then {b,d} is mildly generalized closed set but not αg*p-closed in X.

Theorem 3.23. Every αg*p-closed set is weakly generalized(wg) closed.
Proof. Let A be any αg*p-closed set in X. Let U be any open set containing A.
Since every open set is αg-open and cl(int(A)⊆pcl(A) , we have cl(int(A)) ⊆ U. Hence A is wg-closed.
The converse of above theorem need not be true as seen from the following example.

Example 3.24. Let X = {a,b,c,d} be given the topology τ = {X,ϕ,{a},{b},{a,b}}.
Then {a,c} is a weakly generalized closed set but not αg*p-closed in X.

Theorem 3.25. Every αg*p-closed set is regular weakly generalized(rwg) closed**.**
Proof. Let A be any αg*p-closed set in X. Let U be any regular open set containing A. Since every regular open set is αg-open and cl(int(A)⊆pcl(A) , we have cl(int(A)) ⊆U. Hence A is rwg-closed set.

The converse of above theorem need not be true as seen from the following example.

Example 3.26. Let X = {a,b,c,d} be given the topology τ = {X,ϕ,{a},{b},{d},{a,b},{a,d},{b,d},{a,b,d}}. Then {a} is rwg-closed but not αg*p-closed.

Theorem 3.27. Every αg*p-closed set is pre-semi closed .
Proof. Let A be any αg*p-closed set in X. Let U be any g-open set containing A. Since every g-open set is αg-open and every preclosed set is semi-preclosed, we have spcl(A) ⊆ pcl(A) ⊆ U. Hence A is pre-semi closed. The converse of above theorem need not be true as seen from the following example.

Example 3.28. Let X = {a,b,c,d} be given the topology τ = {X,ϕ,{a},{b},{a,b},{a,b,c}}. Then {a} is a pre-semi closed set but not αg*p-closed in X.

Remark . The following examples show that αg*p-closed sets are independent of g-closed , g*-closed , sg-closed , gs-closed , αg-closed , gα-closed .

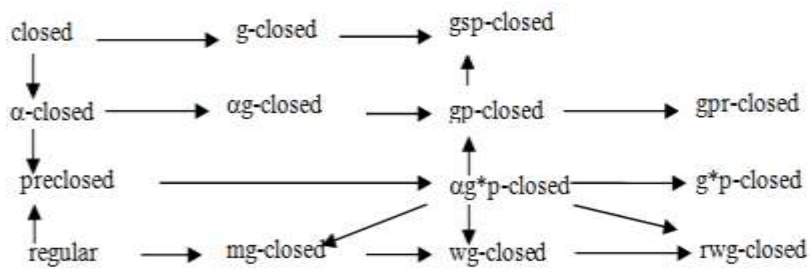
Example 3.31 Let X = {a,b,c,d} . τ = {X,ϕ,{a},{b},{a,b},{a,b,c}}.
The set {a,d} is g-closed and g*-closed but not αg*p-closed .The set {c} is αg*p-closed but not g-closed and g*-closed .

Example 3.32. Let X = {a,b,c,d} , τ = {ϕ,{a,b},X}.
The sets {a},{b} are αg*p-closed but not sg-closed , gs-closed , αg-closed and gα-closed .

Example 3.33. Let X = {a,b,c,d} , τ = {ϕ,{a},{b},{a,b},X}.
The set {a} is sg-closed and gs-closed but not αg*p-closed .

Example 3.34. Let X = {a,b,c,d} , τ = {ϕ,{a},{b},{a,b},{b,c},{a,b,c},X}.
The set {b,d} is αg-closed but not αg*p-closed.

From the above discussions we have the following implications:



IV. CHARACTERISTICS OF αg*p-CLOSED AND αg*p-OPEN SETS

Theorem 4.1. Let A be a αg*p-closed set in (X , τ).
Then i) pcl (A) \ A does not contain any non-empty αg-closed set.
ii) If B is a subset of X such that A⊆ B⊆ pcl(A) then B is also αg*p-closed in (X,τ).
Proof.
i) Let F be a αg-closed set contained in pcl (A) \ A.
Since X\ F is an αg-open set with A ⊆ X \ F and A is αg*p-closed , pcl (A) ⊆ X \ F.
Which implies F ⊆ (X \ pcl(A)) ∩ (pcl(A) \ A) ⊆ (X \ pcl(A)) ∩ pcl(A)= ϕ
Therefore F = ϕ.
ii) Let U be an αg-open set of (X , τ) such that B ⊆ U. Since A ⊆ U and A is αg*p-closed , pcl(A) ⊆ U.
Since B ⊆ pcl(A) , we have pcl(B) ⊆ pcl(pcl(A)) = pcl(A).Thus pcl(B) ⊆ U.
Hence B is a αg*p-closed set of (X , τ).
Corollary 4.2. Let A be a αg*p-closed set in (X , τ).Then pcl (A) \ A does not contain any non-empty α-closed set.

Remark 4.3. Union of two αg*p -closed sets need not be αg*p-closed .

Example 4.4. Let X = {a,b,c,d} with τ = {X,ϕ,{a,b}}.
The sets {a} and {b} are □g*p-closed sets but their union {a,b} is not a □g*p-closed set.

Theorem 4.5. The intersection of two □g*p-closed subsets of X is also □g*p-closed .
Proof. Let A and B be any two □g*p-closed subsets of X.
Then pcl (A) ⊆ U, pcl (B) ⊆ U whenever A ⊆ U and B ⊆ U, U is □g-open.
Let U be an □g-open set in X such that A ∩ B ⊆ U.
Now , pcl (A∩ B) ⊆ pcl (A)∩pcl (B) ⊆ U, U is □g-open in X. Hence A ∩ B is a □g*p-closed set.

Theorem 4.6. For an element $x \in$ X , then the set X-{x} is a □g*p-closed set (or) □g-open set.
Proof. Let $x \in$ X .Suppose that X \ {x} is not □g-open. Then X is the only □g-open set containing X \ {x}.
This implies pcl(X \ {x}) ⊆ X. Hence X \ {x} is □g*p-closed in X.

Theorem 4.7. If A is both □-open and □g*p -closed in X, then A is preclosed.
Proof. Suppose A is □-open and □g*p- closed in X. As every □-open set is □ g-open and A⊆A, pcl(A) ⊆ A.
But Always A ⊆ pcl (A). Therefore A = pcl (A). Hence A is preclosed.

Corollary 4.8. If A is both □ g-open and □g*p-closed in X, then A is preclosed.
Corollary 4.9. Let A be a □g-open set and □g*p-closed set in X.Suppose that F is preclosed in X.Then A∩F is □g*p-closed in X.
Proof.
By corollary 4.8, A is preclosed. So A∩F is preclosed and hence A∩F is an □g*p-closed in X.

Theorem 4.10. Let A be a □g*p-closed set in (X , τ).Then A is preclosed iff pcl (A) \ A is □g-closed.
Proof. Suppose A is preclosed in X. Then pcl(A) = A and so pcl(A) \ A= ϕ which is □ g-closed in X.
Conversely , Suppose pcl(A) \ A is □ g-closed in X.
Since A is □g*p-closed , pcl(A) \ A does not contain any non-empty □g-closed set in X.
That is pcl(A) \ A= ϕ. Hence A is preclosed.

Theorem 4.11. If A is both open and gp-closed in X, then A is □g*p-closed in X.
Proof. Suppose A is open and gp-closed in X. We prove that A is □g*p-closed set in X.
Let U be any □ g-open set in X such that A⊆U. Since A is open and gp-closed , we have pcl(A) ⊆ A⊆ U.
Hence A is □g*p-closed in X.

Theorem 4.12. If A is both open and □g-closed in X, then A is □g*p-closed in X.
Proof. Let A ⊆ U and U be □g-open in X. Now A⊆A. By hypothesis □ cl(A)⊆ A.
Since every □-closed set is preclosed , pcl(A) ⊆ □ cl(A).Thus pcl(A) ⊆ A ⊆ U.
Hence A is □g*p-closed in X.

Definition 4.13. Let B⊆A⊆X. Then we say that B is □g*p-closed relative to A if pcl_A(B) ⊆ U where B⊆ U and U is □g-open in A.

Theorem 4.14. Let B⊆A⊆X and Suppose that B is □g*p-closed in X. Then B is □g*p-closed relative to A .
Proof. Given that B⊆A⊆X and B is □g*p-closed in X.
Let us assume that B ⊆A∩V, where V is □g-open in X.
Since B is □g*p-closed set, B ⊆V implies pcl (B) ⊆ V.
It follows that pcl_A(B) = pcl (B) ∩A ⊆V ∩A.
Therefore B is □g*p-closed relative to A.

Theorem 4.15. Let A and B be □g*p-closed sets such that D(A) ⊂ D_p(A) and D(B) ⊂ D_p(B) then A ∪ B is □g*p-closed .
Proof. Let U be an □g-open set such that A ∪ B⊆ U. Then pcl(A) ⊆ U and pcl(B) ⊆ U
However, for any set E, D_p(E) ⊂ D(E).Therefore cl(A) = pcl(A) and cl(B) = pcl(B) and this shows
cl(A ∪ B) = cl(A) = pcl(A) ∪ pcl(B)
That is pcl(A ∪ B) ⊆ U. Hence A ∪ B is □g*p-closed.

Definition 4.16. A subset A of a topological space (X, τ) is called □g*p-open set if and only if X \ A is □g*p-closed in X. We denote the family of all □g*p-open sets in X by □g*p O(X).

Theorem 4.17. If pint $(A) \subseteq B \subseteq A$ and if A is αg*p-open in X, then B is αg*p-open in X.
Proof. Suppose that pint $(A) \subseteq B \subseteq A$ and A is αg*p-open in X.
Then $X \setminus A \subseteq X \setminus B \subseteq$ pcl $(X \setminus A)$. Since $X \setminus A$ is αg*p-closed in X, we have $X \setminus B$ is αg*p-closed in X.
Hence B is αg*p-open in X.

Theorem 4.18. If A and B are αg*p-open sets in X , then $A \cup B$ is αg*p-open in X .
Proof. Assume that A and B are αg*p-open sets in X. Then $X \setminus A$ and $X \setminus B$ are αg*p-closed sets .
By theorem 4.1 , $X \setminus (A \cup B) = (X \setminus A) \cap (X \setminus B)$ is αg*p -closed in X. Therefore $A \cup B$ αg*p-open in X.

Remark 4.19. The intersection of two αg*p-open sets in X is generally not αg*p-open in X.

Example 4.20. Let X = {a,b,c,d} with τ = {X,ϕ,{a,b}}.
The set {a,c} and {b,c} are αg*p-open sets but their intersection {c} is not αg*p-open set.

Theorem 4.21. A set A is αg*p-open iff $F \subset$ pint(A) whenever F is αg-closed and $F \subset A$.
Proof. Necessity. Let A be αg*p-open set and suppose $F \subset A$ where F is αg-closed.
By definition , $X \setminus A$ is αg*p-closed. Also $X \setminus A$ is contained in the αg-open set $X \setminus F$.
This implies pcl$(X \setminus A) \subset X \setminus F$. Now pcl$(X \setminus A) = X \setminus$ pint(A). Hence $X \setminus$ pint(A) $\subset X \setminus F$.
That is $F \subset$ pint(A)

Suffiency. If F is αg-closed set with $F \subset$ pint(A) where $F \subset A$, it follows that $X \setminus A \subset X \setminus F$
and $X \setminus$ pint(A) $\subset X \setminus F$. That is pcl$(X \setminus A) \subset X \setminus F$.
Hence $X \setminus A$ is αg*p-closed and A becomes αg*p-open.

Theorem 4.22. If A and B are pre-separated αg*p-open sets then $A \cup B$ is αg*p-open.
Proof. By definition, pcl$(A \cap B) = A \cap$ pcl$(B) = \phi$
If F is a αg-closed set such that $F \subset A \cup B$ then $F \cap$ pcl(A) $\subset$ pcl(A) $\cap (A \cup B) \subset A \cup \phi = A$.
Similarly , $F \cap$ pcl(B) $\subset B$. Hence by 4.21, $F \cap$ pcl(A) $\subset$ pint(A) and $F \cap$ pcl(B) $\subset$ pint(B)
Now, $F = F \cap (A \cup B)$
$= (F \cap A) \cup (F \cap B) \subset (F \cap \text{pcl}(A)) \cup (F \cap \text{pcl}(B)) \subset \text{pint}(A) \cup \text{pint}(B) \subset \text{pint}(A \cup B)$
Hence $A \cup B$ is αg*p-open.

□□ αg*p-NEIGHBOURHOODS

In this section we introduce αg*p-neighbourhoods in topological spaces by using the notions of αg*p-open sets and study some of their properties.

Definition 5.1. Let x be a point in a topological space X and let $x \in X$. A subset N of X is said to be a αg*p-nbhd of x iff there exists an αg*p-open set G such that $x \in G \subset N$.

Definition 5.2. A subset N of Space X is called a αg*p-nbhd of $A \subset X$ iff there exists an αg*p-open set G such that $A \subset G \subset N$.

Theorem 5.3. Every nbhd N of $x \in X$ is a αg*p-nbhd of X.
Proof. Let N be a nbhd of point $x \in X$. To prove that N is a αg*p-nbhd of x.
By definition of nbhd, there exists an open set G such that $x \in G \subset N$. Hence N is a αg*p-nbhd of x.

Remark. In general, a αg*p-nbhd of $x \in X$ need not be a nbhd of x in X as seen from the following example.

Example 5.4. Let X = {a, b, c,d} with topology τ = {X, ϕ, {a},{a,b}}.
Then αg*p-O(X) = {X, ϕ,{a},{a,b},{a,c},{a,d},{a,b,c},{a,b,d},{a,c,d}}.
The set {a,c,d} is αg*p-nbhd of point c, since the αg*p-open sets {a,c} is such that $c \in \{a,c\} \subset \{a,c,d\}$. However, the set {a,c,d} is not a nbhd of the point c, since no open set G exists such that $c \in G \subset \{a,c,d\}$.

Remark 5.6. The αg*p-nbhd N of $x \in X$ need not be a αg*p-open set in X.

Example 5.7. Let X = {a, b, c,d} with topology τ = {X, ϕ ,{a},{b},{c},{a,b},{a,c},{b,c},{a,b,c}}.

Then αg*p-O(X) = {X, ϕ,{ a},{b},{c},{a,b},{a,c},{b,c},{a,b,c} }. The set {a,d} is αg*p-nbhd of point a , since a $\in$ {a} $\subset$ {a,d}. But the set {a,d} is not αg*p-open.

Theorem 5.8. If a subset N of a space X is αg*p-open, then N is αg*p-nbhd of each of its points.
Proof. Suppose N is αg*p-open. Let $x \in N$. We claim that N is αg*p-nbhd of x.
For N is a αg*p-open set such that $x \in N \subset N$. Since x is an arbitrary point of N, it follows that N is a αg*p-nbhd of each of its points.

Theorem 5.9. Let X be a topological space. If F is αg*p-closed subset of X and $x \in F^c$. Then there exists a αg*p-nbhd N of x such that $N \cap F = \phi$
Proof. Let F be αg*p-closed subset of X and $x \in F^c$. Then F^c is a αg*p-open set in X.
By theorem 5.8, F^c contains a αg*p-nbhd of each of its points.
Hence there exists a αg*p-nbhd N of x such that $N \subset F^c$. (i.e.) $N \cap F = \phi$.

Definition 5.10. Let x be a point in a topological space X. The set of all αg*p-nbhd of x is called the αg*p-nbhd system at x, and is denoted by αg*p-N(x).

Theorem 5.11. Let N be a αg*p-nbhd of a topological space X and each $x \in X$, Let αg*p-N(X , τ) be the collection of all αg*p-nbhd of x. Then we have the following results.
(i) For every $x \in X$, αg*p-N(x) $\neq \phi$.
(ii) $N \in$ αg*p-N(x) $\Rightarrow x \in N$.
(iii) $N \in$ αg*p-N(x), $M \supset N \Rightarrow M \in$ αg*p-N(x).
(iv) $N \in$ αg*p-N(x), $M \in$ αg*p-N(x) $\Rightarrow N \cup M \in$ αg*p-N(x).
(v) $N \in$ αg*p-N(x) $\Rightarrow$ there exists $M \in$ αg*p-N(x) such that $M \subset N$ and $M \in$ αg*p-N(y) for every $y \in M$.
Proof. (i) Since X is αg*p-open , it is a αg*p-nbhd of every $x \in X$. Hence there exists atleast one αg*p-nbhd (namely-X) for each $x \in X$. Therefore αg*p-N(x) $\neq \phi$ for every $x \in X$.

(ii) If $N \in$ αg*p -N(x), then N is a αg*p-nbhd of x. By definition of αg*p-nbhd , $x \in N$.

(iii) Let $N \in$ αg*p-N(x) and $M \supset N$. Then there is a αg*p-open set G such that $x \in G \subset N$.
Since $N \subset M$, $x \in G \subset M$ and so M is αg*p-nbhd of x. Hence $M \in$ αg*p-N(x).

(iv) Let $N \in$ αg* -N(x), $M \in$ αg*p-N(x). Then by definition of αg*p-nbhd, there exists αg*p-open sets G_1 and G_2 such that $x \in G_1 \subset N$ and $x \in G_2 \subset M$. Hence $x \in G_1 \cup G_2 \subset N \cup M$ -------- (1).
Since $G_1 \cup G_2$ is a αg*p-open set,(being the union of two αg*p-open sets), it follows from (1) that $N \cup M$ is a αg*p-nbhd of x. Hence $N \cup M \in$ αg*p-N(x).

(v) Let $N \in$ αg*p-N(x), Then there is a αg*p-open set M such that $x \in M \subset N$.
Since M is αg*p-open , it is αg*p-nbhd of each of its points. Therefore $M \in$ αg*p-N(y) for every $y \in M$.

V. CONCLUSION

The αg*p-closed set can be used to derive continuity, closed map, open map and homeomorphism, closure and interior and new separation axioms.

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