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International
Journal
Of Modern Engineering Research (IJMER)

A Brief Idea on Fuzzy and Crisp Sets

Rednam SS Jyothi[1], Eswar Patnala[2], K.Asish Vardhan[3]
(Asst.Prof(c),Information Technology, UCEV- JNTUK, INDIA)
(Asst.Prof(c), Information Technology, UCEV- JNTUK, INDIA)
(Research scholar Dept. Of CS & SE, college of Engg.(A),Andhra University , INDIA)

***ABSTRACT**:* Fuzzy logic is the one of computational approach based on the degree of the truth rather Boolean logic i.e., true or false (0 or 1) and for the degree of truth, we use linguistic variable and the membership function and for evaluation of fuzzy input and output is done through a four step process known as fuzzy inference system [3][6].

***KEYWORDS**:* Fuzzy, membership function, crisp, linguistic, fuzzification , defuzzication.

I. INTRODUCTION

The idea of fuzzy logic was first advanced by Dr. Lotfi Zadeh of the University of California at Berkeley in the 1960s. Dr. Zadeh was working on the problem of computer understanding of natural language[6].

1. Fuzzy VS Crisp

1.1 Universal set

Universal set is a collection of elements which has taken under particular context and it is denoted by μ or E or U [3] [1].
Example: the universal set of all natural number system,,the set of alphabets

1.2 Crisp set

Set is a collection of well defined objects called elements or terms or members and the set is denoted by capital letters & elements of set are denoted by small letters[3] [1].
Example:

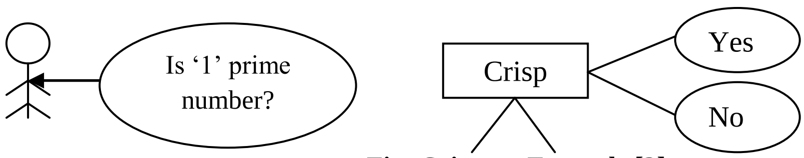

Fig: Crisp set Example [3]

Therefore, the membership function or characteristic function is defined by

$$\mu_{A(x)} = \begin{cases} 1 \text{ if } x \in A \\ 0 \text{ otherwise} \end{cases}$$

In this case there are two possible options member belongs to set, member does not belongs to set
Then the membership function is denoted by ‘1’otherwise ‘0’[3] [1].

2. Fuzzy set

Fuzzy set is a super set of crisp set, in crisp set we only discuss about whether the element there in the set or not. When coming to fuzzy set ,it includes all the elements having degrees of membership. The degree lies between 1 and 0[3] [1].

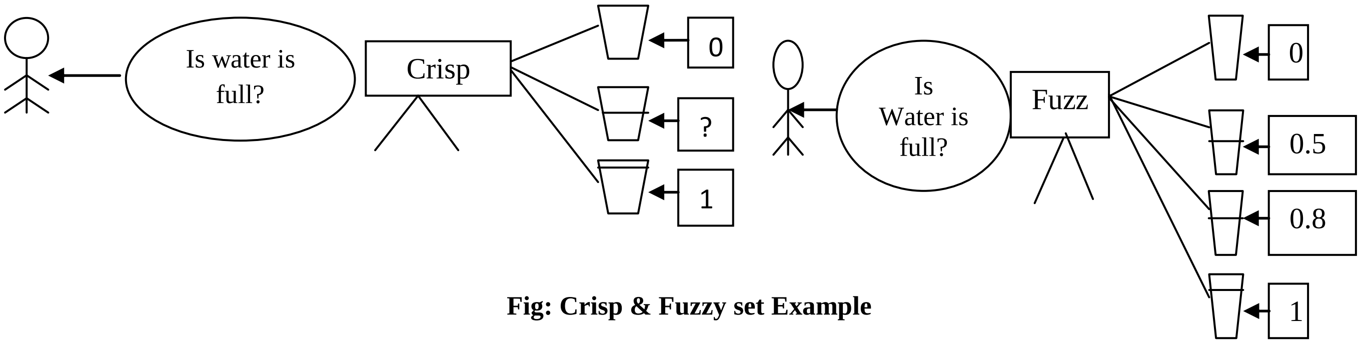

Fig: Crisp & Fuzzy set Example

Therefore, the membership function or characteristic function of fuzzy set is the function that maps every element of the universe of discourse (U) to the interval[0,1]

And it is denoted by $\mu_A(x)$:U→ [0, 1]

Mathematically it is represented as , if U is the universe of discourse and x is a particular element of U
Then a fuzzy set A defined on U may be written collection of ordered pairs A={ (x, $\mu_A(x)$),x∈U}
Another definition of fuzzy set in terms of discrete and continuous.[3] [1]

Discrete $A=\sum_{xi \in U} \mu_A(x_i)/x_i$

Continuous $A=\int_U \mu_A(x)/x$

Where U is universal set or universal discourse,x is particular element of U,A is Fuzzy set
Example for discrete notation:
Consider set of numbers U={1,2,3,4,5,6,7,8,9,10}
A FUZZY SET labeled 'Small number' can be defined as A={(1,1),(2,1),(3,0.8),(7,0.1),(10,0)}

Small number	**membership number**	**comment**
1	1	Small
2	1	almost Small
3	0.8	almost Small
4	0.7	nearest value to Small
5	0.5	nearest value to Small
6	0.4	not a Small in some cases
7	0	definitely not Small
8	0	definitely not Small
9	0	definitely not Small
10	0	not Small

3. Operations

Operations on Crisp & Fuzzy set [3] [1]

OPERATIONS	SET THEORY	Fuzzy set theory
	Let A, B are two finite sets. Then	Let A, B are two finite fuzzy sets. Then
UNION	$A \cup B=\{x/x \in A \vee x \in B\}$	$\mu_{A \cup B}(x)=\max(\mu_A(x), \mu_B(x))$
INTERSECTION	$A \cap B=\{x/x \in A \wedge x \in B\}$	$\mu_{A \cap B}(x)=\min(\mu_A(x), \mu_B(x))$
SET DIFFERENCE	$A-B=\{x/x \in A$ and $x \notin B\}$	$\mu_{A-B}(x)= \mu_{A \cap \bar{B}}(x)$
COMPLIMENT	A^C or $\bar{A}$ $=\{x/x \notin A \wedge x \in U\}$	$\mu_{\bar{A}}(x)=1- \mu_A(x)$

Examples of fuzzy operations:

Let A= {(x1, 0.4), (x2, 0.3), (x3, 1), (x4, 0.5)} and
B= {(x1, 0.6), (x2, 0.8), (x3, 0.9), (x4, 0.3)} are two fuzzy sets Then

3.1 Union

$A \cup B$ = {(x1, 0.6), (x2, 0.8), (x3, 1), (x4, 0.5)}
Since, $\mu_{A \cup B}(x) = \max(\mu_A(x), \mu_B(x))$
$\therefore \mu_{A \cup B}(x1) = \max(\mu_A(x1), \mu_B(x1)) = \max(0.4, 0.6) = 0.6$ respectively

Similarly $\mu_{A \cup B}(x2) = 0.8$, $\mu_{A \cup B}(x3) = 1$, $\mu_{A \cup B}(x4) = 0.5$[3] [1]

3.2 Intersection

$A \cap B$ = {(x1, 0.4), (x2, 0.3), (x3, 0.9), (x4, 0.3)}
Since $\mu_{A \cap B}(x) = \min(\mu_A(x), \mu_B(x))$
$\forall \mu_{A \cap B}(x1) = \min(\mu_A(x1), \mu_B(x1)) = \min(0.4, 0.6) = 0.4$
Similarly $\mu_{A \cap B}(x2) = 0.3$, $\mu_{A \cap B}(x3) = 0.9$, $\mu_{A \cap B}(x4) = 0.3$ respectively [3] [1]

3.3 Compliment

$\bar{A}$ = { (x1, 0.6), (x2, 0.7), (x3, 0), (x4, 0.5)}
Since $\mu_{\bar{A}}(x) = 1- \mu_A(x)$
$\therefore \mu_{\bar{A}}(x1) = 1- \mu_A(x1) = 1-0.4=0.6$
Similarly $\mu_{\bar{A}}(x2) = 0.7$, $\mu_{\bar{A}}(x3) = 0$, $\mu_{\bar{A}}(x4) = 0.5$ respectively [1]

3.4 Set Difference

$A-B = A \cap \bar{B}$ = {(x1, 0.4), (x2, 0.2), (x3, 0.1), (x4, 0.5)}
$\bar{B}$ = {(x1, 0.4), (x2, 0.2), (x3, 0.1), (x4, 0.7)}
Since $\mu_{A-B}(x) = \mu_{A \cap \bar{B}}(x) = \min\{\mu_A(x), \mu_{\bar{B}}(x)\}$
$\therefore \mu_{A \cap \bar{B}}(x1) = \min\{\mu_A(x1), \mu_{\bar{B}}(x1)\} = \min\{0.4, 0.4\} = 0.4$
Similarly $\mu_{A \cap \bar{B}}(x2) = 0.2$, $\mu_{A \cap \bar{B}}(x3) = 0.1$, $\mu_{A \cap \bar{B}}(x4) = 0.5$ respectively [3] [1]

4. Other operations of fuzzy set are

4.1 Product of two fuzzy sets:

A.B(x) = { (x1, 0.24), (x2, 0.24), (x3, 0.9), (x4, 0.15)}
Since $\mu_{A.B}(x) = \mu_A(x) * \mu_B(x)$
$\therefore \mu_{A.B}(x1) = \mu_A(x1) * \mu_B(x1) = (0.4)*(0.6) = 0.24$
Similarly $\mu_{A.B}(x2) = 0.24$, $\mu_{A.B}(x3) = 0.9$, $\mu_{A.B}(x4) = 0.15$ respectively [3] [1]

4.2 Equality:

If two fuzzy sets A,B are said to be equal then $\mu_A(x)=\mu_B(x) \ \forall x$
Let A={(x1,0.2),(x2,0.5)},B = {(x1,0.3),(x2,0.5)},C={(x1,0.2),(x2,0.5)} be three fuzzy sets

From the above example clearly A & C sets are equal sets because $\mu_A(x) = \mu_C(x)\ \forall x$

Whereas A & B sets are not equal sets because $\mu_A(x) \neq \mu_B(x)\ \forall x$ [3] [1]

4.3Product of fuzzy set with a crisp number:
Multiply a fuzzy set A by a crisp number 'a' , results in anew fuzzy set product a. $\overline{A}$ with member function
$\mu_{a.A}(x) = a.\ \mu_A(x)$
$A=\{(x_{1,0.2}), (x_{2,0.5}), (x_{3,0.7})\}$
a=0.2
$\mu_{a.A}(x) = \{(x_{1,0.04}), (x_{2,0.10}), (x_{3,0.14})\}$
Since $\mu_{a.A}(x) = a.\ \mu_A(x_1) = 0.2*0.2 = 0.04$

4.4 Power of a fuzzy set:
The nth power of fuzzy set A is denoted by A^n whose membership function is given by
$\mu_A{}^n(x) = (\mu_A(x))^n$
A= {(x1, 0.2) , (x3,0.6)}
n=2
$\therefore \mu_A{}^2(x) = (\mu_A(x))^2 = \{(x1, 0.04), (x2, 0.36)\}$
Since $\mu_A{}^2(x1) = (\mu_A(x1))^2 = (0.2)^2 = 0.04$
Similarly $\mu_A{}^2(x2) = (\mu_A(x2))^2 = (0.6)^2 = 0.36$ respectively [3] [1]

4.5 Disjunctive sum:
$A \oplus B = (\overline{A} \cap B) \cup (A \cap \overline{B})$
Let A= {(x1, 0.4), (x2, 0.3), (x3, 1), (x4, 0.5)} and
B= {(x1, 0.6), (x2, 0.8), (x3, 0.9), (x4, 0.3)} are two fuzzy sets Then
$\overline{A}$ = {(x1, 0.6), (x2, 0.7), (x3, 0), (x4, 0.5)} {
$(\overline{A} \cap B)$= {(x1, 0.6), (x2, 0.7), (x3, 0), (x4, 0.3)}
$\overline{B}$ = {(x1, 0.4), (x2, 0.2), (x3, 0.1), (x4, 0.7)}
$(A \cap \overline{B})$ = {(x1, 0.4), (x2, 0.2), (x3, 0.1), (x4, 0.5)}
$A \oplus B = (\overline{A} \cap B) \cup (A \cap \overline{B})$ = {(x1, 0.4), (x2, 0.2), (x3, 0), (x4, 0.3)}[3] [1]

5. Properties

Properties of crisp set and fuzzy set[3][1]

Properties	**Crisp set theory**	**Fuzzy set theory**
	Let A, B are two finite sets. Then	Let A, B are two finite fuzzy sets. Then
Commutative	$A \cup B = B \cup A$	$A \cap B = B \cap A$
Associativity	$(A \cup B) \cup C = A \cup (B \cup C)$	$(A \cap B) \cap C = A \cap (B \cap C)$
Distributive	$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$	$A \cap (B \cup C = (A \cap B) \cup (A \cap C)$
Idempotence	$A \cup A = A$ $A \cap A = A$	$A \cup A = A$ $A \cap A = A$
Identity	$A \cup \varnothing = A$ $A \cap U = A$ $A \cup U = U$ $A \cap \varnothing = \varnothing$	$A \cup \varnothing = A$ $A \cap U = A$ $A \cup U = U$ $A \cap \varnothing = \varnothing$
Transitivity	If $A \subseteq B, B \subseteq C$, then $A \subseteq C \subseteq$	If $A \subseteq B, B \subseteq C$, then $A \subseteq C$
Involution	$\overline{\overline{A}} = A$	$\overline{\overline{A}} = A$
De Morgan's laws	$\overline{(A \cup B)} = \overline{A} \cap \overline{B}$ $\overline{(A \cap B)} = \overline{A} \cup \overline{B}$	$\overline{(A \cup B)} = \overline{A} \cap \overline{B}$ $\overline{(A \cap B)} = \overline{A} \cup \overline{B}$
Law Of Extended Middle	$A \cup \overline{A} = U$	$A \cup \overline{A} \neq U$(this property is not holds good in fuzzy set theory)
Law Of Contradiction	$A \cap \overline{A} = \varnothing$	$A \cap \overline{A} \neq \varnothing$(this property is not holds good in fuzzy set theory)

II. WORKING PROCEDURE OF FUZZY INFERENCE SYSTEM

The fuzzy inference process consists of four step process
They are
1. Fuzzification
2. Rule Evaluation
3. Inference
4.Defuzzification

Fuzzification

In this a degree of membership is assigned for each crisp input variable for different conditions. The main intention of fuzzification is to assign or map the inputs from the set of crisp data to the values from 0 to 1 using a set of input membership function [1][5][2].
Eg:
A temperature input might be varied according to its degree of coldness warmth or heat. the process of converting crisp input data to fuzzy set data using linguistic variables fuzzy linguistic terms and membership function this step is known as fuzzification [1][5][3].

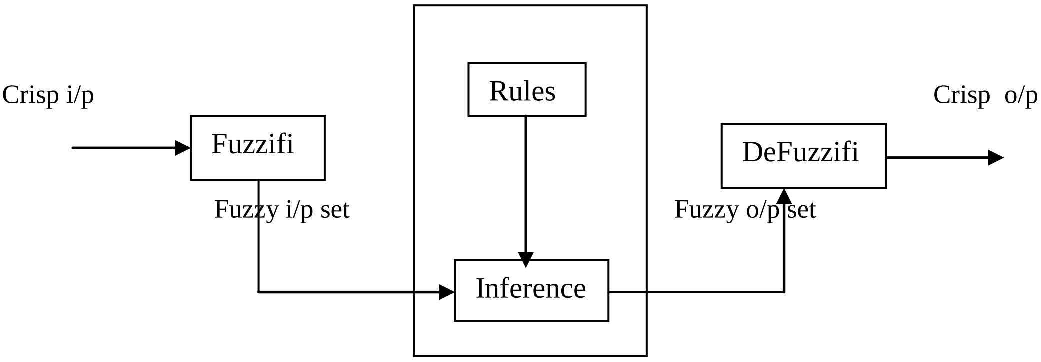

Fig. Fuzzy Inference System

After that we came to one conclusion based on the set of rules then finally resulting fuzzy output is mapped to a crisp output using the membership functions in defuzzification step. In order to understand the fuzzy inference system, consider air condition system.

In this example, the FLS should automatically adjust the room temperature based on the climatic condition (set of target values).

For this purpose FLS, periodically compares the climatic condition (set of target values) and the current room temperature. Based the condition and current room temperature set to the one of the target value[5][3][2].

1. Algorithm:

Step1: Initialize the crisp variables and fuzzy linguistic Variables.
Step 2: Define the statement and their respective conclusion.
Step 3: Assign membership function for each linguistic variable.
Step 4: Implement the fuzzication process.
Step 5: Compare the fuzzy input to the fuzzy inference rules to retrieve respective conclusion.
Step 6: Implement the defuzzification [1][5][3].

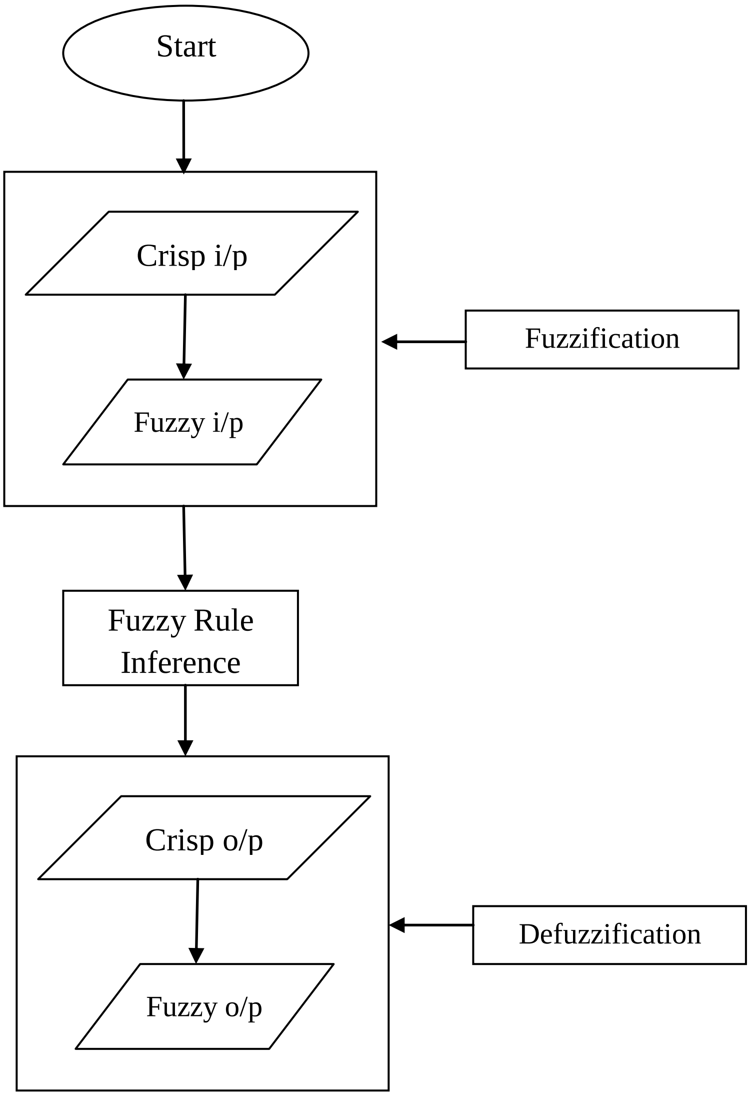

Fig. Flow Chart [1][5][2]

III. FUZZY LINGUISTIC VARIABLE:

Linguistic variables are those values which are words or sentences instead of numerical values. A linguistic variable contains set of linguistic terms [1][5][3] .
Let linguistic variable = temperature.
Linguistic terms ={ cold , hot, warm, very hot}

2. Membership function:

Membership function plays a major role in fuzzification and defuzzification process. For each membership we assign a numerical value [1][5].

- Membership of temperature

μ(temperature)={(too-cold,0) , (cold,0.2),(warm,0.5),(hot,0.8),(too-hot,1)}

3. Fuzzy Rules:

- In fuzzy logic system or FRS, all rules are defined using IF-THEN rules.
- Based on IF condition the corresponding conclusion execution.

Fuzzy rules for air conditioning system [1][5][4]

Temperature	Fan speed
Cold	Stop
Cool	Slow
Just right	Medium
Warm	Fast
Hot	Blast

Rules:

- If temperature is cold then motor speed stop
- If temperature is cool then motor speed slow
- If temperature is just right then motor speed medium
- If temperature is warm then motor speed fast
- If temperature is hot then motor speed blast[2][4]

IV. CONCLUSION

Fuzzy is one of the latest research topic in the current research trend, and it can be applied in almost all research areas like image processing, data mining ,neural networks etc. In this paper we have discussed about, what are the differences between fuzzy and crisp set and their operations. We also came to know the working procedure of fuzzy inference system with an algorithm.

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