OPEN ACCESS

International
Journal
Of Modern Engineering Research (IJMER)

Mhd Flow of A Non-Newtonian Fluid Through A Circular Tube

NASER R. NASER [1], R. K .KHARE [2], AJIT PAUL [3]
*(Department Of Mathematics and Statistics Shiats Allahabad India)
**(Department Of Mathematics and Statistics Shiats Allahabad India)
***(Department Of Mathematics And Statistics Shiats Allahabad India)

***ABSTRACT**:* The work deals with the study of unsteady motion of a dusty fluid through a uniform pipe with sector of a circle as cross-section, the fluid and the dust particle clouds are static at $t = 0$. the influence of time of the flow is dependent to the pressure gradient. We get the solution by analysis of the partial differential equation by variable separable method and Laplace transform. Finally with the aid of graph, the results had been analyzed

Keywords:- unsteady motion, pressure gradient, Laplace transform.

I. INTRODUCTION

The MHD problem becomes more important rapidly during the past few years, it is used in fluidization, use of dust in gas cooling systems for heat transfer process, in environmental pollution, in dust collection, in nuclear reactor cooling, etc. P.G Saffman[12] has discussed the stability of the laminar flow of a dusty gas in which the dust particles are uniformly distributed. Liu[13] has studied the flow induced by an oscillating infinite flat plate in a dusty gas. N.C. Ghosh [3] obtained the solution of MHD flow of a visco-elastic fluid through porous medium. P. Samba siva Rao [13] has got the solution for unsteady flow a dusty viscous liquid through circular cylinder. This paper deals with the study of laminar flow of an unsteady dusty fluid through pipe with sector of circle as cross-section.P.D.Verma and A.K.Mathur[14] studied the unsteady flow of a dusty viscous liquid in a circular tube.Bikash Chandra Ghosh and N.C.Ghosh[15] obtained the solution for MHD flow of a visco-elastic fluid through porous medium.M.H.Hamdan[16] has obtained the dusty gas flow through porous medium. The equation of motion of an unsteady viscous incompressible fluid with uniform distribution of dust particles are given by P. G. Saffman (1962).

II. FORMULATION AND SOLUTION OF THE PROBLEM

$$\nabla.\vec{u} = 0 \quad \text{(continuity)} \tag{1}$$

$$\frac{\partial \vec{u}}{\partial t} + (\vec{u}.\nabla)\vec{u} = -\frac{1}{\rho}\nabla p + \nu\nabla^2\vec{u} + \frac{kN}{\rho}(\vec{v} - \vec{u}) - \frac{\sigma\beta_0{}^2}{\rho(1+c^2)}\vec{u} \quad \text{(Linear Momentum)} \tag{2}$$

$$\nabla.\vec{v} = 0 \quad \text{(continuity)} \tag{3}$$

$$\frac{\partial \vec{u}}{\partial t} + (\vec{v}.\nabla)\vec{v} = \frac{k}{m}(\vec{u} - \vec{v}) \quad \text{(Linear Momentum)} \tag{4}$$

Where $\vec{u}$ and $\vec{v}$ are the velocity of fluid and dust , t is the time,ρ is density of the gas , p is pressure of the fluid, N is the number density of dust particles, ν is kinematic viscosity, k is Stoke's resistance ,m is mass of the dust particle, β_0 is the magnetic field parameter , σ is the conductivity, c is non-Newtonian factor .
We assume an unsteady laminar flow of an incompressible viscous fluid with uniform distribution of dust particles through porous medium in a uniform pipe with sector of a circle of radius a as cross-section .The flow

is due to the influence of time dependent pressure gradient. Both the fluid and the dust particle clouds are supposed to be static at the beginning .It is assumed that the dust particles are spherical in shape and uniform in size and the number density of the dust particles is taken as a constant throughout the flow .The velocity distributions of the fluid and dust particles are given by

$$u_r = 0\,, \quad u_\theta = 0\,, \quad u_z = u_z(r,\theta,t)$$
$$v_r = 0\,, \quad v_\theta = 0\,, \quad v_z = v_z(r,\theta,t) \qquad (5)$$

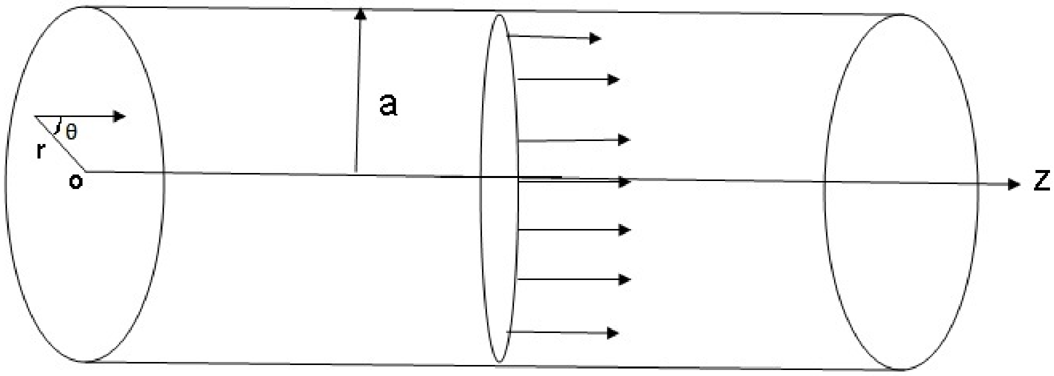


Figure.1: Geometry of the flow in circular channel

By equations (5) , (2) and (4) we get

$$\frac{\partial u_z}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\left(\frac{\partial^2 u_z}{\partial r^2} + \frac{1}{r}\frac{\partial u_z}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}\right) + \frac{kN}{\rho}(v_z - u_z) - \frac{\sigma\beta_0{}^2}{\rho(1+c^2)}u_z$$

. (6)

Consider the following non-dimensional quantities

$$u_z = au\,, \quad v_z = av\,, \quad p = \rho\nu\overline{p}\,, \quad z = a\bar{z}\,,$$
$$r = a\bar{r}\,, t = \frac{a^2}{\nu}\bar{t}\,, k = \frac{mv}{\beta a^2}$$

$$\frac{\partial u}{\partial t} = -\frac{\partial \overline{p}}{\partial \bar{z}} + \left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2}\right) + \frac{l}{\beta}(v - u) - \frac{a^2\sigma\beta_0{}^2}{\nu\rho(1+c^2)}u \qquad (7)$$

$$\frac{\partial v}{\partial t} = \frac{1}{\beta}(u - v) \qquad (8)$$

The initial conditions are

$u(r,\theta,z) = 0$,$v(r,\theta,z) = 0$ when $t = 0$,$r \in (0,1)$ and $\theta \in (-\alpha,\alpha)$
(9)

The no slip boundary conditions are

$t \in (0,\infty)$
$u(r,\theta,z) = 0$,$v(r,\theta,z) = 0$ When $r = 0$ and $r = 1$
$u(r,\theta,z) = 0$,$v(r,\theta,z) = 0$ When $\theta = \pm\alpha$ (10)

Let

$$-\frac{\partial p}{\partial z} = \mathfrak{z}(t)$$

We give the definition for Laplace transformations as follows

$$\bar{u} = \int_0^{\infty} e^{-xt}\, u\, dt \quad \text{and} \quad \bar{v} = \int_0^{\infty} e^{-xt}\, v\, dt \tag{11}$$

Applying the Laplace transformation to equations (7) and (8) and using boundary conditions, we get

$$x\bar{u} = p(X) + \left(\frac{\partial^2 u}{\partial r^2} + \frac{\partial u}{r\partial r} + \frac{\partial^2 u}{r^2 \partial \theta^2}\right) + \frac{l}{\beta}(v - u) - \frac{a^2 \sigma {\beta_0}^2}{v\rho(1+c^2)} u \tag{12}$$

$$x\bar{v} = \frac{1}{\beta}(u - v) \tag{13}$$

The boundary condition now becomes

$\bar{u} = \bar{v} = 0$ When $r = \{0,1\}$ and $\theta = \{-\alpha, \alpha\}$ (14)
Where $p(X)$ is Laplace transform of $\mathfrak{z}(t)$.
Equation (13) implies
$\bar{v} = \frac{\bar{u}}{1+x}$ (15)
By (12) and (15) Then
$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} + \frac{1}{r^2}\frac{\partial^2 u}{\partial \theta^2} - M^2 \bar{u}$ +P(x)= o (16)
Where $M^2 = \frac{a^2 \sigma {\beta_0}^2}{v\rho(1+c^2)} + \frac{lx}{1+x\beta} + x$
e assume the solution of equation (16) in the following form
$\bar{u}(r,\theta) = w_1(r,\theta) + w_2(r)$ (17)
Substitution of $\bar{u}(r,\theta)$ in equation (16)
$\frac{\partial^2 w_1}{\partial r^2} + \frac{\partial^2 w_2}{\partial r^2} + \frac{1}{r}\left(\frac{\partial w_1}{\partial r} + \frac{\partial w_2}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 w_1}{\partial \theta^2} - M^2(w_1 + w_2)$
$+$P(x)$= 0$ (18)
If w_2 satisfies then
$\frac{\partial^2 w_2}{\partial r^2} + \frac{1}{r}\frac{\partial w_2}{\partial r} - M^2 w_2 + \mathrm{P(x)} = 0$ (19)

And $w_1$satisfies

$$\frac{\partial^2 w_1}{\partial r^2} + \frac{1}{r}\frac{\partial w_1}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w_1}{\partial \theta^2} - M^2 w_1 = 0 \tag{20}$$

If $\bar{u}(r,\theta)$ is inserted in no slip boundary conditions, someone can obtain

$\bar{u}(0,\theta) = w_1(0,\theta) + w_2(0) = 0$, $\bar{u}(1,\theta) = w_1(1,\theta) + w_2(1) = 0$,
$\bar{u}(r,\alpha) = w_1(r,\alpha) + w_2(r) = 0$, $\bar{u}(r,-\alpha) = w_1(r,-\alpha) + w_2(r) = 0.$

The equation (19) is the Bessel function whose solution is

$$w_2(r) = C_1 I_0(Mr) + C_2 K_0(Mr) + \frac{P(x)}{M^2} \tag{21}$$

Where I_0 , K_0 are the zeroth order modified Bessel function of first and second kind respectively and C_1, C_2 are constants.
By conditions $w_2(0) = 0$ and $w_2(1) = 0$ in equation (21)
Where (u_r, u_θ, u_z) and (v_r, v_θ, v_z) are velocity components of fluid and dust particles respectively. Above will be shown in the figure.1

We get

$$w_2(r) = \frac{P(x)}{M^2}\left[\frac{I_0(Mr)}{I_0(M)} - 1\right] \tag{22}$$

By this equation we see that the velocity at the centre of the cylinder is zero. Using variable separable method, the solution of the problem (20) With conditions
$w_1(0,\theta) = 0$, $w_1(1,\theta) = 0$, $w_1(r,\alpha) = -w_{2(r)}$
, $w_1(r,-\alpha) = -w_2(r)$ is obtained in the form
$w_1(r,\theta) = \left(AI_\lambda(Mr) + BK_\lambda(Mr)\right)\left(C\ \cos(\lambda\theta) + D\sin(\lambda\theta)\right)$ (23)
Where I_λ and K_λ are the modified Bessel-function of order λ Of first and second kind respectively and A,B,C and D are constants by the same way $B = 0$ since the velocity at the centre of the cylinder is zero , $K_\lambda(Mr) \longrightarrow \infty$ as $r \longrightarrow o$
Then the solution is given by

$$w_1(r,\theta) = -\frac{P(x)}{M^2}\left[\frac{I_0(Mr) - I_0(M)}{I_0(M)}\right]\left[\frac{\cos(\lambda\theta)}{\cos(\lambda\alpha)}\right] \tag{24}$$

Substituting (22) and (24) in (17) we obtain

$$\bar{u}(r,\theta) = -\frac{P(x)}{M^2}\left[\frac{I_0(Mr) - I_0(M)}{I_0(M)}\right]\left[\frac{\cos(\lambda\alpha) - \cos(\lambda\theta)}{\cos(\lambda\alpha)}\right] \tag{25}$$

Using $\bar{u}$ in equation (15) can see that

$$\bar{v}(r,\theta) = \frac{P(x)}{M^2(1+x\beta)}\left[\frac{I_0(Mr) - I_0(M)}{I_0(M)}\right]\left[\frac{\cos(\lambda\alpha) - \cos(\lambda\theta)}{\cos(\lambda\alpha)}\right] \tag{26}$$

Case of Impulsive motion

Let $\mathfrak{z}(t) = p_0\delta(t)$ **Where** $\delta(t)$ is Dirac delta function and p_0 is constant.
Now equation (25) and (26) becomes

$$\bar{u}(r,\theta) = p_0\left[\frac{I_0(Mr) - I_0(M)}{I_0(M)}\right]\left[\frac{\cos(\lambda\alpha) - \cos(\lambda\theta)}{\cos(\lambda\alpha)}\right] \tag{27}$$

$$\bar{v}(r,\theta) = \frac{p_0}{1+x\beta}\left[\frac{I_0(Mr) - I_0(M)}{I_0(M)}\right]\left[\frac{\cos(\lambda\alpha) - \cos(\lambda\theta)}{\cos(\lambda\alpha)}\right] \tag{28}$$

By taking the inverse Laplace transform of $\bar{u}$ and $\bar{v}$ then

$$u = \sum_{\alpha_m=0}^{\infty} \frac{2p_0}{\alpha_m}\left[\left(\frac{I_0(\alpha_m) - I_0(\alpha_m r)}{I_1(\alpha_m)}\right)\left(\frac{\cos(\lambda\alpha) - \cos(\lambda\theta)}{\cos(\lambda\alpha)}\right)\right]$$

$$\times\left[\frac{e^{x_1 t}(1+x_1\beta)^2}{l+(1+x_1\beta)^2} + \frac{e^{x_2 t}(1+x_2\beta)^2}{l+(1+x_2\beta)^2}\right] \tag{29}$$

$$v = \sum_{\alpha_m=0}^{\infty} \frac{2p_0}{\alpha_m}\left[\left(\frac{I_0(\alpha_m) - I_0(\alpha_m r)}{I_1(\alpha_m)}\right)\left(\frac{\cos(\lambda\alpha) - \cos(\lambda\theta)}{\cos(\lambda\alpha)}\right)\right]$$

$$\times\left[\frac{e^{x_1t}(1+x_1\beta)}{l+(1+x_1\beta)^2}+\frac{e^{x_2t}(1+x_2\beta)}{l+(1+x_2\beta)^2}\right] \tag{30}$$

Where (m= $1,2,3,\dots\dots\dots\dots$) are the positive zeros of $I_0(\alpha)=0$

Now $r=0,1$ and $\theta=\pm\alpha$ respectively are

$$\frac{\partial u}{\partial\theta}_{r=0}=\sum_{\alpha_m=0}^{\infty}\frac{2\lambda p_0}{\alpha_m}\left[\left(\frac{I_0(\alpha_m)-I_0(\alpha_m r)}{I_1(\alpha_m)}\right)\left(\frac{\sin(\lambda\theta)}{\cos(\lambda\alpha)}\right)\right]$$

$$\times\left[\frac{e^{x_1t}(1+x_1\beta)^2}{l+(1+x_1\beta)^2}+\frac{e^{x_2t}(1+x_2\beta)^2}{l+(1+x_2\beta)^2}\right]$$

$$\frac{\partial u}{\partial r}_{r=1}=\frac{2p_0[\cos(\lambda\theta)-\cos(\lambda\alpha)]}{\cos(\lambda\alpha)}\times\left[\frac{e^{x_1t}(1+x_1\beta)^2}{l+(1+x_1\beta)^2}+\frac{e^{x_2t}(1+x_2\beta)^2}{l+(1+x_2\beta)^2}\right]$$

$$\frac{\partial u}{\partial\theta}_{\theta=\alpha}=\sum_{\alpha_m=0}^{\infty}\frac{2\lambda p_0\tan(\lambda\alpha)}{\alpha_m}\left[\left(\frac{I_0(\alpha_m)-I_0(\alpha_m r)}{I_1(\alpha_m)}\right)\right]$$
$$\times\left[\frac{e^{x_1t}(1+x_1\beta)^2}{l+(1+x_1\beta)^2}+\frac{e^{x_2t}(1+x_2\beta)^2}{l+(1+x_2\beta)^2}\right]$$

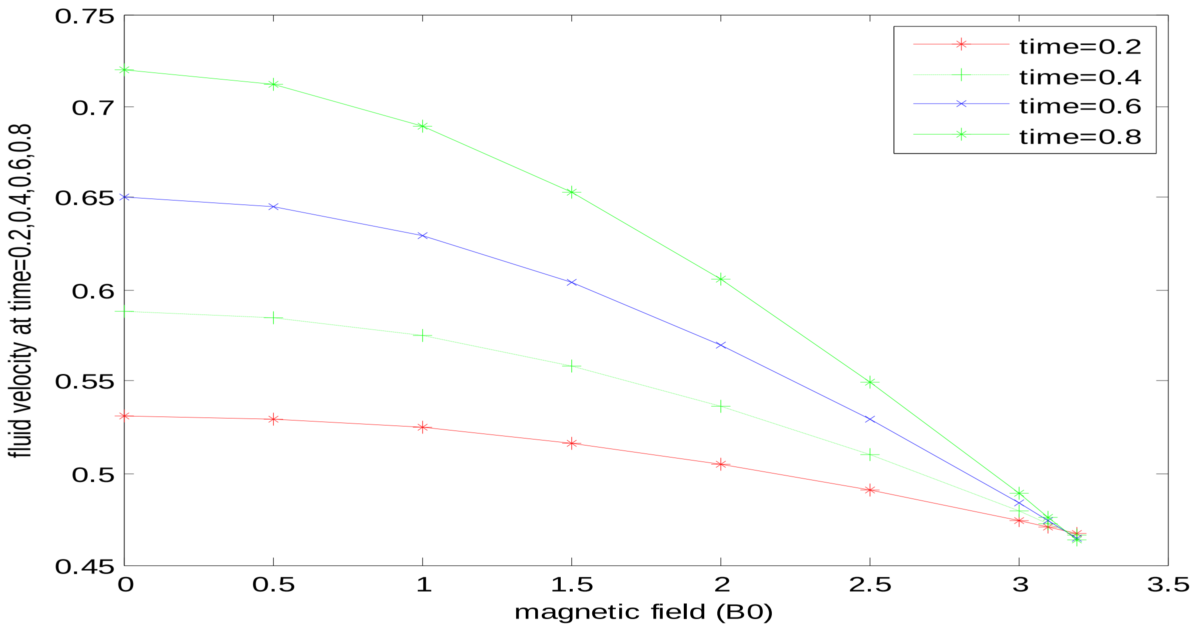
$$\frac{\partial u}{\partial\theta}_{\theta=-\alpha}=-\frac{\partial u}{\partial\theta}_{\theta=\alpha}$$

Where

$$x_1=\frac{-R_1+R_2}{2\rho(1+C^2)\nu\beta}\quad ,x_2=\frac{-(R_1+R_2)}{2\rho(1+C^2)\nu\beta}$$
$$R_1=\rho(1+C^2)\nu(1+l-\beta{\alpha_m}^2)+\sigma{\beta_0}^2\beta a^2$$
$$R_2=\sqrt{{R_1}^2-4\rho(1+C^2)\nu\beta\left(\sigma{\beta_0}^2a^2-\rho(1+C^2)\nu{\alpha_m}^2\right)}$$

Table No.(1)
(T=1 , β=0.05 , m=2 , p_0=0.6 , ρ=2 , c=3 , σ=1 , a=1)

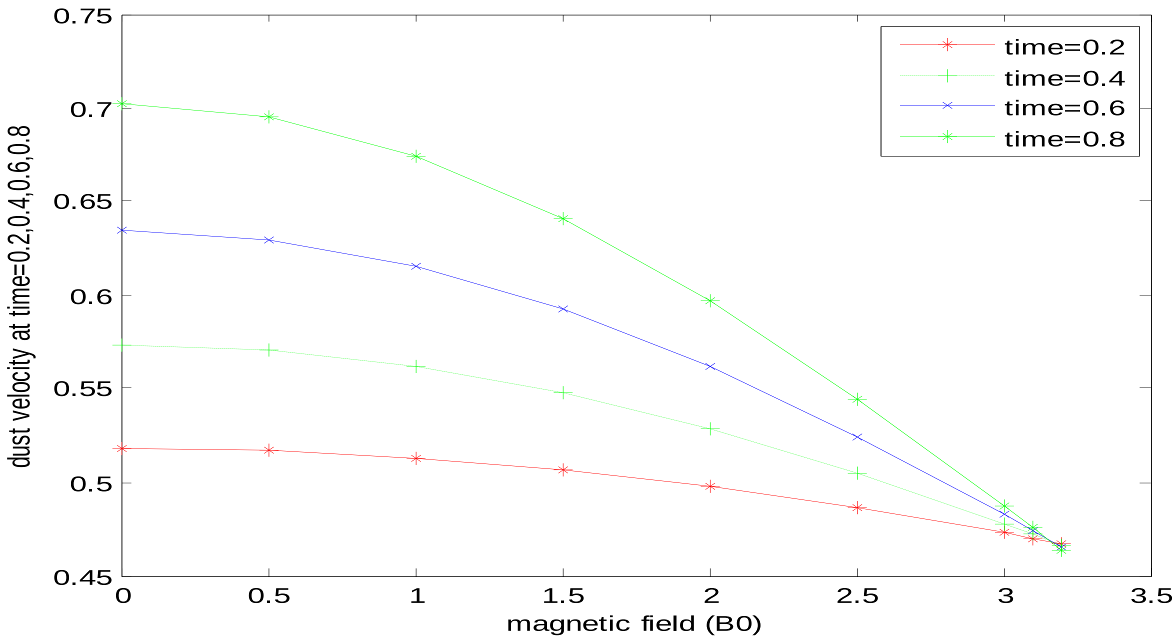
S. No.	β_0	Fluid velocity t=0.2	Fluid velocity t=0.4	Fluid velocity t=0.6	Fluid velocity t=0.8
1	0	0.5314	0.5879	0.6505	0.7198
2	0.5	0.5298	0.5845	0.6451	0.7121
3	1	0.5247	0.5746	0.6293	0.6893
4	1.5	0.5165	0.5584	0.6038	0.6530
5	2	0.5052	0.5365	0.5700	0.6055
6	2.5	0.4911	0.5098	0.5294	0.5497
7	3	0.4745	0.4791	0.4839	0.4888
8	3.1	0.4709	0.4725	0.4744	0.4762
9	3.2	0.4672	0.4659	0.4648	0.4637



We see in above table (1.1) the fluid velocity increase as the magnetic field increase , for every parameter (t) .the reason of this increasing is the value of terms x_1, x_2 which are power for the constant (e) in the numerator then the value for the fluid velocity will be increase

Table No. (2)
(T=1 , $\beta = 0.01$, m=2 , p_0=0.6 , ρ =2 , c=3 , σ=1 , a=1)

S. No.	β_0	Dust velocity t=0.2	Dust velocity at=0.4	Dust velocity t=0.6	Dust velocity t=0.8
1	0	0.5180	0.5734	0.6345	0.7021
2	0.5	0.5167	0.5704	0.6296	0.6949
3	1	0.5127	0.5618	0.6153	0.6739
4	1.5	0.5063	0.5477	0.5922	0.6404
5	2	0.4974	0.5285	0.5615	0.5965
6	2.5	0.4862	0.5050	0.5244	0.5446
7	3	0.4730	0.4779	0.4827	0.4875
8	3.1	0.4701	0.4721	0.4739	0.4758
9	3.2	0.4671	0.4662	0.4650	0.4639



Similarly in this phase we see in above table(1.2) the dust velocity increaseas the magnetic field increase, for every parameter (t) .the reason of this increasing is the value of terms x_1, x_2 which are power for the constant (e) in the numerator then the value for the dust velocity will be increase

Table No. (3)
(p_0=0.6 , β=0.05 , m=2 ,l=1 ,σ=1 , ρ=2 ,a=1)

S.No.	Non-newtonian factor (c)	Fluid velocity t=0.2	Fluid velocity t=0.4	Fluid velocity t=0.6	Fluid velocity t=0.8
1	0	0.0891	0.0261	0.0076	0.0022
2	0.1	0.0904	0.0267	0.0079	0.0023
3	0.2	0.0945	0.0288	0.0088	0.0027
4	0.3	0.1013	0.0324	0.0104	0.0033
5	0.4	0.1107	0.0376	0.0128	0.0043
6	0.5	0.1224	0.0446	0.0162	0.0059
7	0.6	0.1363	0.0535	0.0210	0.0083
8	0.7	0.1518	0.0644	0.0273	0.0116
9	0.8	0.1687	0.0772	0.0354	0.0162
10	0.9	0.1865	0.0918	0.0452	0.0223
11	1	0.2047	0.1080	0.0570	0.0301

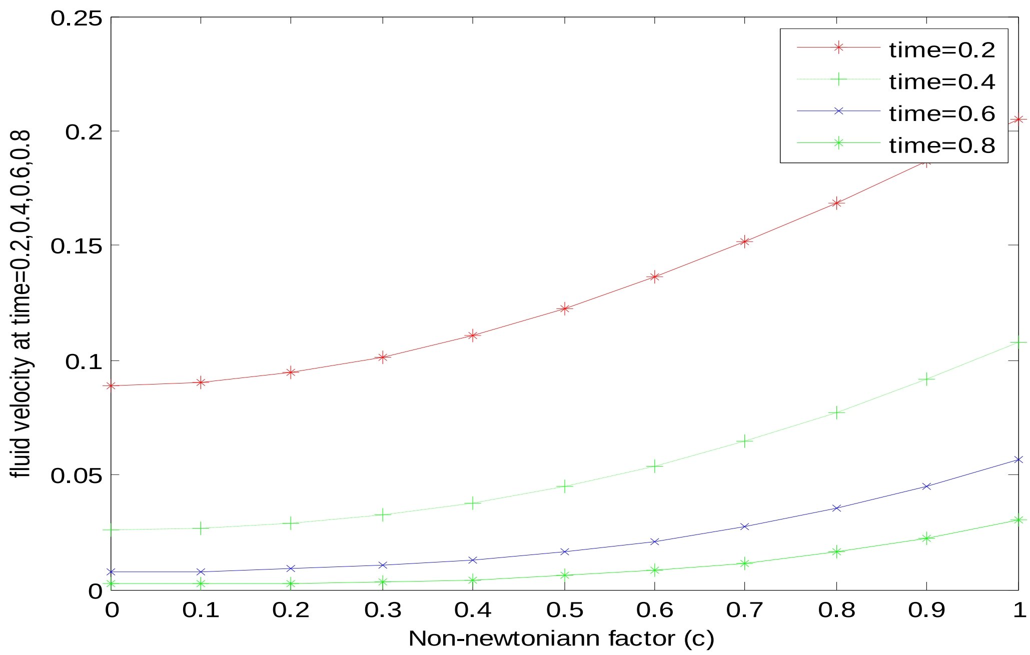
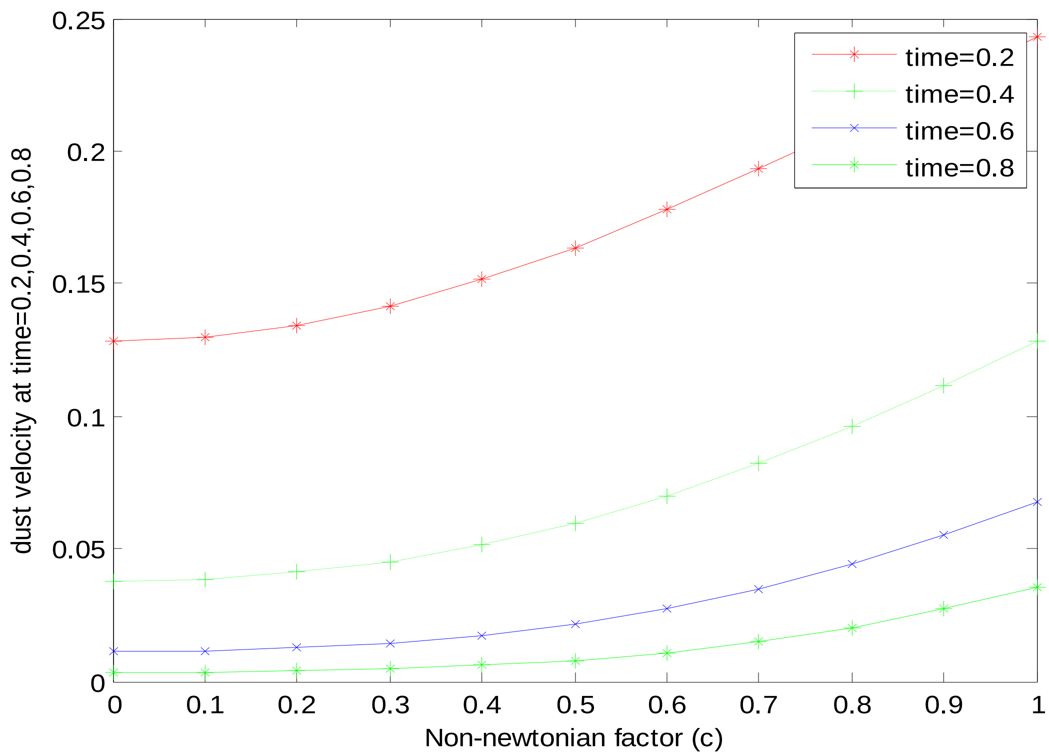


Table (3) reveals as the non-newtonian factor(c) of fluid increases, the velocity of fluid phase increases for every time parameter (t) ,because (c) is in the denominator of x_1, x_2 in (29).

Table No.(4)

S.No.	Non-newtonian factor (c)	Dust velocity t=0.2	Dust velocity t=0.4	Dust velocity t=0.6	Dust velocity t=0.8
1	0	0.1224	0.0376	0.0110	0.0032
2	0.1	0.1299	0.0385	0.0114	0.0034
3	0.2	0.1344	0.0410	0.0125	0.0038
4	0.3	0.1417	0.0453	0.0145	0.0046
5	0.4	0.1515	0.0515	0.0175	0.0059
6	0.5	0.1637	0.0596	0.0217	0.0079
7	0.6	0.1777	0.0698	0.0274	0.0108
8	0.7	0.1931	0.0820	0.0348	0.0148
9	0.8	0.2095	0.0960	0.0440	0.0201
10	0.9	0.2264	0.1116	0.0550	0.0271
11	1	0.2434	0.1285	0.0687	0.0358

(p_0=0.6 , β=0.05 , m=2 , l=1 , σ=1 ,ρ=2 , a=1)



Table(4) reveals as the non-Newtonian factor(c) of dust increases, the velocity of dust phase increases for every time parameter (t) ,because (c) is in the denominator of x_1, x_2 in (30).The graph (1) , (2) ,(3) and(1.4) supported the previous tables

III. RESULT AND DISCUSSION

From the first and second table, we can show that the relation between the fluid velocity and magnetic field is Inverse relation. Similarly for the dust, from the third and fourth table we can show the relation between the fluid velocity and non-newtonian factor is proportional relation .similarly for the dust. By previous we can show the relation between magnetic field and non-newtonian factor is inverse.

REFERENCE

[1] R.M.Barron. "steady two dimensional flow of a dusty gas",*Tensor N.S.,36,* PP 85-86,(1982)
[2] G.S.Bagewadi,B.J.Gireesha, "A study of two dimensional unsteady dusty fluid flow under varying pressure gradient", *Tensor,N.S*., **64** (2003) 232- 240
[3] Bikash Chandra Ghosh, N.C. Ghosh, MHD flow of a visco- elastic fluid through porous medium, Int. J. Num. Meth. Heat and Fluid Flow, 11, No. 7 (2001), 682-698.
[4] Brent E. Sleep, Modelling transient organic vapor transport in porous medi with the dusty gas model, Adv. in Water Resources, 22, No. 3 (1998), 247- 256.
[5] Fathi M. Allan, Naji Qatanani, Imad Barghouthi , Khaled M. Takatka, Dusty gas model of flow through naturally occurring porous media, App. Math. Comp., 148 (2004), 809-821.
[6] B.J. Gireesha, P. Venkatesha , C. S. Bagewadi, Flow of an unsteady conducting dusty fluid through circular cylinder, J. Acta Universitatis Apulensis, 17 (2009), 77-86.
[7] Bekir, S. Y. (2004): Entropy analysis for non Newtonian fluid in annular pipe constant viscosity case. Entropy, pp. 304-315.
[8] Bhattacharya, T.K. (1994): Hydro magnetic flow of a dusty viscous fluid Through an annulus with time dependent pressure gradient. Journal of MACT. Vol. 27, pp. 1-5.
[9] Bjerkness, V. (1900): Meteorologiache Zeitschrift, 17, pp. 97-106.
[10] Chamkha, Ali. J. (1996): Boundary layer laminar flow of a dusty compressible gas over a flat surface. Journal of fluid Engineering, Vol. 118, pp.179-185.
[12] P.G. Saffman,On the stability laminar flow of a dusty gas, Journal of fluid Mechanics ,**13** (1962),120-128.
[13] J. T. C. Liu , Flow induced by an oscillating infinite plat plate in a dusty gas ,Phys. Fluids, 9 (1966), 1716-1720 .
[14] P.D. Verma,A .K. Mathur, Unsteady flow of a dusty viscous liquid in a circularpipe ,Indian Journal of pure and Applied Maths.,4,No. 2(1973), 133-138
[15] Bikash Chandra Ghosh and N.C.Ghosh, MHD flow of a visco-elastic fluid through porous medium,Int.J.Num. Meth.Heat and fluid flow,11,No.7(2001),682-698.
[16] M.H.Hamdan,K.D.Sawalha, Dusty gas flow through porous medium, App. Math. Comp., 75 (1996),59-73.