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On Contra – RW Continuous Functions in Topological Spaces

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ABSTRACT: *In this paper we introduce and investigate some classes of functions called contra-rw continuous functions. We get several characterizations and some of their properties. Also we investigate its relationship with other types of functions.*
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I. Introduction

In 1996, Dontchev[1] presented a new notion of continuous function called contra-continuity. This notion is a stronger form of LC-Continuity. In 2007, Benchalli.S.S. and Wali.R.S.[2] introduced RW-closed set in topological spaces. In 2012, Karpagadevi.M. and Pushpalatha.A.[3] introduced RW-continuous maps and RW-irresolute maps in topological spaces. The purpose this paper is to define a new class of continuous functions called Contra-rw continuous functions and almost Contra rw-Continuous function and investigate their relationships to other functions.

II. Preliminaries

Throughout this paper X and Y denote the topological spaces (X,τ) and (Y,σ) respectively and on which no separation axioms are assumed unless otherwise explicitly stated. For any subset A of a space (X,τ), the closure of A, the interior of A and the complement of A are denoted by cl(A), int(A) and A^c respectively. (X,τ) will be replaced by X if there is no chance of confusion. Let us recall the following definitions as pre requesters.

Definition 2.1: A Subset A of (X,τ) is called
(i) generalized closed set (briefly g-closed) [5] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.
(ii) regular generalized closed set (briefly rg-closed)[6] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular open in X.
The Complement of the above mentioned closed sets are their respective open sets.

Definition 2.2: A Subset A of a Space X is called rw-closed [2] if $cl(A) \subseteq U$ whenever $A \subseteq U$ and U is regular semiopen in X.

Definiton 2.3: A Map $f: (X,\tau) \rightarrow (Y,\sigma)$ is said to be
(i) rw-continuous[3] if $f^{-1}(V)$ is rw-closed in (X,τ) for every closed set V in (Y,σ)
(ii) rw-irresolute[3] if $f^{-1}(V)$ is rw-closed in (X,τ) for every rw-closed set V of (Y,σ)
(iii) rw-closed[2] if f(F) is rw-closed in (Y,σ) for every rw-closed set F of (X,τ)
(iv) rw-open[2] if f(F) is rw-open in (Y,σ) for every rw-open set F of (X,τ)

Definiton 2.4: A Map $f: (X,\tau) \rightarrow (Y,\sigma)$ is said to be contra-continuous [1] if $f^{-1}(V)$ is closed in (X,τ) for every open set V in (Y,σ).

III. Contra RW-Continuous Function

In this section we introduce the notions of contra rw-continuous, contra rw-irresolute and almost contra rw-continuous functions in topological spaces and study some of their properties.

Definition 3.1: A function $f: (X,\tau) \rightarrow (Y,\sigma)$ is called contra rw-continuous if $f^{-1}(V)$ is rw-closed set in X for each open set V in Y.

Example 3.2: Let X=Y={a,b,c} with topologies $\tau = \{X,\phi,\{a\},\{b\},\{a,b\}\}$ and $\sigma = \{Y,\phi,\{a,b\}\}$. Let f: X → Y be a map defined by f (a) = a, f (b) = b and f(c) = c. Clearly f is contra rw-continuous.

Theorem 3.3: Every contra-continuous function is contra rw-continuous.
Proof: The proof follows from the fact that every closed set is rw-closed set.

Remark 3.4: The converse of the above theorem need not be true as seen from the following example.

Example 3.5: Let X=Y={a,b,c} with topologies $\tau = \{X,\phi,\{a\},\{b\},\{a,b\}\}$ and $\sigma = \{Y,\phi,\{a,b\}\}$. Let f: X → Y be a map defined by f (a) = a, f (b) = b and f(c) = c. Clearly f is contra rw-continuous but not contra-continuous since $f^{-1}(\{a,b\}) = \{a,b\}$ which is not closed in X.

Theorem 3.6: If a function f: X → Y is contra rw-continuous, then f is contra-continuous.
Proof: Let V be an open set in Y. Since f is contra rw-continuous, $f^{-1}(V)$ is closed in X. Hence f is contra-continuous.

Remark 3.7: The concept of rw-continuity and contra rw-continuity is independent as shown in the following examples.

Example 3.8 : Let X=Y={a,b,c} with topologies $\tau = \{X,\phi,\{a\},\{b\},\{a,b\}\}$ and $\sigma = \{Y,\phi,\{b,c\}\}$. Let f: X → Y be a map defined by f (a) = a, f (b) = b and f(c) = c. Clearly f is contra rw-continuous but not rw-continuous since $f^{-1}(\{a\}) = \{a\}$ is not rw-closed in X where {a} is closed in Y.

Example 3.9 : Let X=Y={a,b,c} with topologies $\tau = \{X,\phi,\{a\},\{b\},\{a,b\}\}$ and $\sigma = \{Y,\phi,\{b\}\}$. Define f: X → Y identity mapping. Then c learly f is rw-continuous but not contra rw-continuous since $f^{-1}(\{b\}) = \{b\}$ is not rw-closed in X where {b} is closed in Y.

Theorem 3.10: Every contra rw-continuous function is contra rg-continuous.
Proof: Since every rw-closed set is rg-closed, the proof is obvious.

Remark 3.11: The converse of the above theorem need not be true as seen from the following example.

Example 3.12: Let X=Y={a,b,c,d} with topologies $\tau=\{X,\phi,\{a\},\{b\},\{a,b\},\{a,b,c\}\}$ and $\sigma = \{Y,\phi,\{a\},\{b\},\{a,b,c\}\}$. Let f: X → Y be a map defined by f (a) = c, f (b) = d, f(c) = a and f (d) =b. Clearly f is contra rg-continuous but not contra rw-continuous since $f^{-1}(\{a\}) = \{c\}$ is not rw-closed in X where {a} is open in Y.

Remark 3.13: The composition of two contra rw-continuous functions need not be contra rw-continuous as seen from the following example.

Example 3.14: Let X=Y=Z={a,b,c} with topologies $\tau = \{X,\phi,\{a\},\{b\},\{a,b\}\}$ and $\sigma = \{Y,\phi,\{a,b\}\}$ and $\eta = \{Z, \phi,\{a\}\}$. Let f: X → Y be a map defined by f (a) = c, f (b) = b and f(c) = a and g: Y→Z is defined by g (a) =b, g (b) =c and g(c) =a.Then clearly f and g are contra rw-continuous. But g o f: X → Z is not contra rw-continuous since $(g \circ f)^{-1}\{a\} = f^{-1}(g^{-1}\{a\}) = f^{-1}(\{c\}) = \{a\}$ is not rw-closed in X.

Theorem 3.15: If f: (X,τ) → (Y,σ) is contra rw-continuous and g: Y→Z is a continuous function, then g o f : X→Z is contra rw-continuous.
Proof: Let V be open in Z. Since g is continuous, $g^{-1}(V)$ is open in Y. Then $f^{-1}(g^{-1}(V))$ is rw-closed in X since f is contra rw-continuous. Thus $(g \circ f)^{-1}(V)$ is rw-closed in X. Hence g o f is contra rw-continuous.

Theorem 3.16: If f: X → Y is rw-irresolute and g: Y→Z is contra continuous function then g o f: X→Z is contra rw-continuous.
Proof: Since every contra continuous is contra rw-continuous, the proof is obvious.

Theorem 3.17: If f: X → Y is contra rw-continuous then for every $x\in X$, each $F\in C(Y,f(x))$, there exists $U\in RWO(X,x)$ such that $f(U) \subset F$ (ie) For each $x \in X$, each closed subset F of Y with $f(x) \in F$, there exists a rw-open set U of Y such that $x\in U$ and $f(U)\in F$.

Proof: Let f: X → Y be contra rw-continuous. Let F be any closed set of Y and f(x) ∈ F where x∈X. Then $f^{-1}(F)$ is rw-open in X. Also x ∈ $f^{-1}(F)$. Take U = $f^{-1}(F)$. Then U is a rw-open set containing x and f (U) ⊆ F.

Theorem 3.18: Let (X,τ) be a rw-connected space (Y,σ) be any topological space. If X → Y is surjective and contra rw-continuous then Y is not a discrete space.
Proof: Suppose Y is discrete space. Let A be any proper non-empty subset of Y. Then A is both open and closed in Y. Since f is contra rw-continuous, $f^{-1}(A)$ is both rw-open and rw-closed in X. Since X is rw-connected, the only subset of Y which are both rw-open and rw-closed are X andϕ. Hence $f^{-1}(A)$ = X. Then it contradicts to the fact that f: X → Y surjective. Hence Y is not a discrete space.

Definition 3.19: A function f: X → Y is called almost contra rw-continuous if $f^{-1}(V)$ is rw-closed set in X for every regular open set V in Y.

Example 3.20: Let X=Y=Z={a,b,c} with topologies τ = {X,ϕ,{a},{b},{a,b}} and σ ={ Y,ϕ,{a},{c},{a,c}}. Define f: X → Y by f (a) =a, f (b) =b and f(c)=c. Here f is almost contra rw-continuous since f^{-1}({a}) = {a} is rw-closed in X for every regular open set {a} in Y.

Theorem 3.21: Every contra rw-continuous function is almost contra rw-continuous.
Proof: The proof follows from the definition and fact that every regular open set is open.

Definition 3.22: A function f: X → Y is called contra rw-irresolute if $f^{-1}(V)$ is rw-closed in X for each rw-open set V in Y.

Definition 3.23: A function f: X → Y is called perfectly contra rw-irresolute if $f^{-1}(V)$ is rw-closed and rw-open in X for each rw-open set V in Y.

Theorem 3.24: A function f: X → Y is perfectly contra rw-irresolute if and only if f is contra rw-irresolute and rw-irresolute.
Proof: The proof directly follows from the definitions.

Remark 3.25: The following examples shows that the concepts of rw-irresolute and contra rw-irresolute are independent of each other.

Example 3.26: Let X=Y=Z={a,b,c} with topologies τ = {X,ϕ,{a},{b},{a,b}} and σ ={ Y,ϕ,{a},{c},{a,c}}. Define f: X → Y by f (a) = a, f (b) = b and f(c) = c. Here f is contra rw-irresolute but not rw-irresolute since f^{-1}({a,b}) = {a,b} is not rw-open in X.

Example 3.27: Let X=Y=Z={a,b,c} with topologies τ = {X,ϕ,{a},{b},{a,b}} and σ ={ Y,ϕ,{a},{c},{a,c}}. Define f: X → Y by f (a) = a, f (b) = b and f(c) = c. Here f is rw-irresolute but not contra rw-irresolute since f^{-1}({b}) = {b} is not rw-closed in X.

Theorem 3.28: Let f: X → Y and g: Y → Z be a function then
(i) If g is rw-irresolute and f is contra rw-irresolute then g o f is contra rw-irresolute
(ii) If g is contra rw-irresolute and f is rw-irresolute then g o f is contra rw-irresolute
Proof: (i) Let U be a rw-open in Z. Since g is rw-irresolute, $g^{-1}(U)$ is rw-open in Y. Thus $f^{-1}(g^{-1}(U))$ is rw-closed in X. Since f is contra rw-irresolute (ie) $(g \text{ o } f)^{-1}(U)$ is rw-closed in X. This implies that g o f is contra rw-irreolute.
(ii) Let U be a rw-open in Z. Since g is contra rw-irresolute, $g^{-1}(U)$ is rw-closed in Y. Thus $f^{-1}(g^{-1}(U))$ is rw-closed in X since f is rw-irresolute (ie) $(g \text{ o } f)^{-1}(U)$ is rw-closed in X. This implies that g o f is contra rw-irresolute.

Theorem 3.29: Every perfectly contra rw-irresolute function is contra rw-irresolute and rw-irresolute.
Proof: The proof directly follows from the definitions.

Remark 3.30: The following two examples shows that a contra rw-irresolute function may not be perfectly contra rw-irresolute and rw-irresolute function may not be perfectly contra rw-irresolute.

Example 3.31: In example 3.29, f is rw-irresolute but not perfectly contra rw-irresolute and in example 3.28, f is contra rw-irresolute but not perfectly contra rw-irresolute.

Theorem 3.32: A function is perfectly contra rw-irresolute iff f is contra rw-irresolute and rw-irresolute.

Proof: The proof directly follows from the definitions.

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