Analysis of the Effects of Infectious Disease in a Closed Population

R.A. Kareem, T.S. Nurudeen, S.O. Salawu and A.A. Abdurasid
Department of Mathematics, Lagos State Polytechnic, Ikorodu, Nigeria

ABSTRACT: *This work focuses on the analysis of the effects of infectious disease in a closed population. We considered a Drilling rig company in Nigeria positioned several hundred meters from the coast. Because of their remoteness we are worried about the effect of an epidemic among the staff. We formulated a problem base on a typical rig practice in Nigeria. We look at the effect of the epidemic among the staff and the provision the company should make to mitigate the effects of an outbreak of some infectious diseases. Here individual do not come in contact with people from outside the community under consideration.*

I. INTRODUCTION

The diseases and any study about them can be very helpful and important for human beings. Specially, mathematical modeling can be a good and suitable instrument for having a better life. The basic SIR model has a long history. At first Kermak and Mckendrich introduced SIR model in 1927 [8]. Now, SIR model is developing more and more that you can even find it discussed in some introductory calculus text books [5]. SIR model can be very useful and helpful in characterizing some diseases. More numbers of these mathematical models can be seen in [2, 3, 13, 14, and 15].

Epidemics have ever been a great concern to human kind and we are still moved by the dramatic descriptions that arrive to us from the past. A single epidemic outbreak (as opposed to disease endemic) occurs in a time span short enough not to have the demographic changes perturbing the dynamics of the contacts between individuals [7, 9, 10]. The mathematical techniques used to understand, forecast, and control the spread of infectious diseases like influenza are diverse and growing rapidly. Some techniques have been newly developed, whereas others build upon existing methods from diverse fields including dynamical systems, stochastic processes, statistical physics, graph theory, statistics, operations research, and high-performance computing [12]. Here, we considered an epidemic in a fraction of the population in a community especially in a drilling rig company in Nigeria where individual did not come in contact with people from the community.

II. MATHEMATICAL MODEL

At any given time, the population may be divided into three groups: the Susceptible, S, (i.e. those who have not yet been infected by the disease), the Infectives, I, (i.e. those who are currently suffering from the disease, who may pass it on to others), and the Removal, R, (i.e. those who have had the disease and either died, become immune or been isolated from the rest of the population). The rates at which these transfer take place can be assumed to obey the following equations [5, 11]

$$\frac{dS}{dt} = -rSI \qquad (1)$$

$$\frac{dI}{dt} = rSI - aI \qquad (2)$$

$$\frac{dR}{dt} = aI \qquad (3)$$

Let systems (1) - (3) be **Model A**,
Where $r > 0$ is the infection rate and $a > 0$ is the removal rate of infectives.
The population size is constant and is given as

$$\frac{dS}{dt} + \frac{dI}{dt} + \frac{dR}{dt} = 0$$
$$S(t) + I(t) + R(t) = N$$

Where N is the total size of the population
The initial conditions attached to the mathematical formulation of the epidemic problem in (1) - (3) are

$$S(0) = S_0 > 0, I(0) = I_0 > 0, R(0) = R_0 = 0 \qquad (4)$$

III. PROBLEM FORMULATION

In this work, we considered a drilling rig company in Nigeria with staff strength of 100 employees each of whom spend 5 months at a time on the rig. Each month 20 employees leave the rig and are replaced by 20 others. It is these newcomers which could bring infection to the staff of the rig. It is estimated that an infection rate, r, of 0.1 would be typical for most viral diseases under the current conditions of the rig and that current practice on the rigs would lead to a removal rate, a, of 0.5. Both of these rates are based on a time-scale measured in days. Using the initial conditions $S(0)=90, R(0)=0, I(0)=10$, we investigate the effect of more or fewer of the 20 new arrivals being infected, the effect of the removal rate on the spread of the epidemic, effect of adopting safer behavior patterns on the spread of the epidemic and to ensure that there are never fewer than 60% of the staff available for work at any one time.

IV. INVESTIGATIONS

At each month, 20 employees leave the rig and are replaced by 20 others. Then the following cases can be investigated.

(a) 0.1 infection rate within the new 20 arrivals i.e.

$$\frac{10}{100}\times 20 = 2 \text{ People are infected}$$

(b) 0.2 infection rate within the new 20 arrivals i.e.

$$\frac{10}{100}\times 20 = 18 \text{ People are susceptible}$$

(c) 0.3 infection rate within the new 20 arrivals i.e.

$$\frac{10}{100}\times 20 = 2 \text{ People are infected}$$

We investigate three cases of the infection

Case 1: Susceptible = 18, Infected = 2

0 month [0 1]

$$S(0)=90, R(0)=0, I(0)=10$$

[S I R] = [90 10 0]

1st month [1 2]

$$S(1)=0, R(0)=31, I(1)=69$$

S: $0.2\times 0 = 0 \Rightarrow$ home
$0.8\times 0 = 0 \Rightarrow$ remain
I: $0.2\times 69 = 14 \Rightarrow$ home
$0.8\times 69 = 55 \Rightarrow$ remain
R: $0.2\times 31 = 6 \Rightarrow$ home
$0.8\times 31 = 25 \Rightarrow$ remain

[S I R] = [18 57 25]

2nd month [2 3]

$$S(2)=0, R(2)=54, I(2)=46$$

S: $0.2\times 0 = 0 \Rightarrow$ home
$0.8\times 0 = 0 \Rightarrow$ remain
I: $0.2\times 46 = 9 \Rightarrow$ home
$0.8\times 46 = 37 \Rightarrow$ remain
R: $0.2\times 54 = 11 \Rightarrow$ home
$0.8\times 54 = 43 \Rightarrow$ remain

[S I R] = [18 39 43]

3rd month [3 4]

$$S(3)=0, R(3)=64, I(3)=36$$

S: $0.2\times 0 = 0 \Rightarrow$ home
$0.8\times 0 = 0 \Rightarrow$ remain
I: $0.2\times 36 = 7 \Rightarrow$ home
$0.8\times 36 = 29 \Rightarrow$ remain
R: $0.2\times 64 = 13 \Rightarrow$ home
$0.8\times 64 = 51 \Rightarrow$ remain

[S I R] = [18 31 51]

4th month [4 5]

$$S(4)=0, R(4)=69, I(4)=31$$

S: $0.2\times 0 = 0 \Rightarrow$ home

$0.8 \times 0 = 0 \Rightarrow$ remain

I: $0.2 \times 31 = 6 \Rightarrow$ home

$0.8 \times 31 = 25 \Rightarrow$ remain

R: $0.2 \times 69 = 14 \Rightarrow$ home

$0.8 \times 69 = 55 \Rightarrow$ remain

[S I R] = [18 27 55]

Case 2: Susceptible = 16, Infected = 4

0 month [0 1]

$$S(0) = 90, R(0) = 0, I(0) = 10$$

[S I R] = [90 10 0]

1st month [1 2]

$$S(1) = 0, R(1) = 31, I(1) = 69$$

[S I R] = [16 59 25]

2nd month [2 3]

$$S(2) = 0, R(2) = 54, I(2) = 46$$

[S I R] = [16 41 43]

3rd month [3 4]

$$S(3) = 0, R(3) = 64, I(3) = 36$$

[S I R] = [16 33 51]

4th month [0 1]

$$S(4) = 0, R(4) = 69, I(4) = 31$$

[S I R] = [16 29 55]

Case 3: Susceptible = 14, Infected = 6

0 month [0 1]

$$S(0) = 90, R(0) = 0, I(0) = 10$$

[S I R] = [90 10 0]

1st month [1 2]

$$S(1) = 0, R(1) = 31, I(0) = 69$$

[S I R] = [14 61 25]

2nd month [2 3]

$$S(2) = 0, R(2) = 54, I(2) = 46$$

[S I R] = [14 43 43]

3rd month [3 4]

$$S(3) = 0, R(3) = 64, I(3) = 36$$

[S I R] = [14 35 51]

4th month [4 5]

$$S(4) = 0, R(4) = 69, I(0) = 31$$

[S I R] = [14 31 55]

V. OBSERVATION FOR MODEL A

1. A close look at the plots in figures 1 – 4 show that at the beginning the infection rate increase and later fall towards the end with no number of susceptible and the removal rate is increased and reduced over a period of time. So between 0 and 20 people are susceptible and this implies that no number of susceptible people can go home or remain at the rig.

For the infected, 14 people will go home and 55 will remain and this implies that between 0 and 20 people of the new arrivals are infected. That is between 55 and 75 people are infected after the arrival of the new employees at the rig. So if between 1 and 14 people are infected it means we have the same number of people or less infected at the rig. Also more people will be infected at the rig if more than 14 are infected. Assuming no removal will arrive, then six people will go home and 25 remain.

For all the graphs plotted in the epidemic model the following colours represent the lines:

Blue: Susceptible

Green: Infected

Red: Removed

Light blue: Recovery

Yellow: Those available to work

S(0) = 90; I(0) = 10; R(0) = 0

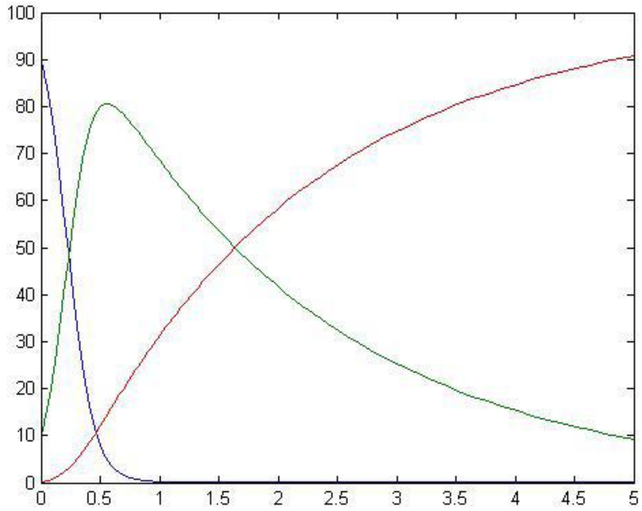

Figure 1: r = 0.1; a = 0.5

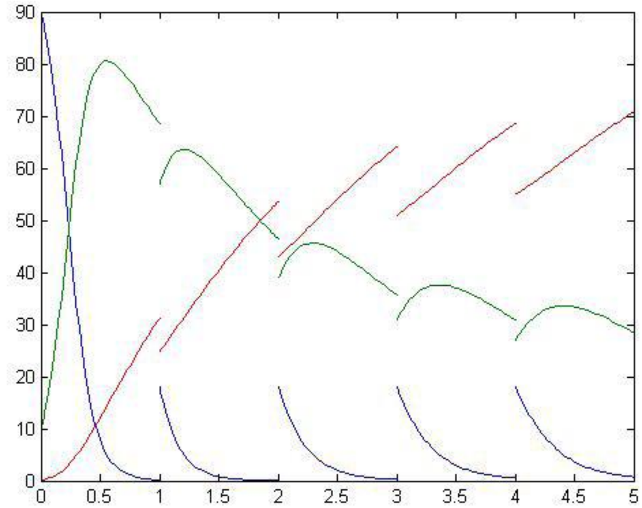

Figure 2: r = 0.1; a = 0.5

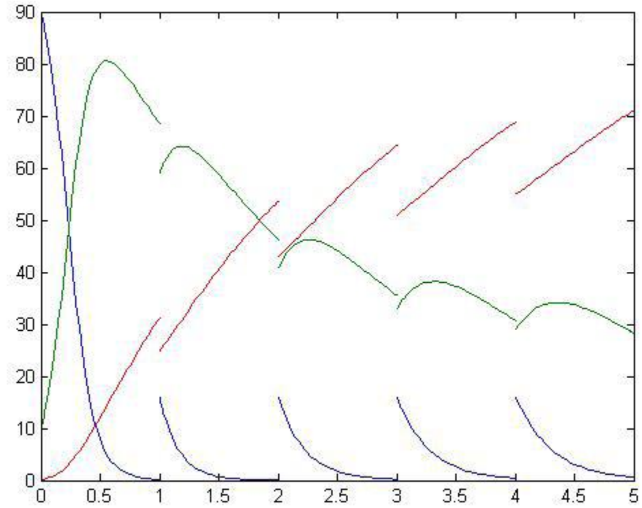

Figure 3: r = 0.1; a = 0.5

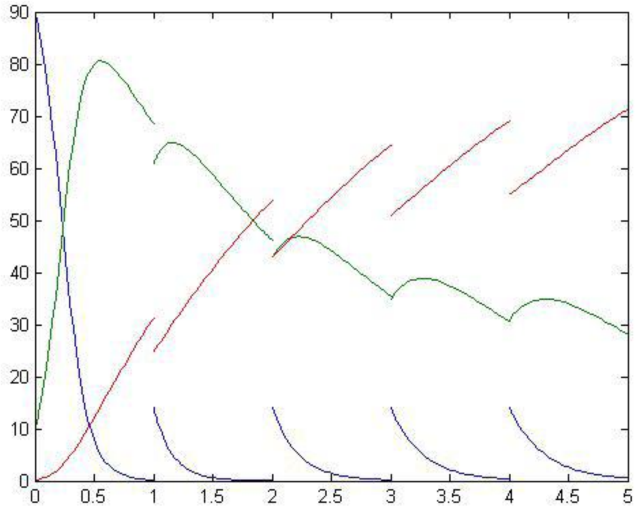

Figure 4: r = 0.1; a = 0.5

2. As there is increase in removal rate it is observed in figures 5 - 8 that the numbers of susceptible is the same for the period of time. For the infected, the rate is the same with reduction in the number of staff infected.

$$S(0) = 90, R(0) = 0, I(0) = 10$$

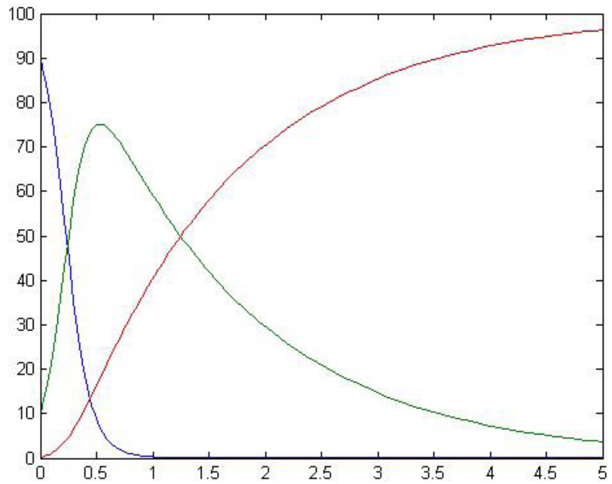

Figure 5: r = 0.1; a = 0.7

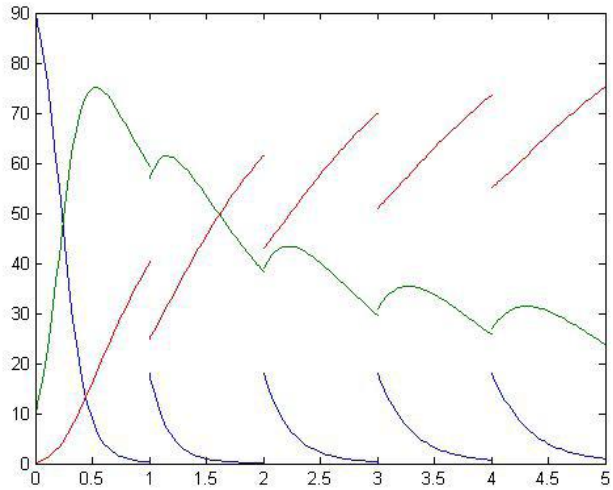

Figure 6: r = 0.1; a = 0.7

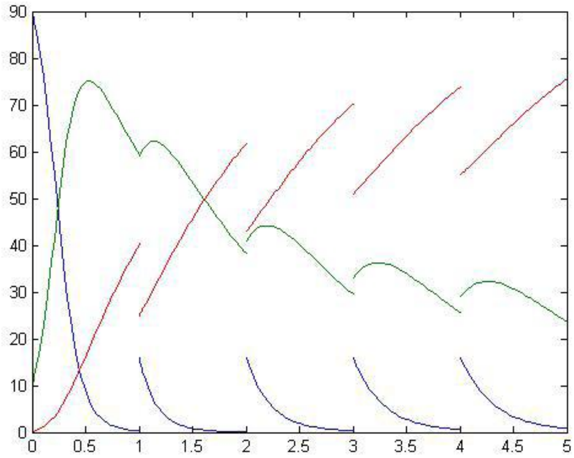

Figure 7: r = 0.1; a = 0.7

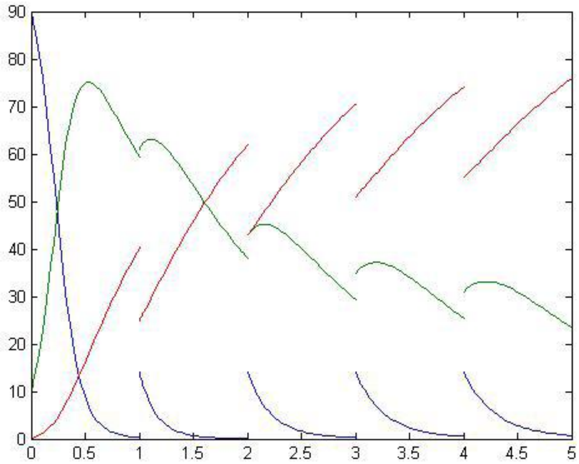

Figure 8: r = 0.1; a = 0.7

3. Figures 9 – 12 show that over a period of time, the infection rate is reduced. This means that there are less people being infected at the beginning. Also the maximum numbers of infected people are smaller to that of figures 1 – 4. Immediately after the maximum point the line is not so steep and number of infected people is a bit higher at the same time. So as there are less people infected at the same time in the beginning, less people will be removed. And towards the end, as more people are infected, more are removed. Hence the lines move closer.

$$S(0) = 90, R(0) = 0, I(0) = 10$$

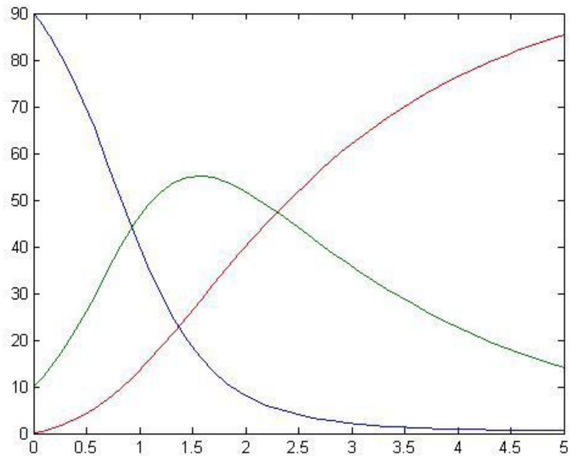

Figure 9: r = 0.03; a = 0.5

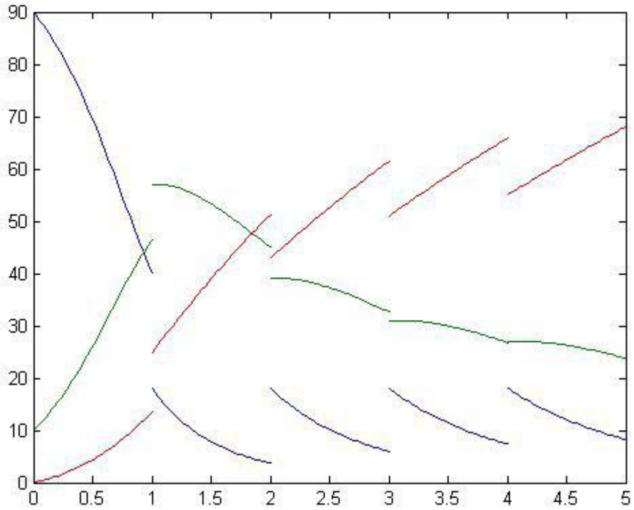

Figure 10: r = 0.03; a = 0.5

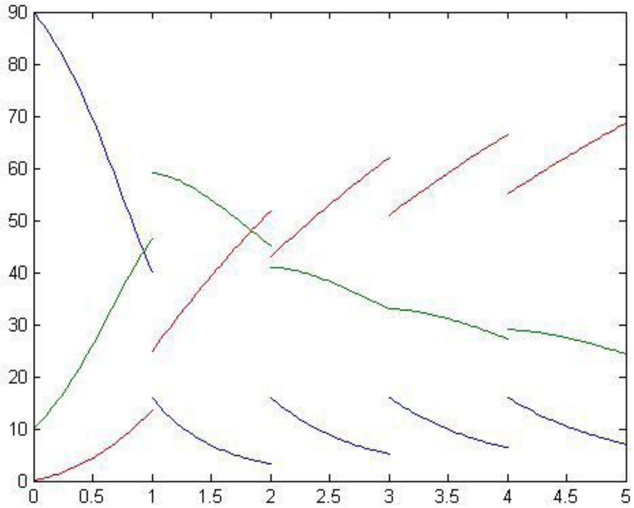

Figure 11: r = 0.03; a = 0.5

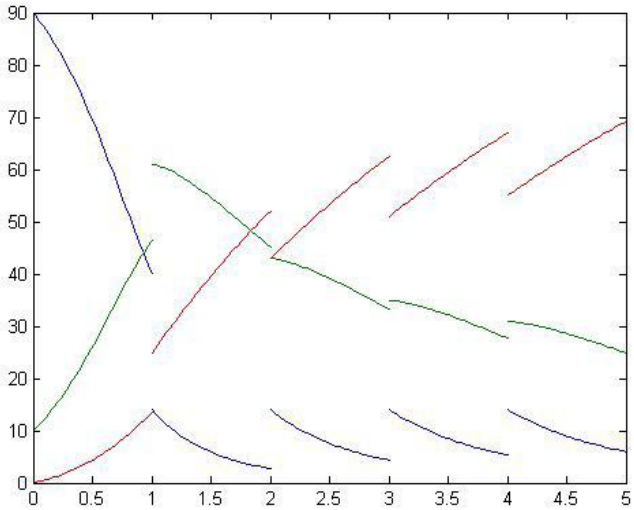

Figure 12: r = 0.03; a = 0.5

To ensure that there are fewer than 60% of the staff available for work at any one time, system 1 – 3 is modified and called **Model B**, i.e.

Model B

$$\frac{dS}{dt} = -rSI$$

$$\frac{dI}{dt} = rSI - aI \qquad \frac{dR}{dt} = aI - vR$$

$$\frac{dC}{dt} = vR$$

Where v is the recovery rate and C is the recovery group.

$r = 0.03, a = 0.7, v = 0.03$

VI. OBSERVATION FOR MODEL B

It is observed in figures 13 – 15 that fewer than 60% of staff will always be available for work assuming a constant recovery rate within the removed group as shown in Model B, and this is not in the best interest of Drilling company as their interest is to ensure that there are never fewer than 60% of the staff available for work at any time.

In figure 13, the recovery group (light blue line) is very low which implies that many of the infected are not recovered within the removed group. In figure 14 the recovery group falls a little bit without much significance on the infected being recovered. The number of susceptible increases a little bit at a constant recovery rate and figure 15 shows that the Drilling company will always have less than 60% of its staff available for work (yellow line) at any time.

S(0) = 90, I(0) = 10, R(0) = 0, C(0) = 0

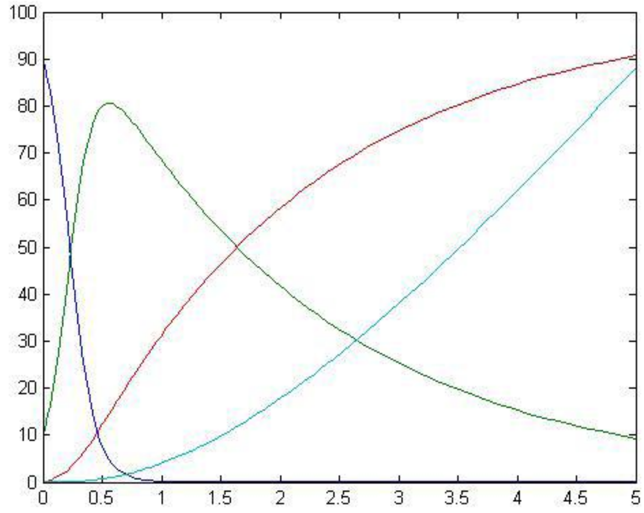

Figure 13: r = 0.1; a = 0.5; v = 0.3

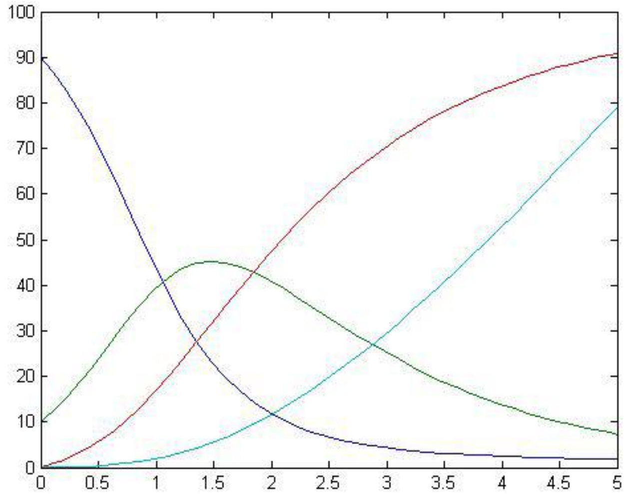

Figure 14: r = 0.03; a = 0.7; v = 0.3

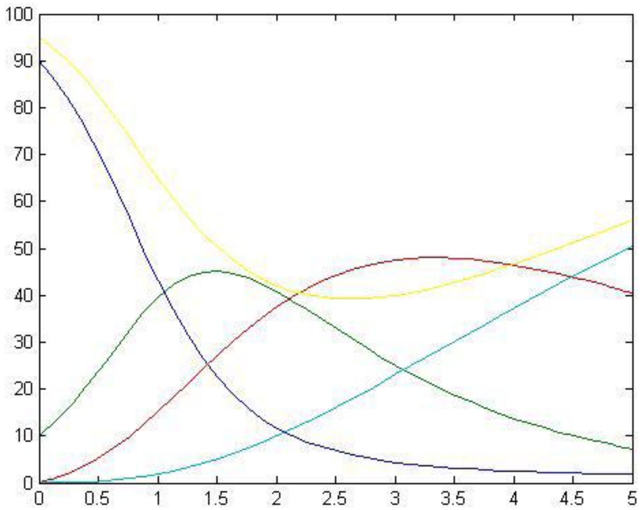

Figure 15: r = 0.03; a = 0.7; v = 0.3

VII. CONCLUSION

As a remedy to Drilling company situation, a thorough medical test should be carried out on the new comers of employees that are to replace the outgoing ones, to screen out those that may possibly have any trace of the disease before being introduced to the other staff working at the rig.

To ensure that more people are available for work, it is recommended that the company should make a research that all the 20 people leaving the rig are in the removed group. That is those who have had the disease and either died or the infected, that is those who are suffering from the disease.

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