International Journal of Modern Engineering Research (IJMER)
www.ijmer.com Vol. 3, Issue. 3, May.-June. 2013 pp-1520-1522 ISSN: 2249-6645

On pairs of Special Polygonal numbers with Unit difference

M. A. Gopalan[1], Manju Somanath[2], K.Geetha[3]
[1]Department of Mathematics, Shrimati Indira Gandhi college, Trichirapalli
[2]Department of Mathematics, National College, Trichirapalli
[3]Department of Mathematics, Cauvery College for Women, Trichirapalli

Abstract: *We obtain the ranks of m-gonal numbers such that the difference between any two special m-gonal numbers is unity. The recurrence relations satisfied by the ranks of each m-gonal numbers are also presented.*
Keyword: *Special polygonal numbers*
MSC classification code: *11D99*

I. INTRODUCTION

Number is the essence of mathematical calculation. Variety of numbers has variety of range and richness. Many numbers exhibit fascinating properties, they form sequences, they form patterns and also and so on [1, 2, 3]. In [4] explicit formulas for the ranks of Triangular number which simultaneously equal to Pentagonal, Octagonal, Decagonal and Dodecagonal numbers in turn are presented. Denoting the ranks of Triangular, Pentagonal, Hexagonal, Octagonal, and Heptagonal, decagonal and Dodecagonal number by the symbols N, P, Q, M, H, D and T respectively. In [5], the following relations are studied:

1. N- P = 1 2. N – M = 1 3. N – H = 1 4. N – D = 1
1. N – T = 1 6. P – M =1 7. P – Q = 1 8. P – H = 1

In [6], the ranks of m-gonal numbers such that the difference between any two m-gonal numbers is unity. The recurrence relations satisfied by the ranks of each m-gonal number in turn are presented. In this communication, we make an attempt to obtain the ranks of other special pairs of m-gonal numbers such that the difference between any two m-gonal numbers is unity. The recurrence relations satisfied by the ranks of m-gonal numbers are also presented.

II. Method of Analysis

1) Centered hexagonal number – Triangular number =1

Denoting the ranks of the centered hexagonal number and Triangular number to be A and M respectively, the identify is given by

Centered hexagonal number – Triangular number = 1 (1)

is written as $$y^2 = 6x^2 - 5 \tag{2}$$

where $$x = 2A+1,\ y = 2M+1 \tag{3}$$

whose initial solution is $x_0 = 3, y_0 = 7$

Let (x_s, y_s) be the general solution of the pellian

$$y^2 = 6x^2 + 1$$

where $$\tilde{x}_s = \frac{1}{2\sqrt{6}}\left(\left(5+2\sqrt{6}\right)^{s+1} - \left(5-2\sqrt{6}\right)^{s+1}\right)$$

$$\tilde{y}_s = \frac{1}{2}\left(\left(5+2\sqrt{6}\right)^{s+1} + \left(5-2\sqrt{6}\right)^{s+1}\right),\ s = 0,1,2...$$

Applying Brahmagupta's lemma between the solutions (x_0, y_0) and $(\tilde{x}_0, \tilde{y}_0)$ the sequence of values of x and y satisfying equation (2) is given by

$$x_s = \frac{1}{2\sqrt{6}}\left(\left(5+2\sqrt{6}\right)^{s+1}\left(7+3\sqrt{6}\right) - \left(5-2\sqrt{6}\right)^{s+1}\left(7-3\sqrt{6}\right)\right)$$

$$y_s = \frac{1}{2}\left(\left(5+2\sqrt{6}\right)^{s+1}\left(7+3\sqrt{6}\right) + \left(5-2\sqrt{6}\right)^{s+1}\left(7-3\sqrt{6}\right)\right),\ s = 0,1,2...$$

Inview of (3), the ranks of centered hexagonal number and Triangular number are respectively given by

$$A_s = \frac{1}{4\sqrt{6}}\left(\left(5+2\sqrt{6}\right)^{s+1}\left(7+3\sqrt{6}\right) - \left(5-2\sqrt{6}\right)^{s+1}\left(7-3\sqrt{6}\right) - 2\sqrt{6}\right)$$

$$M_s = \frac{1}{4}\left(\left(5+2\sqrt{6}\right)^{s+1}\left(7+3\sqrt{6}\right)+\left(5-2\sqrt{6}\right)^{s+1}\left(7-3\sqrt{6}\right)-2\right),\ s=0,1,2...$$

and their corresponding recurrence relations are found to be

$$A_{s+2} - 10A_{s+1} + A_s - 4 = 0$$

$$M_{s+2} - 10M_{s+1} + M_s - 4 = 0,\ s=0,1,2...$$

In similar manner, we present below the ranks of other special Polygonal numbers with unit difference in a tabular form

S. No	M-Gonal number	General forms of ranks
1	Centered heptagonal number(B) Triangular number(M)	$B_s = \frac{1}{4\sqrt{7}}\left(\left(8+3\sqrt{7}\right)^{s+1}\left(13+5\sqrt{7}\right)-\left(8-3\sqrt{7}\right)^{s+1}\left(13-5\sqrt{7}\right)-2\sqrt{7}\right)$ $M_s = \frac{1}{4}\left(\left(8+3\sqrt{7}\right)^{s+1}\left(13+5\sqrt{7}\right)+\left(8-3\sqrt{7}\right)^{s+1}\left(13-5\sqrt{7}\right)-2\right),\ s=0,1,2...$
2	Centered decagonal number(C) Triangular number(M)	$C_s = \frac{1}{4\sqrt{20}}\left(\left(9+2\sqrt{20}\right)^{s+1}\left(31+7\sqrt{20}\right)-\left(9-2\sqrt{20}\right)^{s+1}\left(31-7\sqrt{20}\right)-2\sqrt{20}\right)$ $M_s = \frac{1}{4}\left(\left(9+2\sqrt{20}\right)^{s+1}\left(31+7\sqrt{20}\right)+\left(9-2\sqrt{20}\right)^{s+1}\left(31-7\sqrt{20}\right)-2\right),$ $s=0,1,2...$
3	Centered hendecagonal number(D) Triangular number(M)	$D_s = \frac{1}{4\sqrt{11}}\left(\left(10+3\sqrt{11}\right)^{s+1}\left(23+7\sqrt{11}\right)-\left(10-3\sqrt{11}\right)^{s+1}\left(23-7\sqrt{11}\right)-2\sqrt{11}\right)$ $M_s = \frac{1}{4}\left(\left(10+3\sqrt{11}\right)^{s+1}\left(23+7\sqrt{11}\right)+\left(10-3\sqrt{11}\right)^{s+1}\left(23-7\sqrt{11}\right)-2\right),$ $s=0,1,2...$
4	Centered Tetradecagonal number(E) Triangular number(M)	$E_s = \frac{1}{4\sqrt{14}}\left(\left(15+4\sqrt{14}\right)^{s+1}\left(41+11\sqrt{14}\right)-\left(15-4\sqrt{14}\right)^{s+1}\left(41-11\sqrt{14}\right)-2\sqrt{14}\right.$ $M_s = \frac{1}{4}\left(\left(15+4\sqrt{14}\right)^{s+1}\left(41+11\sqrt{14}\right)+\left(15-4\sqrt{14}\right)^{s+1}\left(41-11\sqrt{14}\right)-2\right),$ $s=0,1,2...$
5	Centered Pentadecagonal number(F) Triangular number(M)	$F_s = \frac{1}{4\sqrt{15}}\left(\left(4+\sqrt{15}\right)^{s+1}\left(11+3\sqrt{15}\right)-\left(4-\sqrt{15}\right)^{s+1}\left(11-3\sqrt{15}\right)-2\sqrt{15}\right)$ $M_s = \frac{1}{4}\left(\left(4+\sqrt{15}\right)^{s+1}\left(11+3\sqrt{15}\right)+\left(4-\sqrt{15}\right)^{s+1}\left(11-3\sqrt{15}\right)-2\right),$ $s=0,1,2...$
6	Centered Icosagonal number(G) Triangular number(M)	$G_s = \frac{1}{4\sqrt{20}}\left(\left(9+2\sqrt{20}\right)^{s+1}\left(49+11\sqrt{20}\right)-\left(9-2\sqrt{20}\right)^{s+1}\left(49-11\sqrt{20}\right)-2\sqrt{20}\right.$ $M_s = \frac{1}{4}\left(\left(9+2\sqrt{20}\right)^{s+1}\left(49+11\sqrt{20}\right)+\left(9-2\sqrt{20}\right)^{s+1}\left(49-11\sqrt{20}\right)+2\right),$ $s=0,1,2...$

The recurrence relations satisfied by the ranks of each of the special polygonal numbers with unit difference presented in the table above are as follows

S. No	Recurrence relations
1	$B_{s+2}=16B_{s+1}-B_s+7;\ B_0=39, B_1=629$ $M_{s+2}=16M_{s+1}-M_s+7;\ M_0=104, M_1=1665$
2	$C_{s+2}=18C_{s+1}-C_s+8;\ C_0=62, C_1=1121$ $M_{s+2}=18M_{s+1}-M_s+8;\ M_0=279, M_1=5015$
3	$D_{s+2}=20D_{s+1}-D_s+9;\ D_0=69, D_1=1386$ $M_{s+2}=20M_{s+1}-M_s+9;\ M_0=230, M_1=4598$
4	$E_{s+2}=30E_{s+1}-E_s+14;\ E_0=164, E_1=4929$ $M_{s+2}=30M_{s+1}-M_s+14;\ M_0=615, M_1=18444$
5	$F_{s+2}=8F_{s+1}-F_s+3;\ F_0=44, F_1=90$ $M_{s+2}=8M_{s+1}-M_s+3;\ M_0=11, M_1=350$
6	$G_{s+2}=18G_{s+1}-G_s+8;\ G_0=98, G_1=1767$ $M_{s+2}=18M_{s+1}-M_s+8;\ M_0=440, M_1=7904$

III. Conclusion

To conclude, one may search for the other M-gonal numbers satisfying the relation under consideration.

References

[1] T.S.Bhanumathy, Ancient Indian Mathematics, New Age Publishers Ltd, New Delhi, 1995.
[2] L.E.Dikson, “History of Theory of Numbers”, Chelsa Publishing Company, New York, Vol.2, 1952.
[3] Shailesh Shirali, “Mathematical Marvels”, A Primer on number sequences, Universities Press, 2001.
[4] M.A.Gopalan and S.Devibala, “Equality of Triangular Numbers with Special m-gonal numbers”, Bulletin of Allahabad Mathematical Society, Vol.21, Pp.25-29, 2006.
[5] M.A.Gopalan, Manju Somanath, N.Vanitha, “ On pairs of m-gonal numbers with unit difference”, Advances in Theorectial and Applied Mathematics, Vol.1, No.3, Pp.197-200, 2006.
[6] M.A.Gopalan, Manju Somanath, N.Vanitha, “On pairs of m-gonal numbers with unit difference”, Reflections des ERA-JMS, Vol.4, Issue4, Pp.365-376, 2009.