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# On Contra-#Rg–Continuous Functions

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***Abstract:*** *In this paper we introduce and investigate some classes of generalized functions called contra-#rg- continuous functions. We get several characterizations and some of their properties. Also we investigate its relationship with other types of functions.*

***Mathematics subject classification:*** *54C08, 54C10*

***Keywords****: #rg- closed, #rg- continuous, #rg- irresolute.*

## I. Introduction

In 1996, Dontchev [1] presented a new notion of continuous function called contra – continuity. This notion is a stronger form of LC – continuity. The purpose of this present paper is to define a new class of generalized continuous functions called contra #rg-continuous functions and almost contra #rg-continuous and investigate their relationships to other functions.

## II. Preliminaries

Throughout the paper X and Y denote the topological spaces (X, $\tau$) and (Y, $\sigma$) respectively and on which no separation axioms are assumed unless otherwise explicitly stated. For any subset A of a space (X,τ), the closure of A, interior of A and the complement of A are denoted by cl (A), int (A) and $A^c$ or X\A respectively. (X, τ) will be replaced by X if there is no chance of confusion. Let us recall the following definitions as pre requesters.

**Definition 2.1.** A subset A of a space X is called
1) a preopen set [2] if A $\subseteq$ intcl (A) and a preclosed set if clint (A)$\subseteq$ A.
2) a semi open set [3] if A $\subseteq$ clint (A) and a semi closed set if intcl (A) $\subseteq$A.
3) a regular open set [4] if A = intcl (A) and a regular closed set if A = clint (A).
4) a regular semi open [5] if there is a regular open U such U $\subseteq$A$\subseteq$cl(U).

**Definition 2.2**. A subset A of (X,τ) is called
1) generalized closed set (briefly, g-closed)[6] if cl (A)$\subseteq$ U whenever A$\subseteq$U and U is open in X.
2) regular generalized closed set (briefly, rg-closed)[7] if cl (A)$\subseteq$U whenever A$\subseteq$U and U is regular open in X.
3) generalized preregular closed set (briefly, gpr-closed)[8] if pcl (A)$\subseteq$U whenever A $\subseteq$ U and U is regular open in X.
4) regular weakly generalized closed set (briefly, rwg-closed)[9] if clint(A)$\subseteq$U whenever A $\subseteq$U and U is regular open in X.
5) rw-closed [10] if cl(A)$\subseteq$U whenever A$\subseteq$U and U is regular semi open.

The complements of the above mentioned closed sets are their respective open sets.

**Definition 2.3.** A subset A of a space X is called #rg-closed[11] if cl(A)$\subseteq$U whenever A$\subseteq$U and U is rw-open. The complement of #rg-closed set is #rg-open set. The family of #rg-closed sets and #rg-open sets are denoted by #RGC(X) and #RGO(X).

**Definition 2.4**. A map f :( X,τ) →(Y,σ) is said to be
(i) #rg-continuous [12] if $f^{-1}$(V) is #rg-closed in (X,τ) for every closed set V in (Y,σ).
(ii) #rg-irresolute [12] if $f^{-1}$ (V) is #rg-closed in (X,τ) for each #rg-closed set V of (Y,σ).
(iii) #rg-closed [12] if f(F) is #rg-closed in (Y,σ) for every #rg-closed set F of (X,τ).
(iv) #rg-open[12] if f(F) is #rg-open in (Y,σ) for every #rg-open set F of (X,τ).
(v) #rg-homeomorphism [13] if f is bijection and f and $f^{-1}$ are #rg-continuous.

**Definition 2.5.** A map f:(X,τ)→(Y,σ) is said to be contra- continuous [1] if $f^{-1}$(V) is closed in (X,τ) for every open set V in (Y,σ).

**Definition 2.6.** A space X is called a $T_{\#rg}$-space [11] if every #rg-closed set in it is closed.

## III. Contra #rg–Continuous Function

In this section, we introduce the notions of contra #rg- continuous, contra #rg-irresolute and almost contra #rg-continuous functions in topological spaces and study some of their properties.

**Definition 3.1**
A function f:(X,τ ) → (Y,σ) is called contra #rg- continuous if $f^{-1}$ (V) is #rg-closed set in X for each open set V in Y.

**Example 3.2**
Let X = Y = { a,b,c } with topologies τ = { X, ϕ, {a} } and σ = { Y , ϕ, {b , c} }.Define f:( X,τ )→(Y,σ) by an identity function. Clearly f is contra #rg – continuous.

**Example 3.3**
Let X = Y = {a,b,c} with τ = { X,ϕ,{a} ,{b},{a,b} } and σ = { Y,ϕ,{a,b} }. Define f:X→Y by f(a) = c, f(b) = b and f(c) = a. Clearly f is contra #rg – continuous.

**Remark 3.4.**
The family of all #rg-open sets of X is denoted by #RGO(X). The set #RGO(X,x)={V∈#RGO(X) / x∈V }for x ∈ X.

**Theorem 3.5**
Every contra – continuous function is contra #rg-continuous.
**Proof:** It follows from the fact that every closed set is #rg-closed set.
The converse of the above theorem is not true as seen from the following example.

**Example 3.6**
Let X = Y = {a,b,c,d } with τ = { X,ϕ,{a},{b},{a,b},{a,b,c}} and σ = { Y,ϕ, {a,b}}. Define f: X→Y by f(a) = a, f(b) = d, f(c)=c and f(d)=b. Here f is contra #rg-continuous but not contra continuous since $f^{-1}$ ({a, b}) = {a, d} which is not closed in X.

**Theorem 3.7**
If a function f: X →Y is contra #rg-continuous and X is $T_{\#rg}$- Space. Then f is contra continuous.
**Proof**: Let V be an open set in Y. Since f is contra #rg-continuous, $f^{-1}$(V) is closed in X. Hence f is contra –continuous.

**Remarks 3.8**
The concept of #rg – continuity and contra #rg continuity are independent as shown in the following examples.

**Example 3.9**
Let X = Y = {a, b, c, d }. τ = {X,ϕ,{a},{b},{a,b},{a,b,c}}and σ= {Y,ϕ,{a},{b},{a,b}}. Define f: X→Y by identity mapping then clearly f is #rg – continuous. Since $f^{-1}$({a}) = {a} is not #rg-closed in X where {a} is open in X.

**Example 3.10**
Let X = Y = {a,b,c,d } , τ = {x,ϕ,{a},{b},{a,b},{a,b,c} }and σ={Y,ϕ,{a,b}}. Define f: X→Y by f (a)=a, f (b) = d, f (c)=c and f(d) = b. Here f is contra #rg-continuous, but not #rg-continuous, because $f^{-1}$({c,d}) = {b,c} is not #rg-closed in X, where {a,d} is closed in Y.

**Theorem 3.11**
Every contra - #rg- continuous function is contra g – continuous.
**Proof.** Since every #rg – closed set is g- closed, the proof follows.
The converse of the theorem is need not be true as shown in the following example.

**Example 3.12**
Let X = Y = { a,b,c,d },τ = {X,ϕ,{a},{b},{a,b } } and σ= {Y,ϕ,{a},{b}, {a,b,c} }. A function f: X→Y defined by f(a)=c, f(b)=d, f(c)=a and f(d)=b. Clearly f is contra g – continuous but not contra #rg –continuous since $f^{-1}${c} = {c} is not #rg-closed.

**Remark 3.13**
1. Every contra #rg-continuous is contra *g –continuous
2. Every contra #rg –continuous is contra rg – continuous
3. Every contra #rg –continuous is contra –gpr – continuous
4. Every contra #rg – continuous is contra – rwg-continuous.

**Remark 3.14**
The composition of two contra - #rg-continuous functions need not be contra #rg –continuous as seen from the following example.

**Example 3.15**

Let X = Y = Z = { a,b,c}, $\tau$ = { X,$\phi$,{a},{b},{a,b} }, $\sigma$= {Y,$\phi$,{a,b}} and $\eta$={Z,$\phi$,{a}}. Let f:X $\rightarrow$ Y defined by f(a)=c, f(b)=b, f(c)=a and g:Y$\rightarrow$Z is defined by g(a)=b, g(b)=c and g(c)=a. Then clearly f and g are contra #rg – continuous. But gof:X$\rightarrow$Z is not contra #rg continuous, since $(gof)^{-1}$ {a} = $f^{-1}(g^{-1}\{a\})=f^{-1}(\{c\})=\{a\}$ which is not #rg – closed in X.

**Theorem 3.16**

If f: (X,$\tau$) $\rightarrow$ (Y,$\sigma$ ) is contra #rg–continuous and g: Y $\rightarrow$ Z is a continuous function, then gof: X$\rightarrow$Z is contra #rg-continuous.

**Proof:** Let V be open in Z. Since g is continuous, $g^{-1}$(V) is open in Y. Then $f^{-1}(g^{-1}$ (V)) is #rg- closed in X, since f is contra #rg – continuous. Thus $(gof)^{-1}$ (V) is #rg – closed in X. Hence gof is contra #rg – continuous.

**Corollary 3.17**

If f : X $\rightarrow$ Y is #rg – irresolute and g : Y $\rightarrow$ Z is contra – continuous function then gof:X$\rightarrow$Z is contra #rg – continuous.

**Proof.** Using the fact that every contra – continuous is contra #rg – continuous

**Theorem 3.18**

Let f: X $\rightarrow$Y be surjective, #rg – Irresolute and #rg – open and g: Y $\rightarrow$Z be any function then gof is contra #rg – continuous iff g is contra #rg – continuous.

**Proof.** Suppose gof is contra #rg continuous. Let F be an open set in Z. Then $(gof)^{-1}(F) =f^{-1}(g^{-1}(F) )$ is #rg – open in X. Since f is #rg – open and surjective $f(f^{-1}(g^{-1}(V) )$ is #rg –open in Y. (i.e.) $g^{-1}(V)$ is #rg –open in Y. Hence g is contra #rg – continuous. Conversely, suppose that g is contra #rg – continuous. Let V be closed in Z. Then $g^{-1}(V)$ is #rg –open in .Since f is #rg – irresolute, $f^{-1}(g^{-1}(V) )$ is #rg – open. (i.e.) $(gof)^{-1}$ (V) is #rg – open in X. Hence gof is contra #rg – continuous.

**Theorem 3.19**

Let f : X $\rightarrow$ Y be a map. Then the following are equivalent.

(i) f is contra #rg – continuous.

(ii) The inverse image of each closed set in Y is #rg – open in X.

**Proof :** (i) $\Rightarrow$(ii) & (ii) $\Rightarrow$ (i) are obvious.

**Theorem 3.20**

If f : X $\rightarrow$ Y is contra #rg- continuous then for every x $\in$ X, each F $\in$ C(Y,f(x) ), there exists U $\in$ #RGO (X,x), such that f(U)$\subset$F (i.e.) For each x$\in$X, each closed subset F of Y with f(x) $\in$ F, there exists a #rg- open set U of X such that x $\in$U and f(U)$\in$F.

**Proof .**Let f: X $\rightarrow$ Y be contra #rg – continuous. Let F be any closed set of Y and f(x) $\in$ F where x $\in$ X. Then $f^{-1}$ (F) is #rg – open in X, also x$\in f^{-1}$(F ). Take U = $f^{-1}$(F). Then U is a #rg-open set containing x and f(U) $\subseteq$ F.

**Theorem 3.21**

If a function f: X $\rightarrow$Y is contra #rg –continuous and X is $T_{\#rg}$ – space then f is contra continuous.

**Proof.** Let V be an open set in Y. Since f is contra #rg- continuous, $f^{-1}$(V) is #rg- closed in X .Then $f^{-1}$(V) is closed in X , since X is $T_{\#rg}$- space . Hence f is contra – continuous.

**Corollary 3.22**

If X is a $T_{\#rg}$-Space then for a function f:X$\rightarrow$Y the following are equivalent.

(i) f is contra continuous

(ii) f is contra #rg-continuous.

**Proof:** It is obvious.

**Theorem 3.23**

Let (X,$\tau$ ) be a #rg-connected space and (Y,$\sigma$) be any topological space. If f: X$\rightarrow$Y is surjective and contra #rg-continuous, then Y is not a discrete space.

**Proof.** Suppose Y is discrete space. Let A be any proper non empty subset of Y. Then A is both open and closed in Y. Since f is contra #rg-continuous $f^{-1}$ (A) is both #rg open and #rg-closed in X. Since X is #rg- connected, the only subsets of X which are both #rg-open and #rg-closed are X and $\phi$. Hence $f^{-1}$(A) = X, then it contradicts to the fact that f: X $\rightarrow$ Y is surjective. Hence Y is not a discrete space.

**Definition 3.24**

A function f: X $\rightarrow$Y is called almost contra #rg-continuous if $f^{-1}$(V) is #rg-closed set in X for every regular open set V in Y.

**Theorem 3.25**

Every contra #rg-continuous function is almost contra #rg-continuous but not conversely.

**Proof**: Since every regular open set is open, the proof follows.

**Definition 3.26**
A function f: X → Y is called contra #rg-irresolute if $f^{-1}$(V) is #rg-closed in X for each #rg-open set V in Y.

**Definition 3.27**
A function f: X → Y is called perfectly contra #rg-irresolute if $f^{-1}$(V) is #rg-closed and #rg-open in X for each #rg-open set V in Y.

**Theorem 3.28**
A function f: X→Y is perfectly contra #rg-irresolute if and only if f is contra #rg-irresolute and #rg-irresolute.
**Proof:** It directly follows from the definitions.

**Remark 3.29**
The following example shows that the concepts of #rg irresolute and contra #rg – irresolute are independent of each other.

**Example 3.30**
Let X = Y = { a,b,c,d }, τ={X,ϕ,{c},{a,b},{a,b,c } } and σ= {Y,ϕ,{a},{b}, {a,b,} }. A function f: X→Y defined by f(a)= f(b)=a, f(c)=d and f(d)=b. Clearly f is contra #rg–irresolute but not #rg –irresolute, since $f^{-1}$({b}) = {d} which is not #rg-open in X.

**Example 3.31**
Let X = Y = {a,b,c,d}, τ={X,ϕ,{c},{a,b},{a,b,c}} and σ= {Y,ϕ,{a},{b}, {a,b,}}. Define f: X→Y by an identity function. Clearly f is #rg–irresolute but not contra #rg –irresolute, since $f^{-1}$({b}) = {b} which is not #rg-closed in X.

**Remark 3.32**
Every contra #rg-irresolute function is contra #rg-continuous. But the converse need not be true as seen from the following example.

**Example 3.33**
Let X = Y = {a,b,c,d } with τ = { X,ϕ,{a},{b},{a,b},{a,b,c}} and σ = { Y,ϕ, {a,b}}. Define f:X→Y by f(a) = a, f(b) = d, f(c)=c and f(d)=b. Here f is contra #rg-continuous but not contra #rg- irresolute.

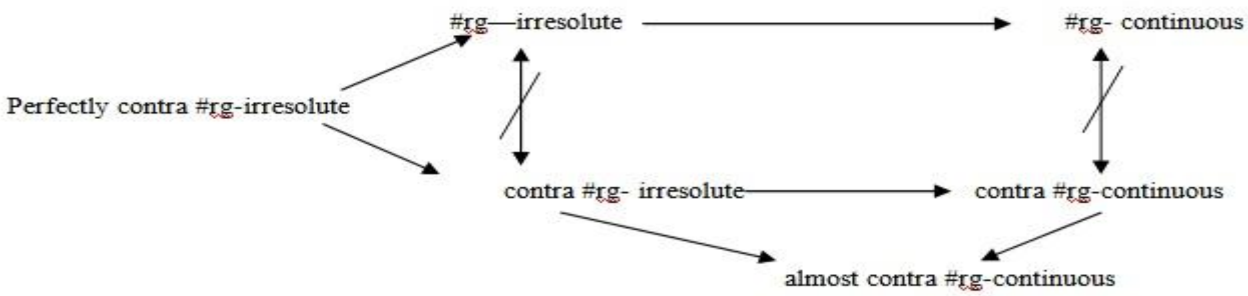
**Theorem 3.34**
Let f:X→Y and g:Y→Z be a function then
(i) if g is #rg-irresolute and f is contra #rg-irresolute then gof is contra #rg-irresolute.
(ii) If g is contra #rg-irresolute and f is #rg- irresolute then gof is contra #rg-irresolute.
**Proof.**(i) Let U be a #rg-open in Z. Since g is #rg-irresolute, $g^{-1}$(U) is #rg-open in Y. Thus $f^{-1}(g^{-1}(U))$ is #rg-closed in X, since f is contra #rg-irresolute. (i.e.) $(gof)^{-1}$(U) is #rg-closed in X. This implies that gof is contra #rg-irresolute.
(ii) Let U be a #rg-open in Z. Since g is contra #rg-irresolute, $g^{-1}$(U) is #rg-closed in Y. Thus $f^{-}(g^{-1}(U))$ is #rg-closed in X, since f is #rg-irresolute. (i.e.) $(gof)^{-1}$(U) is #rg-closed in X. This implies that gof is contra #rg-irresolute.

**Theorem 3.35**
If f: X→Y is contra #rg-irresolute and g:Y→Z is #rg-continuous then gof is contra #rg-continuous.
**Proof.** Let U be an open set in Z. Since g is #rg- continuous, $g^{-1}$(U) is #rg-open in Y. Thus $f^{-1}(g^{-1}(U))$ is #rg-closed in X, since f is contra #rg-irresolute. (i.e.) $(gof)^{-1}$(U) is #rg-closed in X. This implies that gof is contra #rg-irresolute.

**Remark 3.36**
Every perfectly contra #rg-irresolute function is contra #rg-irresolute and #rg-irresolute. The following two examples shows that a contra #rg-irresolute function may not be perfectly contra #rg-irresolute and a #rg-irresolute function may not be perfectly contra #rg-irresolute.
In example.3.30, f is contra #rg-irresolute but not perfectly contra #rg-irresolute.
In example 3.31, f is #rg-irresolute but not perfectly contra #rg-irresolute.

**Theorem 3.37.**
A function is perfectly contra #rg-irresolute iff f is contra #rg-irresolute and #rg-irresolute.
**Proof.** It directly follows from the definitions.

**Remark 3.37** From the above results we have the following diagram where A→B represent A implies B but not conversely.

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