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On wgr □-Continuous Functions in Topological Spaces

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Abstract: *In this paper, we introduce new type of continuous functions called strongly wgrα-continuous and perfectly wgrα-continuous and study some of its properties. Also we introduce the concept of wgrα-compact spaces and wgrα-connected spaces and some their properties are analyzed.*

Subject Classification: *54C05, 54C10.*

Keywords: *perfectly wgrα-continuous, strongly wgrα-continuous, wgrα-compact spaces and wgrα-connected spaces.*

I. Introduction

Balachandran et al in [9, 10] introduced the concept of generalized continuous maps of a topological space. A property of gpr continuous functions was discussed by Y.Gnanambal and Balachandran K [5]. Strong forms of continuity and generalization of perfect functions were introduced and discussed by T.Noiri [11, 12]. Regular α-open set is introduced by A.Vadivel and K. Vairamanickam [14]. Rg-compact spaces and rg-connected spaces, τ^*-generalized compact spaces and τ^*-generalized connected spaces, gb-compactness and gb-connectedness introduced by A.M.Al.Shibani [1], S.Eswaran and A.Pushpalatha [4], S.S.Benchalli and Priyanka M.Bansali [2] respectively. In this paper we establish the relationship between perfectly wgrα-continuous and strongly wgrα-continuous. Also we introduce the concept of wgrα-compact spaces and wgrα-connected spaces and study their properties using wgrα-continuous functions.

II. Preliminary Definitions

Definition: 2.1
A subset A of a topological space (X, τ) is called α-closed [10] if a ⊂ int (cl (int (A)).

Definition:2.2
A subset A of a topological space (X, τ) is called gα-closed [9] if αcl (A) ⊂ U,when ever A ⊂ U and U is α-open in X.

Definition: 2.3
A subset A of a topological space (X, τ) is called rwg-closed [14] if cl (int (A)) ⊂ U, whenever A ⊂ U and U is regular-open in X.

Definition: 2.4
A map f: X→Y is said to be continuous [3] if $f^{-1}(V)$ is closed in X for every closed set V in Y.

Definition: 2.5
A map f: X→Y is said to be wgrα- continuous [6] if $f^{-1}(V)$ is wgrα-closed in X for every closed set V in Y.

Definition: 2.6
A map f: X→Y is said to be perfectly-continuous [12] if $f^{-1}(V)$ is clopen in X for every open set V in Y.

Definition: 2.7
A map f: X→Y is said to be strongly-continuous [8] if $f^{-1}(V)$ is clopen in X for every subset V in Y.

Definition: 2.8
A function f: X →Y is called wgrα- irresolute [6] if every $f^{-1}(V)$ is wgrα-closed in X for every wgrα-closed set V of Y.

Definition: 2.9
A function f: X → Y is said to be wgrα-open [7] if f(V) is wgrα-open in Y for every open set V of X.

Definition: 2.10
A function f: X → Y is said to be pre wgrα-open [7] if f(V) is wgrα-open in Y for every wgrα-open set V of X.

Definition: 2.11
A space (X,τ) is called wgrα-$T_{1\backslash 2}$ space[7] if every wgrα-closed set is α-closed.

Definition: 2.12
A space (X,τ) is called $T_{wgr\alpha}$-space[7] if every wgrα-closed set is closed.
The complement of the above mentioned closed sets are their respective open sets.

III. Strongly Wgr □-Continuous and Perfectly Wgr □-Continuous Functions

Definition: 3.1
A function f :(X,τ) → (Y,σ) is called strongly wgrα-continuous if $f^{-1}(V)$ is open in (X,τ) for every wgrα-open set V of (Y,σ).

Definition: 3.2
A function f :(X,τ) → (Y,σ) is called perfectly wgrα-continuous if $f^{-1}(V)$ is clopen in (X,τ) for every wgrα-open set V of (Y,σ).

Definition: 3.3

A function $f:(X,\tau)\rightarrow(Y,\sigma)$ is called strongly wgrα- irresolute if $f^{-1}(V)$ is open in (X,τ) for every wgrα-open set V of (Y,σ).

Definition: 3.4

A function $f:(X,\tau)\rightarrow(Y,\sigma)$ is called strongly rwg-continuous if $f^{-1}(V)$ is open in (X,τ) for every rwg-open set V of (Y,σ).

Definition: 3.5

A function $f:(X,\tau)\rightarrow(Y,\sigma)$ is called perfectly rwg-continuous if $f^{-1}(V)$ is clopen in (X,τ) for every rwg-open set V of (Y,σ).

Theorem: 3.6

If a function $f:(X,\tau)\rightarrow(Y,\sigma)$ is perfectly wgrα-continuous, then f is perfectly continuous.

Proof

Let F be any open set of (Y,σ).Since every open set is wgrα-open.We get that F is wgrα-open in (Y,σ).By assumption , we get that $f^{-1}(F)$ is clopen in (X,τ). Hence f is perfectly continuous.

Theorem: 3.7

If f: $(X,\tau)\rightarrow(Y,\sigma)$ is strongly wgrα-continuous, then it is continuous.

Proof

Let U be any open set in (Y,σ).Since every open set is wgrα-open, U is wgrα-open in (Y,σ).Then $f^{-1}(U)$ is open in (X,τ). Hence f is continuous.

Remark: 3.8

Converse of the above theorem need not be true as seen in the following example.

Example: 3.9

Let $X=\{a,b,c,d\},\tau=\{\phi,X,\{a\},\{c,d\},\{a,c,d\}\}$ and $\sigma=\{\phi,Y,\{a\},\{b,c\},\{a,b,c\}\}$. Define $f:X\rightarrow Y$ by f(a)=a, f(b)=d, f(c)=c, f(d)=b. Here f is continuous, but it is not strongly wgrα-continuous.

Theorem: 3.10

Let (X,τ) be any topological space and (Y,σ) be a $T_{wgr\alpha}$-space and f: $(X,\tau)\rightarrow(Y,\sigma)$ be a map. Then the following are equivalent:

(i) f is strongly wgrα-continuous.

(ii) f is continuous.

Proof

(i) $\Rightarrow$(ii) Let U be any open set in (Y,σ).Since every open set is wgrα-open,U is wgrα-open in (Y,σ).Then $f^{-1}(U)$ is open in (X,τ). Hence f is continuous.

(ii)$\Rightarrow$(i) Let U be any wgrα-open set in (Y,σ).Since (Y,σ) is a $T_{wgr\alpha}$-space,U is open in (Y,σ).Since f is continuous. Then $f^{-1}(U)$ is open in (X,τ). Hence f is strongly wgrα-continuous.

Theorem: 3.11

If f: $(X,\tau)\rightarrow(Y,\sigma)$ is strongly rwg -continuous, then it is strongly wgrα- continuous.

Proof

Let U be any wgrα-open set in (Y,σ).By hypothesis,$f^{-1}(U)$ is open and closed in (X,τ). Hence f is strongly wgrα-continuous.

Remark: 3.12

Converse of the above theorem need not be true as seen in the following example.

Example: 3.13

Let $X=\{a,b,c\},\tau=\sigma=\{\phi,X,\{a\},\{b\},\{a,b\}\}$.Define map $f:X\rightarrow Y$ is an identity map. Here f is strongly wgrα-continuous, but it is not strongly rwg-continuous.

Theorem: 3.14

Let f: $(X,\tau)\rightarrow(Y,\sigma)$ be a map.Both (X,τ) and (Y,σ) are $T_{wgr\alpha}$-space . Then the following are equivalent:

(i)f is wgrα-irresolute.

(ii)f is strongly wgrα-continuous.

(iii) f is continuous.

(iv) f is wgrα-continuous.

Proof

Straight forward.

Theorem: 3.15

If $f:(X,\tau)\rightarrow(Y,\sigma)$ is strongly wgrα-continuous and A is open subset of X, then the restriction $f|A:A\rightarrow Y$ is strongly wgrα-continuous.

Proof

Let V be any wgrα-closed set of Y. Since f is strongly wgrα-continuous,then $f^{-1}(V)$ is open in (X,τ).Since A is open in X,$(f|A)^{-1}(V)=A\cap f^{-1}(V)$ is open in A. Hence f|A is strongly wgrα-continuous.

Theorem: 3.16

If a function $f:(X,\tau)\rightarrow(Y,\sigma)$ is perfectly wgrα-continuous, then f is strongly wgrα-continuous.

Proof

Let F be any wgrα-open set of (Y,σ).By assumption, we get that $f^{-1}(F)$ is clopen in(X,τ),which implies that $f^{-1}(F)$ is closed and open in (X,τ).Hence f is strongly wgrα-continuous.

Remark: 3.17

Converse of the above theorem need not be true as seen in the following example.

Example: 3.18

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Let X={a,b,c},τ={ϕ,X,{a},{c},{a,c}}= σ. Define f:X → Y by f(a)=a, f(b)=b, f(c)=c. Here f is strongly wgrα-continuous, but it is not perfectly wgrα-continuous.

Theorem: 3.19

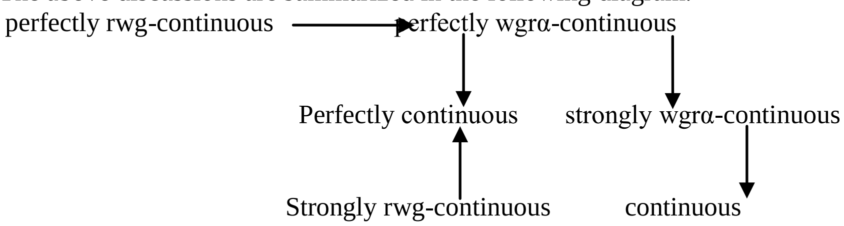
If f: (X,τ) → (Y,σ) is perfectly rwg-continuous, then it is perfectly wgrα-continuous.

Proof

As f is strongly continuous,$f^{-1}(U)$ is both open and closed in (X,τ) for every wgrα-open set U in (Y,σ). Hence f is perfectly wgrα-continuous.

Remark: 3.20

The above discussions are summarized in the following diagram.



Theorem: 3.21

Let (X,τ) be a discrete topological space and (Y,σ) be any topological space .Let f: (X,τ) → (Y,σ) be a map. Then the following statements are equivalent:

(i) f is strongly wgrα-continuous.

(ii) f is perfectly wgrα-continuous.

Proof

(i) ⟹ (ii) Let U be any wgrα-open set in (Y,σ).By hypothesis $f^{-1}(U)$ is open in (X,τ).Since(X,τ) is a discrete space,$f^{-1}(U)$ is also closed in (X,τ). $f^{-1}(U)$ is both open and closed in (X,τ). Hence f is perfectly wgrα- continuous.

(ii) ⟹ (i) Let U be any wgrα-open set in (Y,σ).Then $f^{-1}(U)$ is both open and closed in (X,τ). Hence f is strongly wgrα-continuous.

Theorem:3.22

If f:(X,τ) → (Y,σ) and g:(Y,σ) → (Z,μ) are perfectly wgrα-continuous ,then their composition

g ∘ f:(X,τ) → (Z,μ) is also perfectly wgrα-continuous.

Proof

Let U be a wgrα-open set in (Z,μ).Since g is perfectly wgrα-continuous, we get that $g^{-1}(U)$ is open and closed in (Y,σ).As any open set is wgrα-open in (X,τ) and f is also strongly wgrα-continuous, $f^{-1}(g^{-1}(U))= (g \circ f)^{-1}(U)$ is both open and closed in (X,τ). Hence g ∘ f is perfectly wgrα-continuous.

Theorem:3.23

If f:(X,τ) → (Y,σ) and g:(Y,σ) → (Z,μ) be any two maps. Then their composition g ∘ f: (X,τ) → (Z,μ) is

(i) wgrα-irresolute if g is strongly wgrα-continuous and f is wgrα-continuous.

(ii) Strongly wgrα-continuous if g is perfectly wgrα-continuous and f is continuous.

(iii) Perfectly wgrα-continuous if g is strongly wgrα-continuous and f is perfectly wgrα-continuous.

Proof

(i)Let U be a wgrα-open set in (Z,μ). Then $g^{-1}(U)$ is open in (Y,σ).Since f is wgrα-continuous, $f^{-1}(g^{-1}(U))=(g \circ f)^{-1}(U)$ is wgrα-open in (X,τ). Hence g ∘ f is wgrα-irresolute.

(ii)Let U be any wgrα-open set in (Z,μ).Then $g^{-1}(U)$ is both open and closed in (Y,σ) and therefore $f^{-1}(g^{-1}(U))= (g \circ f)^{-1}(U)$ is both open and closed in (X,τ). Hence g ∘ f is strongly wgrα-continuous.

(iii) Let U be any wgrα-open set in (Z,μ).Then $g^{-1}(U)$ is open and closed in (Y,σ).By hypothesis, $f^{-1}(g^{-1}(U))$ is both open and closed in (X,τ).Hence g ∘ f is perfectly wgrα-continuous.

Theorem: 3.24

If f:(X,τ) → (Y,σ) and g:(Y,σ) → (Z,μ) are strongly wgrα-continuous ,then their composition g ∘ f:(X,τ) → (Z,μ) is also strongly wgrα-continuous.

Proof

Let U be a wgrα-open set in (Z,μ).Since g is strongly wgrα-continuous, we get that $g^{-1}(U)$ is open in (Y,σ).It is wgrα-open in (Y,σ) As f is also strongly wgrα-continuous,$f^{-1}(g^{-1}(U))= (g \circ f)^{-1}(U)$ is open in (X,τ). Hence g ∘ f is continuous.

Theorem:3.25

If f:(X,τ) → (Y,σ) and g:(Y,σ) → (Z,μ) be any two maps. Then their composition g ∘ f :(X,τ) → (Z,μ) is (i) strongly wgrα-continuous if g is strongly wgrα-continuous and f is continuous.

(ii) wgrα-irresolute if g is strongly wgrα-continuous and f is wgrα-continuous.

(iii) Continuous if g is wgrα-continuous and f is strongly wgrα-continuous.

Proof

(i) Let U be a wgrα-open set in (Z,μ).Since g is strongly wgrα-continuous, $g^{-1}(U)$ is open in (Y,σ).Since f is continuous, $f^{-1}(g^{-1}(U))= (g \circ f)^{-1}(U)$ is open in (X,τ). Hence g ∘ f is strongly wgrα-continuous.

(ii) Let U be a wgrα-open set in (Z,μ).Since g is strongly wgrα-continuous, $g^{-1}(U)$ is open in (Y,σ).As f is wgrα-continuous, $f^{-1}(g^{-1}(U))= (g\circ f)^{-1}(U)$ is wgrα-open in (X,τ). Hence $g\circ f$ is wgrα-irresolute.

(iii)Let U be any open set in (Z,μ).Since g is wgrα-continuous, $g^{-1}(U)$ is wgrα-open in (Y,σ).As f is strongly wgrα-continuous, $f^{-1}(g^{-1}(U))= (g\circ f)^{-1}(U)$ is open in (X,τ). Hence $g\circ f$ is continuous.

Theorem: 3.26

Let f: $(X,\tau)\rightarrow(Y,\sigma)$ and g: $(Y,\tau)\rightarrow(Z,\eta)$ be two mappings and let $g\circ f :(X,\tau)\rightarrow(Z,\eta)$ be wgrα-closed.If g is strongly wgrα-irresolute and bijective, then f is closed.

Proof

Let A be closed in (X,τ),then $(g\circ f)(A)$ is wgrα-closed in (Z,η).Since g is strongly wgrα-irresolute,$g^{-1}(g\circ f)(A)=f(A)$ is closed in (Y,σ).Hence f(A) is closed.

Theorem: 3.27

If f :$(X,\tau)\rightarrow(Y,\sigma)$ is perfectly wgrα-continuous and A is any subset of X, then the restriction
$f|A:A\rightarrow Y$ is also perfectly wgrα-continuous.

Proof

Let V be any wgrα-closed set in(Y,σ). Since f is perfectly wgrα-continuous, $f^{-1}(V)$ is both open and closed in (X,τ).$(f|A)^{-1}(V)=A\cap f^{-1}(V)$ is both open and closed in A. Hence f|A is perfectly wgrα-continuous.

IV. Wgr -Compact Spaces

Definition: 4.1

A collection $\{A_\alpha:\alpha\in\nabla\}$ of wgrα-open sets in a topological space X is called wgrα-open cover of a subset B of X if $B\subset\cup\{A_\alpha:\alpha\in\nabla\}$ holds.

Definition: 4.2

A topological space (X,τ) is wgrα-compact if every wgrα-open cover of X has a finite subcover.

Definition: 4.3

A subset B of X is called wgrα-compact relative of X if for every collection $\{A_\alpha:\alpha\in\nabla\}$ of wgrα-open subsets of X such that $B\subset\{A_\alpha:\alpha\in\nabla\}$,there exists a finite subset $\nabla_\circ$ of ∇ such that $B\subset\cup\{A_\alpha:\alpha\in\nabla_\circ\}$.

Definition: 4.4

A subset B of X is said to be wgrα-compact if B is wgrα-compact subspace of X.

Theorem: 4.5

Every wgrα-closed subset of a wgrα-compact space is wgrα-compact space relative to X.

Proof

Let A be wgrα-closed subset of X, then A^c is wgrα-open. Let $O=\{G_\alpha:\alpha\in\nabla\}$ be a cover of A by wgrα-open subsets of X.Then $W=O\cup A^C$ is an wgrα-open cover of X. That is $X=(\cup\{G_\alpha:\alpha\in\nabla\})\cup A^C$.By hypothesis, X is wgrα-compact. Hence W has a finite subcover of X say $(G_1\cup G_2\cup G_3\cup\cdots\cup G_n)\cup A^C$. But A and A^C are disjoint, hence $A\subset G_1\cup G_2\cup\cdots\cup G_n$. So O contains a finite subcover for A, therefore A is wgrα-compact relative to X.

Theorem: 4.6

Let f: $X\rightarrow Y$ be a map:

(i) If X is wgrα-compact and f is wgrα-continuous bijective, then Y is compact.

(ii)If f is wgrα-irresolute and B is wgrα-compact relative to X, then f(B) is wgrα-compact relative to Y.

Proof

(i)Let f: $X\rightarrow Y$ be an wgrα-continuous bijective map and X be an wgrα-compact space. Let $\{A_\alpha:\alpha\in\nabla\}$ be an open cover for Y.Then $\{f^{-1}(A_\alpha):\alpha\in\nabla\}$ is an wgrα-open cover of X. Since X is wgrα-compact, it has finite subcover say $\{f^{-1}(A_1),f^{-1}(A_2),\ldots,f^{-1}(A_n)\}$,but f is surjective,so $\{A_1,A_2,\ldots,A_n\}$ is a finite subcover of Y.
Therefore Y is compact.

(ii) Let $B\subset X$ be wgrα-compact relative to X,$\{A_\alpha:\alpha\in\nabla\}$ be any collection of wgrα-open subsets of Y such that $f(B)\subset\cup\{A_\alpha:\alpha\in\nabla\}$.Then $B\subset\cup\{f^{-1}(A_\alpha):\alpha\in\nabla\}$. By hypothesis, there exists a finite subset $\nabla_\circ$ of ∇ such that $f(B)\subset\cup\{A_\alpha: \alpha\in\nabla\}$.Then $B\subset\cup\{ f^{-1}(A_\alpha):\alpha\in\nabla\}$.By hypothesis, there exists a finite subset $\nabla_\circ$ of ∇ such that $B\subset\cup\{f^{-1}(A_\alpha):\alpha\in\nabla_\circ\}$.Therefore, we have $f(B)\subset\cup\{A_\alpha:\alpha\in\nabla_\circ\}$ which shows that f(B) is wgrα-compact relative to Y.

Theorem: 4.7

If f: $X\rightarrow Y$ is prewgrα-open bijection and Y is wgrα-compact space, then X is a wgrα-compact space.

Proof

Let $\{U_\alpha : \alpha\in\nabla\}$ be a wgrα-open cover of X.So X= $\bigcup_{\alpha\in\nabla} U_\alpha$ and then Y=f(X) =f($\bigcup_{\alpha\in\nabla} U_\alpha$) = $\bigcup_{\alpha\in\nabla} f(U_\alpha)$ .Since f is prewgrα-open, for each $\alpha\in\nabla$,f(U_α) is wgrα-open set. By hypothesis, there exists a finite subset $\nabla_\circ$ of ∇ such that Y= $\bigcup_{\alpha\in\nabla_\circ} f(U_\alpha)$

Therefore, X=f^{-1}(Y)=f^{-1}($\bigcup_{\alpha\in\nabla_\circ} f(U_\alpha)$)= $\bigcup_{\alpha\in\nabla_\circ} U_\alpha$ This shows that X is wgrα-compact.

Theorem: 4.8

If f:X $\rightarrow$ Y is wgrα-irresolute bijection and X is wgrα-compact space, then Y is a wgrα-compact space.

Proof

Let $\{U_\alpha:\alpha\in\nabla\}$ be a wgrα-open cover of Y. So Y= $\bigcup_{\alpha\in\nabla} U_\alpha$ and then X=f^{-1}(Y)= f^{-1}($\bigcup_{\alpha\in\nabla} U_\alpha$) = $\bigcup_{\alpha\in\nabla} f^{-1}(U_\alpha)$ .Since f is wgrα-irresolute, it follows that for each $\alpha\in\nabla$, $f^{-1}(U_\alpha)$ is wgrα-open set. By wgrα-compactness of X, there exists a finite subset $\nabla_\circ$ of ∇ such that X= $\bigcup_{\alpha\in\nabla_\circ} f^{-1}(U_\alpha)$. Therefore, Y=f(X)=f($\bigcup_{\alpha\in\nabla_\circ} f^{-1}(U_\alpha)$)= $\bigcup_{\alpha\in\nabla_\circ} U_\alpha$ .This shows that Y is wgrα-compact.

Theorem: 4.9

A wgrα-continuous image of a wgrα-compact space is compact.

Proof

Let f: X $\rightarrow$ Y be a wgrα-continuous map from a wgrα-compact space X onto a topological space Y. let $\{A_i:i\in\nabla\}$ be an open cover of Y.Then $\{f^{-1}(A_i):i\in\nabla\}$ is wgrα-open cover of X. Since X is wgrα-compact,it has finite subcover,say$\{f^{-1}(A_1),f^{-1}(A_2),\ldots,f^{-1}(A_n)\}$.Since f is onto,$\{A_1,A_2,\ldots,A_n\}$ and so Y is compact.

Theorem: 4.10

A space X is wgrα-compact if and only if each family of wgrα-closed subsets of X with the finite intersection property has a non-empty intersection.

Proof

X is wgrα-compact and A is any collection of wgrα-closed sets with F.I.P. Let A =$\{F_\alpha:\alpha\in\nabla\}$ be an arbitrary collection of wgrα-closed subsets of X with F.I.P, so that $\bigcap\{F_{\alpha_i}:i\in\nabla_o\}\neq\phi$ →(1)**,**we have to prove that the collection A has non-empty intersection ,that is, $\bigcap\{F_\alpha:\alpha\in\nabla\}\neq\phi$→(2).Let us assume that the above condition does not hold and hence $\bigcap\{F_\alpha:\alpha\in\nabla\}=\phi$.Taking complements of both sides,we get $\bigcup\{F_\alpha^C:\alpha\in\nabla\}$=X→(3).But each F_α being wgrα-closed,which implies that F_α^C is wgrα-open and hence from (3),we conclude that C=$\{F_\alpha^C:\alpha\in\nabla\}$ is an wgrα open cover of X. Since X is wgrα-compact, this cover C has a finite subcover. C =$\{F_{\alpha_i}:i\in\nabla_o\}$ is also an open subcover. Therefore X=$\bigcup\{F_{\alpha_i}^C:i\in\nabla_o\}$.Taking complement , we get $\phi=\bigcap\{F_{\alpha_i}:i\in\nabla_o\}$which is a contradiction of (1).Hence $\bigcap\{F_\alpha:\alpha\in\nabla\}\neq\phi$. Conversely, suppose any collection of wgrα-closed sets with F.I.P has a empty intersection.Let C =$\{G_\alpha:\alpha\in\nabla\}$,where G_α is a wgrα-open cover of X and hence X=$\bigcup\{G_\alpha:\alpha\in\nabla\}$**.**Taking complements, we have $\phi=\bigcap\{G_\alpha^C:\alpha\in\nabla\}$**.**But G_α^C is wgrα-closed. Therefore the class A of wgrα-closed subsets with empty intersection.So that it does not have F.I.P. Hence there there exists a finite number of wgrα-closed sets $G_{\alpha_i}^C$ such that $i\in\nabla_o$ with empty intersection. That is,$\{G_{\alpha_i}:i\in\nabla_o\}=\phi$.Taking complement, we have $\{G_{\alpha_i}^C:i\in\nabla_o\}$= X **.**Therefore C of X has an open subcover C***=**$\{G_{\alpha_i}:i\in\nabla_o\}$.Hence (X,τ) is compact.

Theorem :4.11

If f: (X,τ)→(Y,σ) is a strongly wgrα-continuous onto map, where (X,τ) is a compact space, then (Y,σ) is wgrα-compact.

Proof

Let $\{A_i : i\in\nabla\}$ be a wgrα-open cover of (Y,σ).Since f is strongly wgrα-continuous,$\{f^{-1}(A_i : i\in\nabla\}$ is an open cover (X,τ).As (X,τ) is compact, it has a finite subcover say,$\{f^{-1}(A_1),f^{-1}(A_2),\ldots, f^{-1}(A_n)\}$ and since f is onto, $\{A_1,A_2,\ldots,A_n\}$ is a finite subcover of (Y,σ) and therefore (Y,σ) is wgrα-compact.

Theorem :4.12

If a map f: (X,τ)→(Y,σ) is a perfectly wgrα-continuous onto map, where (X,τ) is compact, then (Y,σ) is wgrα-compact.

Proof

Since every perfectly wgrα-continuous function is strongly wgrα-continuous. Therefore by theorem 4.11, (Y,σ) is wgrα-compact .

V. Wgr -Connected Spaces

Definition: 5.1

A Space X is said to be wgrα-connected if it cannot be written as a disjoint union of two non-empty wgrα-open sets.

Definition: 5.2
A subset of X is said to be wgrα-connected if it is wgrα-connected as a subspace of X.
Definition: 5.3
A function f: (X,τ) $\rightarrow$ (Y,σ) is called contra wgrα-continuous if $f^{-1}(V)$ is wgrα-closed in (X,τ) for each open set V in (Y,σ).
Theorem: 5.4
For a space X, the following statements are equivalent
(i)X is wgrα-connected.
(ii)X and φ are the only subsets of X which are both wgrα-open and wgrα-closed.
(iii)Each wgrα-continuous map of X into some discrete space Y with atleast two points is a constant map.
Proof
(i) $\Rightarrow$(ii) Let X be wgrα-connected. Let A be wgrα-open and wgrα-closed subset of X. Since X is the disjoint union of the wgrα-open sets A and A^C, one of these
sets must be empty. That is, A =φ or A=X.
(ii) $\Rightarrow$(i) Let X be not wgrα-connected, which implies X=A $\cup$ B ,where A and B are disjoint non-empty wgrα-open subsets of X. Then A is both wgrα-open and wgrα-closed. By assumption A=φ or A=X, therefore X is wgrα-connected.
(ii)$\Rightarrow$(iii) Let f:X$\rightarrow$Y be wgrα –continuous map from X into discrete space Y with atleast two points, then{ f^{-1}(y):y∊ Y} is a cover of X by wgrα-open and wgrα-closed sets. By assumption, f^{-1}(y)=φ or X for each y∊ Y. If f^{-1}(y)=φ for all y∊ Y, then f is not a map. So there exists a exactly one point y∊ Y such that $f^{-1}(y)\neq \varphi$ and hence f^{-1}(y)
=X. This shows that f is a constant map.
(iii)$\Rightarrow$(ii) Let O≠φ be both an wgrα-open and wgrα-closed subset of X.Let f:X$\rightarrow$Y be wgrα-continuous map defined by f(O)={y} and f(O^C)={ω} for some distinct
points y and ω in Y. By assumption f is constant, therefore O=X.
Theorem: 5.5
Let f: X$\rightarrow$Y be a map:
(i) If X is wgrα-connected and f is wgrα-continuous surjective, then Y is connected.
(ii) If X is wgrα-connected and f is wgrα-irresolute surjective, then Y is wgrα-connected.
Proof
(i) If Y is not connected, then Y= A$\cup$B, where A and B are disjoint non-empty open subsets of Y. Since f is wgrα-continuous surjective, therefore X=$f^{-1}(A) \cup f^{-1}(B)$, where f^{-1}(A) and f^{-1}(B) are disjoint non-empty wgrα-open subsets of X. This contradicts the fact that X is wgrα-connected. Hence, Y is connected.
(ii)Suppose that Y is not wgrα-connected, then Y=A$\cup$B, where A and B are disjoint non-empty wgrα-open subsets of Y. Since f is wgrα-irresolute surjective, therefore X=$f^{-1}(A) \cup f^{-1}(B)$,where f^{-1}(A),f^{-1}(B) are disjoint non-empty wgrα-open subsets of X. So X is not wgrα-connected, a contradiction.
Theorem: 5.6
A contra wgrα-continuous image of a wgrα-connected space is connected.
Proof
Let f: (X,τ)$\rightarrow$(Y,σ) be a contra wgrα-continuous from a wgrα-connected space X onto a space Y. Assume Y is not connected. Then Y=A$\cup$B, where A and B are non-empty closed sets in Y with A$\cap$B=φ. Since f is contra wgrα-continuous,we have that f^{-1}(A) and f^{-1}(B) are non-empty wgrα-open sets in X with $f^{-1}(A) \cup f^{-1}(B)= f^{-1}(A \cup B)$
=f^{-1}(Y)=X and $f^{-1}(A) \cap f^{-1}(B)=f^{-1}(A \cap B)= f^{-1}(\varphi)$.This means that X is not wgrα-connected, which is a contradiction. This proves the theorem.
Theorem: 5.7
Every wgrα-connected space is connected.
Proof
Let X be an wgrα-connected space. Suppose X is not connected. Then there exists a proper non-empty subset B of X which is both open and closed in X. Since every closed set is wgrα-closed,B is a proper non-empty subsets of X which is both wgrα-open and wgrα-closed in X. Therefore X is not wgrα-connected. This proves the theorem.
Remark: 5.8
Converse of the above theorem need not be true as seen in the following example.
Example: 5.9
Let X={a,b,c} ,τ={φ,{a,b},X}.{X,τ} is connected. But {a} and {b} are both wgrα-closed and wgrα-open, X is not wgrα-connected.
Theorem: 5.10
Let X be a $T_{wgr\alpha}$-space. Then X is wgrα-connected if X is connected.
Proof
Suppose X is not wgrα-connected. Then there exists a proper non-empty subset B of X which is both wgrα-open and wgrα-closed in X. Since X is $T_{wgr\alpha}$-space, B is both open and closed in X and hence X is not connected.
Theorem: 5.11
Suppose X is wgrα-$T_{1\backslash 2}$ space. Then X is wgrα-connected if and only if X is gα-connected
Proof
Suppose X is wgrα-connected. X is gα-connected.

Conversely, we assume that X is gα-connected. Suppose X is not wgrα-connected. Then there exists a proper non-empty subset B of X which is both wgrα-open and wgrα-closed in X .Since X is wgrα-$T_{1\backslash 2}$ -space is both α-open and α-closed in X. Since α-closed set is gα-closed in X, B is not gα-connected in X, which is a contradiction. Therefore X is wgrα-connected.

Theorem: 5.12

In a topological space (X,τ) with at least two points , if αO(X,τ)=αC(X,τ),then X is not wgrα-connected.

Proof

By hypothesis, we have αO(X,τ)=αC(X,τ) and by the result,we have every α-closed set is wgrα-closed, there exists some non-empty proper subset of X which is both wgrα-open and wgrα-closed in X . So by theorem 5.4, we have X is not wgrα-connected.

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