Statistical Optimization for Generalised Fuzzy Number

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Abstract: *In this paper an attempt has been made to propose a method based on Pascal triangular graded mean representation to solve the two-objective fuzzy assignment problem under fuzzy environment. In this, costs and times are to be fuzzy variables. And we apply statistical interpretation for new approach to solve the fuzzy assignment problem. Here we take the coefficients of fuzzy numbers from Pascal triangles and develop a new procedure to solve fuzzy assignment problem. Given through the numerical example*

Keywords: *Assignment problem, Triangular Fuzzy number, Pascal triangular graded mean representation.*

I. Introduction

Assignment problem is a common problem in the real system. For this problem in deterministic environment, a lot of models and algorithms have been presented up to now. In recent years, many researchers began to investigate this kind of problem under the uncertain environment .Tadei and Ricciardi [13] given the solutions of the multilevel assignment problems. Bogomolnaia.A and Moulin.A (1&2) given a new solution for random assignment problem and developed a procedure for a simple assignment problem with unique solution. The another uncertainty in the real world is fuzziness. In order to deal with fuzziness, Zadeh [14] gave an information about the the fuzzy set theory in 1965. For the assignment problem, it may be also considered under the fuzzy environment, which causes the research of fuzzy assignment problem. Constructive bounds and exact expectation of the random assignment problem was discussed by Coppersmith and Sorkin (4). Lin and Wen [7] framed and designed a labeling algorithm for it. Ridwan [8] did a simple preference based traffic assignment problem in fuzzy approach. A multi-criteria fuzzy assignment method was introduced by Belacela and Boulasselb (3).

In a linear programming problem, the assignment problem is a particular case to assign the different jobs to different workers. It has been discussed with the situation in which jobs are to be assigned to a persons for execution. The assignment problem is derived in terms of Linear programming formulation was given by D. Konig [5]. Let a number of n jobs be given that must be performed by m workers, where the costs and the times depend on the specific assignments. Each job must be assigned one worker and each worker has to perform one and only one job so that the total cost and the total time is minimized after all the jobs are completed. S.Muruganandam et.al given a special procedure to solve the two-objective assignment problem through the graded mean integration representation. The Hungarian method [6], which is the most popular, a very convenient and efficient iterative procedure for solving an assignment problem.

Based on the uncertainity, in decision-making, we always treat some parameters as uncertain variables, which cannot obtain their concrete values. In this paper, we consider a two objective assignment problem under fuzzy environment in which the costs and the times are supposed to be fuzzy variables. This paper is organized as follows: In section 2, we construct the mathematical model for the problem. In section3, the methodology is introduced. In section 4, a statistical implementations are given. In section 5, a numerical example is given and in section 6, we give a conclusion for our problem.

II. Mathematical Model

Assume there are n jobs and m workers. For the convenience of description, the notations i = 1, 2, ……….m and j = 1, 2,………..n are used to denote the indexes of the different workers and jobs. The purpose of the two objective assignment problem in this paper is n jobs and m workers, so that the total cost and the total time is minimized after all the jobs are completed. If worker i has ability to undertake some jobs and we think that he will probably produce much less profit or consume very long time, in such conditions worker i may be deprived of opportunity to undertake these jobs.

In the process of decision making, if job j is assigned to worker i, then the corresponding costs and consumed times, denoted by $\tilde{c}_{ij}$ and $\tilde{t}_{ij}$, $i = 1,2,3,……..m$, $j = 1,2,3……n$, respectively. Generally, the optimal plan is made before the jobs are completed, thus it is impossible for us to know the concrete values of $\tilde{c}_{ij}$ and $\tilde{t}_{ij}$ in advance. In order to obtain a decision, $\tilde{c}_{ij}$ and $\tilde{t}_{ij}$ ($i = 1,2,3,……..m$, $j = 1,2,3……n$) have been given. Then the cost matrix and the consumed time matrix may be denoted by

$$\tilde{C}=(\tilde{c}_{ij})_{nxn}=\begin{pmatrix} \bar{c}_{11} & \bar{c}_{12} & . & . & . & \bar{c}_{1n} \\ \bar{c}_{21} & \bar{c}_{22} & . & . & . & \bar{c}_{2n} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ \bar{c}_{m1} & \bar{c}_{m2} & . & . & . & \bar{c}_{mn} \end{pmatrix} \qquad \tilde{T}=(\tilde{t}_{ij})_{nxn}=\begin{pmatrix} \bar{t}_{11} & \bar{t}_{12} & . & . & . & \bar{t}_{1n} \\ \bar{t}_{21} & \bar{t}_{22} & . & . & . & \bar{t}_{2n} \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ . & . & . & . & . & . \\ \bar{t}_{m1} & \bar{t}_{m2} & . & . & . & \bar{t}_{mn} \end{pmatrix}$$

In order to model the above fuzzy assignment problem, the following variables are employed:

$$x_{ij}=\begin{cases} 1 & if\ wor\ker i\ is\ to\ assigned\ to\ job\ j \\ 0 & otherwise \end{cases}$$

Where $i = 1,2,\ldots\ldots\ldots,m,$ $j = 1,2,\ldots\ldots\ldots\ldots\ldots\ldots,n$

The two objective fuzzy assignment problem can now be stated as equation:

$$\text{Minimize} \quad \sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{c}_{ij}\bar{x}_{ij}$$

$$\text{Maximize} \quad \sum_{i=1}^{m}\sum_{j=1}^{n}\tilde{t}_{ij}\bar{x}_{ij}$$

$$\text{Subject to} \quad \sum_{i=1}^{m}\bar{x}_{ij}=1 \quad j = 1, 2, 3\ldots\ldots\ldots\ldots m$$

$$\sum_{i=1}^{n}\bar{x}_{ij}=1 \quad i = 1, 2, 3\ldots\ldots\ldots\ldots n$$

$$\bar{x}_{ij} = 0 \text{ or } 1$$

Since the elements of the cost matrix and the consumed time matrix are fuzzy variables, it follows that the total cost and the total consumed time are also fuzzy variables. In order to optimize the objective, we introduce the methodology in the next section.

III. Methodology

3.1 Triangular Fuzzy Number

A fuzzy number A is a triangular fuzzy number denoted by (a_1, a_2, a_3) and its membership function $\mu_A(x)$ is given below:

$$\mu_A(x)=\begin{cases} \dfrac{x-a_1}{a_2-a_1}, & a_1 < x < a_2 \\ \dfrac{x-a_3}{a_2-a_3}, & a_2 < x < a_3 \end{cases}$$

3.2 Triangular Fuzzy Number

Chen [9] has been introduced function principle for fuzzy arithmetical operations. We shall use this principle as the operation of addition, multiplication, subtraction and division of fuzzy numbers.

Suppose $\tilde{a}=(a_1,a_2,a_3)$ and $\tilde{b}=(b_1,b_2,b_3)$ are two triangular fuzzy numbers. Then

(i) $\tilde{a}+\tilde{b}=(a_1,a_2,a_3)+(b_1,b_2,b_3)=(a_1+b_1,a_2+b_2,a_3+b_3)$

(ii) $\tilde{a}-\tilde{b}=(a_1,a_2,a_3)-(b_1,b_2,b_3)=(a_1-b_1,a_2-b_2,a_3-b_3)$

(iii) The multiplication of $\tilde{a}$ and $\tilde{b}$ is $\tilde{a}\times\tilde{b}=(c_1,c_2,c_3)$ where $T=\{a_1b_2,a_1b_3,a_3b_1,a_3b_3\}$

$$C_1=\min T,\ C_2=a_2b_2,\ C_1=\max T.$$

If a_1, a_2, a_3,b_1, b_2, b_3 are non-zero positive real numbers, then

$$\tilde{a}\times\tilde{b}=(a_1,a_2,a_3)\times(b_1,b_2,b_3)=(a_1b_1,a_2b_2,a_3b_3)$$

(iv) $\frac{1}{\tilde{b}} = \tilde{b}^{-1} = \left(\frac{1}{b_3}, \frac{1}{b_2}, \frac{1}{b_1}\right)$ Where b_1, *b2*, *b3are* all non-zero positive real numbers, then the division of $\tilde{a}$ and $\tilde{b}$ is

$$\frac{\tilde{a}}{\tilde{b}} = \left(\frac{a_1}{b_1}, \frac{a_2}{b_2}, \frac{a_3}{b_1}\right)$$

(v) Let $k \in R.$ Then for $k(a_1, a_2, a_3) = (ka_1, ka_2, ka_3)$ for $k \geq 0$.

3.3 Graded Mean Integration Representation Method

Chen and Hseih [10, 11, 12] propose graded mean integration representation for representing generalized fuzzy number. Then S.Muruganandam described for generalized fuzzy number.

Suppose L^{-1} and R^{-1} are inverse functions of functions *L* and *R*, respectively and the graded mean h-level of generalized fuzzy number is $A = (a_1, a_2, a_3, a_4 : w)$ is $h[L^{-1}(h) + R^{-1}]/2$. Then the defuzzified value P (A) by graded mean integration representation of generalized fuzzy number based on the integral value of graded mean *h* - level is

$$P(A) = \frac{\int_0^h [\frac{L^{-1}(h) + R^{-1}(h)}{2}]\, dh}{\int_0^w h\, dh}$$

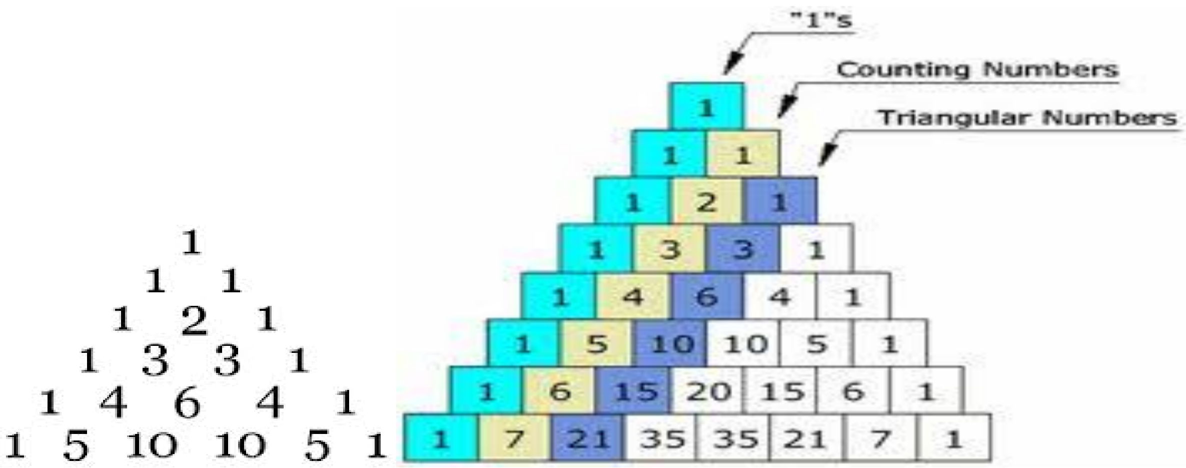
Where *h* is between 0 and *w*, $0 < w \leq 1$.

If $A = (a_1, a_2, a_3)$ is a triangular fuzzy number. Chen and Hsieh [10, 11, 12] already find the general formulae of the representation of generalized triangular fuzzy number as follows:

$$P(A) = \frac{1}{2} \frac{\int_0^1 \int h[a_1 + h(a_2 - a_1) - h(a_3 - a_2)]dh}{\int_0^1 h dh} \quad ; P(A) = \frac{a_1 + 4a_2 + a_3}{6}$$

3.4. Pascal triangular graded mean approach

Chen and Hsieh [10,11,12] proposed graded mean integration representation for representing generalized fuzzy number and S.Muruganandam etal described one more graded mean representation for generalized fuzzy number. This is useful to obtain the solution of the generalized fuzzy variables for assignment problem. But, the present approach is very simple way of analyzing multi objective fuzzy variables for assignment problem then we apply Hungarian method we get the optimum solution. Here, we apply simple statistical tests to analyze and compare the above two methods. This approach is also applicable for multi-objective fuzzy assignment problem. But , this procedure is simply taken from the following Pascal triangles. These are useful to take the coefficients of fuzzy variables are Pascal triangular numbers and we just add and divided by the total of Pascal numbers, then we call it as Pascal's triangular probability approach. This is also the alternative procure for graded mean representation.



Let the are two triangular fuzzy numbers then we can take the coefficient of fuzzy numbers as pascal triangular type and apply the simple probability approach.

Let the $\tilde{a}=(a_1,a_2,a_3)$ and $\tilde{b}=(b_1,b_2,b_3)$ are two triangular fuzzy numbers then we can take the coefficient of fuzzy numbers from pascal's triangles and apply the simple probability approach then we get the following formula:

$$P(A)=\frac{a_1+2a_2+a_3}{4}$$

The coefficients of a_1, a_2, a3 are 1, 2, 1. This approach can be extended for n dimensional Pascal Traingular fuzzy order also.

IV. Statistical Implementations

4.1 For Seducers F-Test

Let us assume X is for getting values from Graded integration mean method and Y be the Variable is for getting values from Pascal Triangle approach. Then obtain variances of the both variables and find F calculated value and then compare with F table values at 5% level of significance. If F cal (Value) > F tab (Value), then we reject hypothesis (Ho) otherwise we do not.

4.2 For Student t-test

Apply t-test for the above two variables X and Y then Calculate t value and compare to t table value at 5% level of significance. If calculated t > t- table value, we reject hypothesis (Ho) otherwise we do not reject Ho.

5. Results and Discussions

In this section, we give a some simple results and discussions though the following numerical example to show the efficiency of the methodology. The cost and time matrix are given in triangular fuzzy numbers.

$$\tilde{C}=\begin{pmatrix}(11,12,14) & (7,9,10) & (12,14,16)\\ (5,7,9) & (8,10,12) & (7,10,12)\\ (10,12,15) & (6,9,11) & (4,6,9)\end{pmatrix}$$

$$\tilde{T}=\begin{pmatrix}(6,9,10) & (1,3,5) & (3,6,8)\\ (2,4,6) & (7,9,11) & (5,7,9)\\ (4,6,8) & (6,8,10) & (2,5,6)\end{pmatrix}$$

$$\tilde{C}+\tilde{T}=\begin{pmatrix}(17,21,24) & (8,12,15) & (15,20,24)\\ (7,11,15) & (15,19,23) & (12,17,21)\\ (14,18,23) & (12,17,21) & (6,11,15)\end{pmatrix}$$

Applying the graded mean integration representation for defuzzification

$$P(A)=\frac{a_1+4a_2+a_3}{6}$$

$$C+T=\begin{pmatrix}21 & 12 & 20\\ 11 & 19 & 17\\ 18 & 17 & 11\end{pmatrix}$$

By Pascal triangular approach

$$P(A)=\frac{a_1+2a_2+a_3}{4}$$

5.1 Statistical interpretation

For F –Test : Let we stated the hypothesis

Ho: There is no significance difference between the variances of the above two variables

X : 20.83 11.83 19.83 11 19 16.83 18.16 16.83 10.83

Y : 20.75 11.75 19.75 11 19 16.75 18.25 16.75 10.75

Variance(X) = 15.2518 ; Variance(Y) = 15.2969 and Fcal Value = 1.0029:F tab Value =3.44(at 5%level of significance); F cal value is very least, so we do not reject hypothesis.

For t-test : Let the Hypothesis

Ho: There is no significance difference between the means of the two variables.

Mean of X is 16.1267 and Mean of Y is 16.0833 and t-cal Value = 0.0222 ,t-tab Value is 2.120 then we reject Hypothesis.

Based on statistical tests, mean and variances are the same ,then we say that these two grouped values are came from the normal population. And for the solution of these two methods are comparatively same. Therefore, this approach is alternative for the previous one.

Applying the Hungarian method, the solution is

$$\begin{pmatrix} 21 & 12^* & 20 \\ 11^* & 19 & 17 \\ 18 & 17 & 11^* \end{pmatrix}$$

Optimal allocation:

Therefore, the optimum feasible solution is 12+11+11=34.

V. Conclusion

This measure is more applicable for generalized fuzzy number by comparing graded mean average method and it is also useful to solve the assignment problem by Hungarian method. For statistical tests, there is no significance difference between the two methods. By comparison, Pascal triangular method is also applicable for fuzzy assignment problem. It is also applicable to solve the multi-objective fuzzy assignment problems also.

Based on statistical tests, mean and variances are the same, then we say that these two grouped values are came from the normal population. And for the solution of these two methods are comparatively same. Therefore, this approach is alternative for the previous one.

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