

## Elastic Properties of RCC under Flexural Loading

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**ABSTRACT :** In structural analysis, especially in indeterminate structures, it becomes essential to know material and geometrical properties of members. The codal provisions recommend elastic properties of concrete and steel and these are fairly accurate enough. The stress-strain curve for concrete cylinder or a cube specimen is plotted. The slope of this curve is modulus of elasticity of plain concrete. Another method of determining modulus of elasticity of concrete is by flexural test of a beam specimen. The modulus of elasticity most commonly used for concrete is secant modulus. The modulus of elasticity of steel is obtained by performing a tension test of steel bar. While performing analysis by any software for high rise building, cross area of plain concrete is taken into consideration whereas effects of reinforcement bars and concrete confined by stirrups are neglected. Present aim of study is to determine elastic properties of reinforced cement concrete material. Two important stiffness properties such as AE and EI play important role in analysis of high rise RCC building idealized as plane frame. The experimental program consists of testing of beams (model size 150 x 150 x 700 mm) with percentage of reinforcement varying from 0.54 to 1.21%.

**Keywords:** indeterminate structures, stiffness properties.

### I. Introduction

In order to design an RC structural element, it is important to gain a general overview of reinforced concrete structures and an understanding of the basic material properties. It is also important to get acquainted with the basic concepts relating to performance criteria in reinforced concrete design. The aim of structural design is to design a structure so that it fulfils three criteria namely safety in terms of strength, stability and structural integrity; adequate serviceability in terms of stiffness, durability, deformation and economy. The behaviour of section at various stages of loading can be studied in two parts i.e. initial un-cracked phase and the ultimate condition at collapse.

For a simply supported beam subjected to gradually increasing load, the applied moment at any section is less than cracking moment  $M_{cr}$  and the maximum tensile stress  $f_{ct}$  in the concrete is less than its flexural tensile strength  $f_{cr}$ . This phase is called the un-cracked phase. In this case, the entire section is effective in resisting the moment. The un-cracked phase reaches its limit when the applied moment  $M$  becomes equal to the cracking moment  $M_{cr}$ . In the stress-strain curve, uncracked phase falls within the initial linear portion.

As the applied moment exceeds  $M_{cr}$ , the maximum tensile stress in concrete exceeds the flexural tensile strength of Concrete and the propagation of crack enhances. The cracks get developed in the bottom fibers of the beam. As

the load increases, the cracks get widened and propagate towards the neutral axis. The cracked portion of the concrete becomes ineffective in resisting tensile stress. Hence, the effective concrete section is reduced. For further increment in the applied moment, strain in tension steel increases which results in an upward shift of the neutral axis and ultimately there is an increase in curvature and collapse occurs.

### II. Current Codal Provisions in IS: 456-2000 [1]

For the analysis purpose, modular ratio concept makes it possible to transform the composite section into an equivalent homogeneous section made up entirely of one material. The use of transformed section concept may be limited in determining the neutral axis as the centroidal axis of the transformed section. The stress can be computed in transformed section by applying the flexure formula. In this case the second moment of area  $I_g$  of the transformed section has to be considered.

Applying the concept of transformed section, the area of tension reinforcement steel  $A_{st}$  is transformed into equivalent concrete area as  $m A_{st}$ . This transformation is valid in reinforced concrete not only for flexural members but also for members subjected to axial forces.

The long-term effects of creep and shrinkage of concrete and non-linearity at high stresses result in much larger compressive strains in the compression steel. Hence the Code recommends that the transformed area of the compression steel  $A_{sc}$  can be taken as  $1.5m A_{sc}$  rather than  $m A_{sc}$ . While considering the area of concrete under compression in the transformed section, net area  $A_c$  i.e. gross area  $A_g$  minus  $A_{sc}$  making allowance for the concrete area displaced by the steel area should be considered.

Considering the current practice of RC design, it is necessary to carry out one more cycle of analysis and design. IS Code specifies E value for plain concrete and not for the composite reinforced concrete. Second cycle of analysis can be done with available design from the first cycle. In this second cycle, as steel area is known for both beam and column, it becomes important to make the use of this steel area in defining effective area of cross section as well as effective moment of inertia considering the amount of steel. Then there exists two parts in the analysis i.e. pre-cracking and post-cracking of RC members. For this, one should know the material and geometrical properties such as E, A and I for RCC section. This may also vary for pre-cracking and post-cracking conditions. The effect of the reinforcement ratio was considered in a model which was used to estimate the effective moment of inertia of reinforced concrete beams [2]. A parametric study was also carried out to determine effective stiffness of concrete and beams [3]. Further, a procedure was developed analytically for moment-curvature behavior of beams [4]. Effect of material properties was studied on behavior of over-reinforced concrete beams [5]. From all these studies, it was observed that effective modulus of elasticity and effective

moments of inertia of RC sections are not addressed much in the literature.

**III. Rationale and Significance of the Study**

In structural analysis, especially in indeterminate structures, it becomes essential to know material and geometrical properties of members. The IS codal provisions recommend elastic properties of concrete and steel and these are fairly accurate enough. These elastic properties are known independently for concrete and steel. But reinforced concrete is a composite material containing steel and concrete. While performing analysis by any software for high rise building, cross area of plain concrete is taken into consideration whereas effects of reinforcement bars and concrete confined by stirrups are neglected. Present aim of study is to determine elastic properties of reinforced cement concrete material. Two important stiffness properties such as AE and EI play important role in analysis of high rise RCC building idealized as plane frame. It is also hoped that knowing exact material and geometrical properties, analysis may become more accurate and economical design can be achieved. The proposed experimental program consists of testing of beams (model size 150 x 150 x 700 mm) with percentage of reinforcement varying from 0.5 to 1.2%. The experimental results have been verified by using 3D finite element techniques. The present paper reviews critically codal provisions of Indian as well as other international codes such as ACI, BS, Australian and Canadian codes.

**IV. Shortcomings of the traditional method of RC design**

In pre-cracked state, area of concrete below neutral axis is to be considered. Hence value of  $I_g$  is known. In post-cracked state, area of concrete below neutral axis is to be ignored. If the cracks developed below neutral axis are within acceptable limits i.e. hairline cracks, then this area should not be ignored. But if these cracks are wider and deeper, then question arises to what limit for depth and width of cracks is to be considered. Another problem is concerned with E value of RCC. In conventional design, we take into account E value of plain concrete given by code.

Beam section is considered to be safe if ultimate moment of resistance  $M_{ur}$  is greater than or equal to the factored moment  $M_u$ . The ductile failure is accepted to be desirable. The excessive cover to the reinforcement causes cracks at the early stages of loading. In over-reinforced sections, the strength requirement may be satisfied but not the ductility requirement. The ductility requirement may be partly satisfied in case of mild steel, even if  $x_u$  slightly exceeds  $x_{u,max}$ .

**V. Experimental program**

In the present study, experimental investigation of the RCC models under flexural loadings was carried out. Possible combinations of reinforcement for flexure test for  $M_{20}$  and  $M_{25}$  grade of concrete are shown in Table 1 and Table 2 respectively. Model size 150x150x700mm is used as per IS 516:1959 (Reaffirmed 1999) Methods of Tests for Strength of Concrete) [6].

For each combination, 3 models are tested and average results are presented.

TABLE 1 Combinations of reinforcement for flexure test ( $M_{20}$ )

Model No.	Diameter (mm)	No of Bars	Ast (sq.mm)	pt (%)
1	-	-	0.00	0.00
2	8	2	100.53	0.54
3	8 + 6	2 + 1	128.81	0.60
4	8	3	150.79	0.80
5	10	2	157.08	0.84

TABLE 2 Combinations of reinforcement for flexure test ( $M_{25}$ )

Model No.	Diameter (mm)	No of Bars	Ast (sq.mm)	pt (%)
1	-	-	0.00	0.00
2	8	2	100.53	0.54
3	8 + 6	2 + 1	128.81	0.60
4	8	3	150.79	0.80
5	10	2	157.08	0.84



Fig. 1: Typical Beam Specimen Showing First Crack During Loading



Fig. 2: Typical Beam Specimen Showing Crack at Failure



Fig. 3: Typical Beam Specimen Showing Shear Crack at Failure

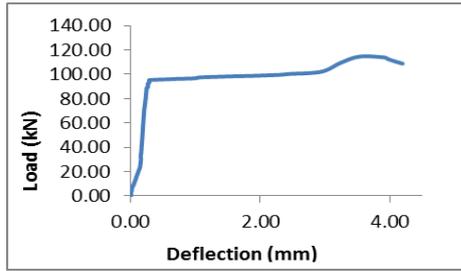


Fig.4 Typical Load Deflection Curve for Model 4 (M<sub>20</sub>)

### VI. Experimental Results

For a beam subjected to two-point loading the deflection formula is given in equation (1)

$$\delta = \left(\frac{5}{384}\right) \times \left(\frac{wl^4}{EI}\right) + \left(\frac{23}{648}\right) \left(\frac{WI^3}{EI}\right) \quad (1)$$

where  $w$  is self-weight of beam in N/mm and  $W$  is half the load at failure in N

TABLE 3 Percentage of Steel and Corresponding E and  $f_{cr}$  values of RCC for M<sub>20</sub>

Model No	Percentage of Reinforcement	Modulus of Elasticity (N/mm <sup>2</sup> )	
		M20	M25
1	0.00	24066.81	25994.86
2	0.54	28288.60	29161.02
3	0.69	28453.85	29502.47
4	0.80	31013.40	29716.23
5	0.84	32478.33	31438.35
6	1.21	-	35510.70

### VII. Analytical Investigation

The realistic and practical modeling of steel and surrounded concrete is one of the most challenging problems in structural analysis. Such a modeling is unavoidable in studying structural behavior. The grades of concrete selected in the present study are M<sub>20</sub> and M<sub>25</sub>. For concrete, the element used is SOLID65. The element is defined by eight nodes having three degrees of freedom at each node i.e. translation in x, y, and z directions. This element is a highly non-linear element and specifically used for materials like concrete and it takes into account cracking in three orthogonal directions due to tension, compression and plastic deformation. Strain ratio between the concrete and steel is supposed to be equal assuming perfect bond, therefore it is accepted that there is a unique adherence between the concrete and steel [7]. Hence there is no element defined between concrete and steel.

For the main reinforcement and stirrups, LINK8 spar element with three degrees of freedom at each node is used. It considers the parameters like plasticity, creep, swelling, stress stiffening and large deflection.

It was observed that the region of load and the supports are facing a problem of stress concentration. Hence in order to arrest this problem, the supports and loading region are modeled with a steel block. The element used for this is SOLID45 with three degrees of freedom (u, v, w) at each node. This element is treated as a linear element which handles plasticity, creep, bulge, stress stiffening and big deflection [8].

### 7.1 Load step and load increment

For the prediction of non-linear behaviour, the total load was divided into series of load increments or load steps as required by Newton-Raphson method. The number of load steps, minimum and maximum step sizes was determined after number of trials [9]. For each load increment, the crack depth in the section was observed. By considering only the effective area of cross-section and tension reinforcement, effective moment of inertia was determined. This value of moment of inertia was used for calculation of modulus of elasticity of RCC section. Fig. 5 and 6 show variation of modulus of elasticity of each model for various crack depths for M<sub>20</sub> and M<sub>25</sub> grades of concrete respectively.

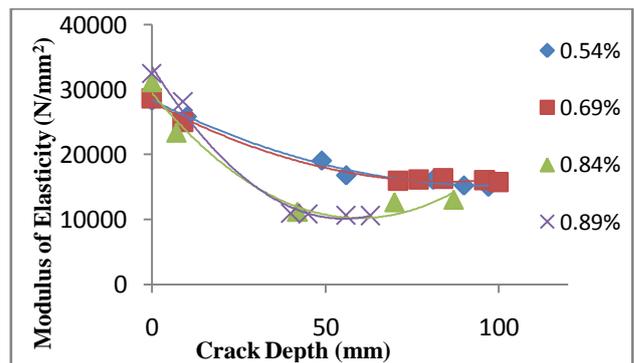


Fig. 5 Modulus of Elasticity vs Crack Depth for M<sub>20</sub> grade of Concrete

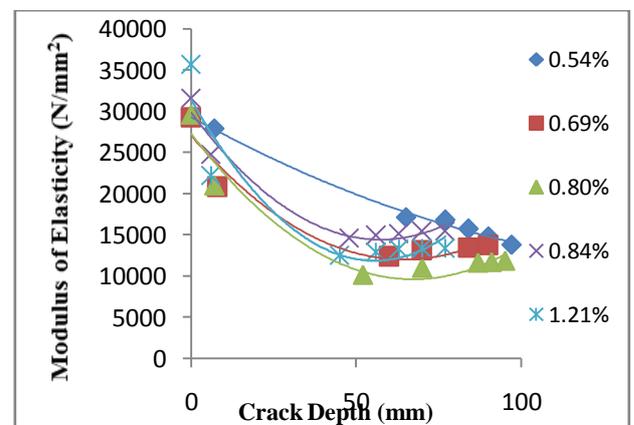


Fig. 6 Modulus of Elasticity vs Crack Depth for M<sub>20</sub> grade of Concrete

## VIII. Observations

### 8.1 M<sub>20</sub> Grade

- For  $A_{st} = 0.54$  to  $0.69\%$  E reduction variation is almost identical as crack depth increases. E remains constant from crack depth 50mm onwards.
- More drop of E for  $A_{st} = 0.84\%$ . E remains same almost from 40mm crack depth.
- E remains same for  $A_{st} = 0.89\%$  after crack depth of 40mm.
- $E = 18000 \text{ N/mm}^2$  for  $A_{st} = 0.54$  to  $0.69\%$  and  $E = 12000 \text{ N/mm}^2$  for  $A_{st} = 0.84$  and  $0.89\%$  after crack depth of 40mm.

### 8.2 M<sub>25</sub> Grade

- E lies in the range of  $12000 - 17000 \text{ N/mm}^2$  for  $A_{st} = 0.69\%$
- E for  $A_{st}$  beyond  $0.69\%$  is as follows-  $17000 \text{ N/mm}^2$  for  $A_{st} = 0.84\%$ ,  $12000 \text{ N/mm}^2$  for  $A_{st} = 0.80\%$  and  $13000 \text{ N/mm}^2$  for  $A_{st} = 1.21\%$

## IX. Conclusions

- For both grades of concrete, 120mm is the effective depth of section to be considered for calculation of modulus of elasticity.
- For no crack depth in the section, modulus of elasticity increases with increase in percentage of reinforcement for both the grades.
- Equation for effective moment of inertia of the RC section can be proposed. Equation of  $I_{eff}$  given in IS: 456-2000 is for plain section which neglects tension reinforcement.
- Also in the same equation of  $I_{eff}$  given in IS: 456-2000, the code remains silent about moment of inertia of cracked section  $I_r$ .

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