Supply Chain Production Inventory Model: Innovative Study for Shortages Allowed With Partial Backlogging

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Abstract: *In this paper, we have strived to combine all the above mentioned factors into a single problem. We shall undertake to explore a two echelon supply chain, comprising of a vendor and a buyer. The whole environment of business dealings has been assumed to be progressive credit period, which conforms to the practical market situation. The whole combination is very unique and very much practical. The variable holding cost and variable setup has been explored numerically as well; an optimal solution has been reached. The final outcome shows that the model is not only economically feasible, but stable also.*

Keywords: *Inventory model, partial backlogging, Progressive permissible delay, Supply chain, Shortages, EOQ (Economics Order Quantity).*

I. INTRODUCTION

Inventory represents one of the most significant possessions that most businesses possess. It is in direct touch with the user department in its day today activities. Inventory management is playing a key role in setting up efficient closed loop supply chains. A supply chain is a network of facilities and distribution options that performs the functions of procurement of materials, transformation of these materials into intermediate and finished products, and the distribution of these finished products to customers. It consists of a network of companies which are dependent on each other while making independent decisions. The supply chain not only includes the manufacturer and suppliers, but also transporters, warehouses, retailers, and customers themselves. Therefore, supply chain analysis tools and methodologies have become more and more important. It can be a source of great efficiency and cost-savings gains. Supply chain speed and flexibility have become key levers for competitive differentiation and increased profitability. The faster the supply chain, the better a company can respond to changing market situation and the less it needs inventory which resulting in higher return on capital employed. Supply chain management offers a large potential
or organizations to reduce costs and improve customer service performance. In the existing literature, most of the inventory models studies only aimed at the determination of the optimum solutions that minimized cost or maximized profit from the vendor's and vendor's side. However, in the modern global competitive market, the buyer and vendor should be treated as strategic partners in the supply chain with a long term cooperative relationship. Recently, many researchers have considered the buyer and vendor as a unit to find the optimal EOQ in achieving the minimum total cost. In today's business transactions, it is more and more common to see that the customers are allowed some grace Period before they settle the account with the supplier. This provides an advantage to the customers, due to the fact that they do not have to pay the supplier immediately after receiving the product, but instead, can defer their payment until the end of the allowed period. The customer pays no interest during the fixed period they are supposed to settle the account; but if the payment is delayed beyond that period, interest will be charged. The customer can start to accumulate revenues on the sale or use of the product and earn interest on that revenue. So it is to the advantage of the customer to offer the payment to the supplier until the end of the period.

The two famous formulae of EOQ and EPQ are treated separately for a buyer and a vendor respectively. From the traditional point of view, the vendor and the buyer are two individual entities with different objectives and self-interest. Due to rising costs, the globalization trend, shrinking resources, shortened product life cycle and quicker response time, increasing attention has been placed on the collaboration of the whole supply chain system. An effective supply chain network requires a cooperative relationship between the vendor and the buyer. It assumes that the buyer must pay off as soon as the items are received. Suppliers often offer trade credit as a marketing strategy to increase sales and reduce on-hand stock is reduced, and that leads to a reduction in the buyer's holding cost of finance. In addition, during the time of the credit period, buyers may earn interest on the money. In fact, buyers, especially small businesses which tend to have a limited number of financing opportunities rely on trade credit as a source of short-term funds. The classical inventory models have considered demand rates which were either constant or depended upon a single factor only, like, stock, time etc. But changing market conditions have rendered such a consideration quite unfruitful, since in real life situation, a demand cannot depend exclusively on a single parameter. A combination of two or more factors grants more authenticity to the formulation of the model. Many delivery policies have been proposed in literature for this problem. **Clark and Scarf (1960)** presented the concept of serial multi-echelon structures to determine the optimal policy. **Goyal (1985)** considered a mathematical models with a permissible delay in payments to determine the optimal order quantities. **Ha and Kim (1997)** used a graphical method to analyze the integrated vendor-buyer inventory status to derive an optimal solution. **Hwang and Shinn (1997)** studied effects of permissible delay in payments on retailer's pricing and lot sizing policy for exponentially deteriorating products. **Yang and Wee (2000)** developed an integrated economic ordering policy of deteriorating items for a vendor and a buyer. **Wang et al. (2000)** analyzed supply chain models for perishable products under inflation and permissible delay in payment.

Teng (2002) modified **Goyal (1985)** model by considering the selling price, instead of purchasing cost, as the base to calculate the interest. **Abad and Jaggi (2003)** studied a seller-buyer model with a permissible delay in payments by game theory to determine the optimal unit price and the credit period, considering that the demand rate is a function of retail price. **Huang, Y.F. et al. (2005)** considered the optimal inventory policies under permissible delay in payments depending on the ordering quantity. **Song and Cai (2006)** has been taken on optimal payment time for a retailer under permitted delay of payment by the wholesaler. **Liao (2007)** assumed on an EPQ model for deteriorating items under permissible delay in payments.

In the present study, we have strived to combine all the above mentioned factors into a single problem. We shall undertake to explore a two echelon supply chain, comprising of a vendor and a buyer. The whole environment of business dealings has been assumed to be progressive credit period, which conforms to the practical market situation. The whole combination is very unique and very much practical. The variable holding cost and variable setup has been explored numerically as well; an optimal solution has been reached. The final outcome shows that the model is not only economically feasible, but stable also.

II. PROPOSED ASSUMTIONS & NOTATIONS

1. ASSUMPTIONS

The following assumptions are used to develop aforesaid model:

1.1 The demand rate, D(t), is deterministic, the demand function D(t) is given by D(t) = $\lambda_0 e^{\alpha t}$, a and b are positive constants.

1.2 Shortages are allowed with partial backlogging.

1.3 If the retailer pays by M, then the supplier does not charge to the retailer. If the retailer pays after M and before N (N > M), he can keep the difference in the unit sale price and unit purchase price in an interest bearing account at the rate of I_e/unit/year. During [M, N], the supplier charges the retailer an interest rate of Ic_1/unit/year on unpaid balance. If the retailer pays after N, then supplier charges the retailer an interest rate of Ic_2/unit/year ($Ic_1 > Ic_2$) on unpaid balance.

2. NOTATIONS:

2.1 P = the selling price / unit.

2.2 KD = the production rate per year, where K>1

2.3 C = the unit purchase cost, with C < P.

2.4 M = the first offered credit period in settling the account without any charges.

2.5 N = the second permissible credit period in settling the account with interest charge Ic2 on unpaid balance and N > M.

2.6 Ic1 = the interest charged per \$ in stock per year by the supplier when retailer pays during [M, N].

2.7 Ic2 = the interest charged per \$ in stock per year by the supplier when retailer pays during [N, T]. (Ic1 > Ic2)

2.8 Ie = the interest earned / \$ / year.

2.9 T = the replenishment cycle.

2.10 r = the discount rate (r > α)

2.11 IE = the interest earned / time unit.

2.12 IC = the interest charged / time unit.

2.13 $(C_{vs} + \beta_2 t)$ = the setup cost for each production cycle for vendor.

2.14 $(C_{bs} + \beta_1 t)$ = the setup cost per order for buyer.

2.15 $(C_{hv} + \alpha_2 t)$ = holding cost per unit time for vendor.

2.16 $(C_{bh} + \alpha_1 t)$ = holding cost per unit time for buyer.

2.17 C_v = the unit cost for vendor.

2.18 C_b = the unit purchase cost for buyer.

2.19 S_b = shortage cost per unit time for buyer.

2.20 L_b= lost sale cost per unit time for buyer.

2.21 VC = the cost of vendor per unit time.

2.22 BC = the cost of buyer per unit time.

2.23 TC(T) = total cost of an inventory system / time unit.

2.24 B= Backlogging rate.

2.25 The deterioration function

$\theta(t,\delta) = \theta_0(\delta)t,$ 0< $\theta_0(\delta)$ <<1, t>0

This is a special form of the two parameter weibull function considered by Covert and Philip. The function is some functions of the random variable α which range over a space and in which a p.d.f. p(δ) is defined such that $\int_\Gamma p(\delta)d\delta = 1$

III. INDENTATIONS AND EQUATIONS MATHEMATICAL FORMULATION

The actual vendor's average inventory level in the integrated two-echelon inventory model is difference between the vendor's total average inventory level and the buyer's average inventory level. Since the inventory level is depleted due to a constant deterioration rate of the on-hand stock, the buyer's inventory level is represented by the following differential equation:

$$I_b'(t) + \theta_0(\delta)tI_b(t) = -\lambda_0 e^{\alpha t}, 0 \le t \le t_1 \quad (1)$$

$$I_b'(t) = -B\lambda_0 e^{\alpha t}, \quad t_1 \le t \le T \quad (2)$$

The vendor's total inventory system consisting of production period and non-production period can be described as follows:

$$I_{v1}'(t) + \theta_0(\delta)tI_{v1}(t) = (K-1)\lambda_0 e^{\alpha t}, \quad 0 \le t \le T_1 \quad (3)$$

$$I_{v2}'(t) + \theta_0(\delta)tI_{v2}(t) = -\lambda_0 e^{\alpha t}, \quad 0 \le t \le T_2 \quad (4)$$

The boundary conditions are

$$I_{v1}(t) = 0, \quad t = 0 \quad (5)$$

$$I_{v2}(t) = 0, \quad t = T_2 \quad (6)$$

$$I_b(t) = I_0, \quad t = 0 \quad (7)$$

$$I_b(t) = 0, \quad t = t_1 \quad (8)$$

$$I_{v1}(T_1) = I_{v2}(0) \quad (9)$$

And,

$$T = \frac{T_2}{n} \tag{10}$$

The solutions of the above differential equations obtained are

$$I_b(t) = I_0 e^{-\frac{\theta_0(\delta)t^2}{2}} - \lambda_0[t + \frac{\alpha t^2}{2} + \frac{\theta_0(\delta)t^3}{6}]e^{-\frac{\theta_0(\delta)t^2}{2}}, \qquad 0 \le t \le t_1\partial \tag{11}$$

$$I_b(t) = -\frac{B\lambda_0}{\alpha}\left[e^{\alpha t_1} - e^{\alpha t}\right], \qquad t_1 \le t \le T \tag{12}$$

$$I_{v1}(t) = (K-1)\lambda_0[t + \frac{\alpha t^2}{2} + \frac{\theta_0(\delta)t^3}{6}]e^{-\frac{\theta_0(\delta)t^2}{2}}, 0 \le t \le T_1 \tag{13}$$

$$I_{v2}(t) = \lambda_0[(T_2 - t) + \frac{\alpha}{2}(T_2^2 - t^2) + \frac{\theta_0(\delta)}{6}(T_2^3 - t^3)], \qquad 0 \le t \le T_2 \tag{14}$$

Using the condition that one can get,

$$I_0 = -\lambda_0[t_1 + \frac{\alpha t_1^2}{2} + \frac{\theta_0(\delta)t_1^3}{6}] \tag{15}$$

If the product of the deterioration rate and the replenishment interval is much smaller than one, the buyer's and the vendor's actual average inventory level, $\overline{I_b}$ *and* $\overline{I_v}$, are

$$\overline{I_b} = \frac{1}{T}\int_0^{t_1} e^{-rt} I_b(t)dt$$

$$= \frac{I_0}{T}[t_1 - \frac{rt_1^2}{2} + (r^2 - \theta_0(\delta))\frac{t_1^3}{6}] - \frac{\lambda_0}{T}[\frac{t_1^2}{2} + (\alpha - 2r)\frac{t_1^3}{6}$$

$$+ r(r-\alpha)\frac{t_1^4}{8} + \frac{\alpha r^2 t_1^5}{20} - \frac{\theta_0(\delta)t_1^4}{12} - (3\alpha + 2r)\frac{\theta_0(\delta)t_1^5}{30}] \tag{16}$$

and

$$\overline{I_v} = \frac{1}{T_2}\left[\int_0^{T_1} e^{-rt} I_{v1}(t)dt + e^{-rT_1}\int_0^{T_2} e^{-rt} I_{v2}(t)dt\right] - \overline{I_b}$$

$$= \frac{1}{T_2}[(K-1)\lambda_0(\frac{T_1^2}{2} + \frac{T_1^3}{6}(\alpha - 2r) + \frac{T_1^4}{8}r(r-\alpha) + \frac{\alpha T_1^5 r^2}{20} - \frac{\theta_0(\delta)T_1^4}{12} - \frac{\theta_0(\delta)T_1^5(3\alpha + 2r)}{60})$$

$$+ e^{-rT_1}\lambda_0(\frac{T_2^2}{2} - \frac{T_2^3}{6}(\alpha - 2r) + \frac{r^2T_2^4}{24} - \frac{\alpha r T_2^4}{8} + \frac{\theta_0(\delta)T_2^4}{24} + \frac{\theta_0(\delta)T_2^5(4\alpha + 3r)}{60})] - \frac{I_0}{T}[t_1$$

$$- \frac{rt_1^2}{2} + (r^2 - \theta_0(\delta))\frac{t_1^3}{6}] + \frac{\lambda_0}{T}[\frac{t_1^2}{2} + (\alpha - 2r)\frac{t_1^3}{6} + r(r-\alpha)\frac{t_1^4}{8} + \frac{\alpha r^2 t_1^5}{20} - \frac{\theta_0(\delta)t_1^4}{12}$$

$$-(3\alpha + 2r)\frac{\theta_0(\delta)t_1^5}{30}] \tag{17}$$

Respectively.

The annual total holding cost for the buyer and the vendor are

$$HC_b = \frac{1}{T}\left[\int_0^{t_1} (C_{bh} + \alpha_1 t)e^{-rt} I_{b1}(t)dt\right]$$

$$= -\frac{\lambda_0}{T}C_{bh}[\frac{t_1^2}{2} + \frac{t_1^3}{6}(\alpha - 2r) - \frac{t_1^4}{8}r(r-\alpha) + \frac{\alpha t_1^5 r^2}{20} - \frac{\theta_0(\delta)t_1^4}{12} - \frac{\theta_0(\delta)t_1^5(3\alpha + 2r)}{60}]$$

$$-\frac{\alpha_1\lambda_0}{T}[\frac{t_1^3}{3} + \frac{t_1^4}{8}(\alpha - 2r) - \frac{t_1^5}{10}r(r-\alpha) - \frac{\theta_0(\delta)t_1^5}{15}] + \frac{C_{bh}I_0}{T}[t_1 - \frac{rt_1^2}{2}$$

$$+ \frac{(r^2 - \theta_0(\delta))t_1^3}{6}] + \frac{\alpha_1 I_0}{T}[\frac{t_1^2}{2} - \frac{rt_1^3}{3} + \frac{(r^2 - \theta_0(\delta))t_1^4}{8}]$$

(18)

And

$$HC_v = \frac{1}{T_2}\left[\int_0^{T_1} (C_{hv} + \alpha_2 t)e^{-rt} I_{v1}(t)dt + e^{-rT_1}\int_0^{T_2} (C_{hv} + \alpha_2 t)e^{-rt} I_{v2}(t)dt\right] - \overline{I_b}$$

$$= \frac{1}{T_2}[(K-1)\lambda_0 C_{hv}\{\frac{T_1^2}{2} + \frac{T_1^3}{6}(\alpha - 2r) - \frac{T_1^4}{8}r(r-\alpha) + \frac{\alpha T_1^5 r^2}{20} - \frac{\theta_0(\delta)T_1^4}{12} - \frac{\theta_0(\delta)T_1^5(3\alpha + 2r)}{60}\}]$$

$$+ \frac{\alpha_2(K-1)\lambda_0}{T_2}[\frac{T_1^3}{3} + \frac{T_1^4}{8}(\alpha - 2r) + \frac{T_1^5}{10}r(r-\alpha) - \frac{\theta_0(\delta)T_1^5}{15}] + \frac{C_{bh}\lambda_0 e^{-rT_1}}{T}[\frac{T_2^2}{2} - \frac{T_2^3(\alpha - 2r)}{6}$$

$$+ \frac{r^2T_2^4}{24} - \frac{\theta_0(\delta)T_2^4}{24} + \frac{\theta_0(\delta)T_2^5(4\alpha + 3r)}{60}] + \frac{\lambda_0 e^{-rT_1}\alpha_2}{T_2}[\frac{T_2^3}{6} + \frac{T_2^4(3\alpha - 2r)}{24}] - \frac{I_0}{T}[t_1$$

$$- \frac{rt_1^2}{2} + (r^2 - \theta_0(\delta))\frac{t_1^3}{6}] + \frac{\lambda_0}{T}[\frac{t_1^2}{2} + (\alpha - 2r)\frac{t_1^3}{6} + r(r-\alpha)\frac{t_1^4}{8} + \frac{\alpha r^2 t_1^5}{20} - \frac{\theta_0(\delta)t_1^4}{12}$$

$$-(3\alpha + 2r)\frac{\theta_0(\delta)t_1^5}{30}] \tag{19}$$

respectively.

The annual deterioration cost for the buyer and the vendor are

$$DC_b = \frac{C_b}{T}\left[\int_0^{t_1} \theta_0(\delta) t e^{-rt} I_{b1}(t)dt\right] =$$

$$[\frac{I_0\theta_0(\delta)}{T}\{\frac{t_1^2}{2} - \frac{rt_1^3}{6} + \frac{r^4t_1^4}{8}\} - \frac{\lambda_0\theta_0(\delta)}{T}\{\frac{t_1^3}{3} + \frac{t_1^4(\alpha - 2r)}{8} - \frac{t_1^5}{10}r(r-\alpha)\}] \tag{20}$$

and

$$DC_v = \frac{C_v}{T_2}\left[\int_0^{T_1}\theta_0(\delta)te^{-rt}I_{v1}(t)dt + e^{-rT_1}\int_0^{T_2}\theta_0(\delta)te^{-rt}I_{v2}(t)dt\right]$$

$$= C_v[\frac{(K-1)\lambda_0\theta_0(\delta)}{T_2}\{\frac{T_1^3}{3}+\frac{T_1^4(\alpha-2r)}{8}+\frac{T_1^5 r(r-\alpha)}{10}\}+\frac{e^{-rT_1}\lambda_0\theta_0(\delta)}{T_2}\{\frac{T_2^3}{6}$$

$$+\frac{T_2^4(3\alpha-2r)}{24}\}] \tag{21}$$

respectively.

The annual set-up cost for the buyer and the vendor are

$$OC_b = \frac{1}{T}[\int_0^{t_1}(C_{bs}+\beta_1 t)dt + \int_{t_1}^{T}(C_{bs}+\beta_1 t)dt]$$

$$= C_{bs} + \frac{\beta_1 T}{2} \tag{22}$$

and

$$OC_v = \frac{1}{T_2}[\int_0^{T_1}(C_{vs}+\beta_2 t)dt + \int_0^{T_2}(C_{vs}+\beta_2 t)dt]$$

$$= [C_{vs}(T_1+T_2) + \frac{\beta_2(T_1^2+T_2^2)}{2}] \tag{23}$$

respectively.

The annual shortage cost for the buyer is

$$SC_b = \frac{S_b e^{-rt_1}}{T}\int_{t_1}^{T}\frac{\lambda_0}{\alpha}\left[e^{\alpha t_1}-e^{\alpha t}\right]e^{-rt}dt$$

$$= \frac{S_b\lambda_0 e^{-rt_1}}{T\alpha}[\frac{\alpha e^{(\alpha-r)t_1}}{r(\alpha-r)} - \frac{e^{(\alpha t_1 - rT)}}{r} - \frac{e^{(\alpha-r)T}}{(\alpha-r)}] \tag{24}$$

The annual lost sale cost for the buyer is

$$LC_b = \frac{L_b e^{-rt_1}}{T}\int_{t_1}^{T}(1-B)\lambda_0 e^{\alpha t}e^{-rt}dt$$

$$= \frac{L_b e^{-rt_1}\lambda_0(1-B)}{T(\alpha-r)}[e^{(\alpha-r)T} - e^{(\alpha-r)t_1}] \tag{25}$$

The different costs associated with the system are set-up costs, holding costs, deterioration cost and shortage cost. Our aim is to minimize the total cost.

From (9), one can derive the following condition:

$$(K-1)\lambda_0[T_1+\frac{\alpha T_1^2}{2}+\frac{\theta_0(\delta)T_1^3}{6}]e^{-\frac{\theta_0(\delta)T_1^2}{2}} = \lambda_0[T_2+\frac{\alpha T_2^2}{2}+\frac{\theta_0(\delta)T_2^3}{6}]e^{-\frac{\theta_0(\delta)T_2^2}{2}} \tag{26}$$

By Taylor's series expansion, (4.26) is derived as

$$T_1 = \frac{1}{K-1}T_2\left[1+\frac{\alpha}{2}T_2\right] \tag{27}$$

Regarding interest charged and interest earned based on the length of the cycle time t_1, three cases arise:

IV. Figures and Tables

Regarding interest charged and interest earned based on the length of the cycle time t_1, three cases arise:

Case I: M □ t_1

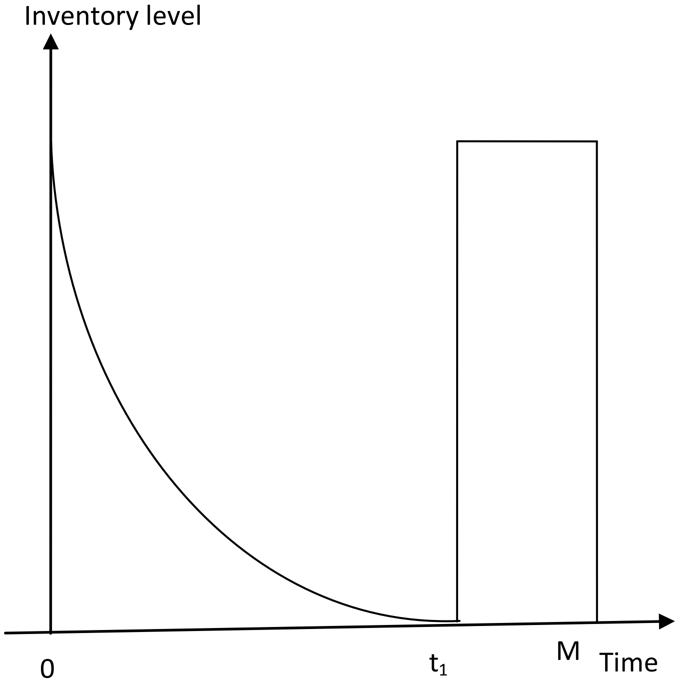

Fig 1: t_1 □ M

In the first case, retailer does not pay any interest to the supplier. Here, retailer sells I_0 units during $(0, t_1)$ time interval and paying for CI_0 units in full to the supplier at time $M \geq t_1$, so interest charges are zero, i.e.

$$IC_1 = 0 \tag{28}$$

Retailers deposits the revenue in an interest bearing account at the rate of Ie / \$ / year. Therefore, interest earned IE_1, per year is

$$IE_1 = \frac{PI_e}{T_2}[\int_0^{t_1}e^{-rt}D(t)tdt + (M-t_1)\int_0^{t_1}e^{-rt}D(t)dt]$$

$$= \frac{PI_e\lambda_0}{T_2}[(M-t_1)t_1 + \{1+(\alpha-r)(M-t_1)\}\frac{t_1^2}{2}+\frac{t_1^3(\alpha-r)}{3}] \tag{29}$$

Total cost per unit time of an inventory system is

$$TC_b(t_1,\delta) = OC_b + HC_b + DC_b + SC_b + IC_1 - IE_1$$

$$= [C_{bs}+\frac{\beta_1 T}{2}-\frac{\lambda_0}{T}C_{bh}[\frac{t_1^2}{2}+\frac{t_1^3}{6}(\alpha-2r)-\frac{t_1^4}{8}r(r-\alpha)+\frac{\alpha t_1^5 r^2}{20}-\frac{\theta_0(\delta)t_1^4}{12}$$

$$-\frac{\theta_0(\delta)t_1^5(3\alpha+2r)}{60}]-\frac{\alpha_1\lambda_0}{T}[\frac{t_1^3}{3}+\frac{t_1^4}{8}(\alpha-2r)-\frac{t_1^5}{10}r(r-\alpha)-\frac{\theta_0(\delta)t_1^5}{15}]$$

$$+\frac{C_{bh}I_0}{T}[t_1-\frac{rt_1^{\,2}}{2}+\frac{(r^2-\theta_0(\delta))t_1^{\,3}}{6}]+\frac{\alpha_1 I_0}{T}[\frac{t_1^{\,2}}{2}-\frac{rt_1^{\,3}}{3}+\frac{(r^2-\theta_0(\delta))t_1^{\,4}}{8}]$$

$$+[\frac{I_0\theta_0(\delta)}{T}\{\frac{t_1^{\,2}}{2}-\frac{rt_1^{\,3}}{6}+\frac{r^4t_1^{\,4}}{8}\}-\frac{\lambda_0\theta_0(\delta)}{T}\{\frac{t_1^{\,3}}{3}+\frac{t_1^{\,4}(\alpha-2r)}{8}-\frac{t_1^{\,5}}{10}r(r-\alpha)\}]$$

$$+\frac{S_b\lambda_0 e^{-rt_1}}{T\alpha}[\frac{\alpha e^{(\alpha-r)t_1}}{r(\alpha-r)}-\frac{e^{(\alpha t_1-rT)}}{r}-\frac{e^{(\alpha-r)T}}{(\alpha-r)}]+\frac{L_b e^{-rt_1}\lambda_0(1-B)}{T(\alpha-r)}[e^{(\alpha-r)T}-e^{(\alpha-r)t_1}]$$

$$-\frac{PI_e\lambda_0}{T_2}[(M-t_1)t_1+\{1+(\alpha-r)(M-t_1)\}\frac{t_1^{\,2}}{2}+\frac{t_1^{\,3}(\alpha-r)}{3}]] \tag{30}$$

Hence the mean cost

$$< TC_b >= \int TC_b(t_1,\delta)p(\delta)d\delta \tag{31}$$

$$< TC_b >=[C_{bs}+\frac{\beta_1 T}{2}-\frac{\lambda_0}{T}C_{bh}[\frac{t_1^{\,2}}{2}+\frac{t_1^{\,3}}{6}(\alpha-2r)-\frac{t_1^{\,4}}{8}r(r-\alpha)+\frac{\alpha t_1^{\,5}r^2}{20}-\frac{At_1^{\,4}}{12}$$
$$-\frac{At_1^{\,5}(3\alpha+2r)}{60}]-\frac{\alpha_1\lambda_0}{T}[\frac{t_1^{\,3}}{3}+\frac{t_1^{\,4}}{8}(\alpha-2r)-\frac{t_1^{\,5}}{10}r(r-\alpha)-\frac{At_1^{\,5}}{15}]$$
$$+\frac{C_{bh}I_0}{T}[t_1-\frac{rt_1^{\,2}}{2}+\frac{(r^2-A)t_1^{\,3}}{6}]+\frac{\alpha_1 I_0}{T}[\frac{t_1^{\,2}}{2}-\frac{rt_1^{\,3}}{3}+\frac{(r^2-A)t_1^{\,4}}{8}]$$
$$+[\frac{I_0A}{T}\{\frac{t_1^{\,2}}{2}-\frac{rt_1^{\,3}}{6}+\frac{r^4t_1^{\,4}}{8}\}-\frac{\lambda_0 A}{T}\{\frac{t_1^{\,3}}{3}+\frac{t_1^{\,4}(\alpha-2r)}{8}-\frac{t_1^{\,5}}{10}r(r-\alpha)\}]$$
$$+\frac{S_b\lambda_0 e^{-rt_1}}{T\alpha}[\frac{\alpha e^{(\alpha-r)t_1}}{r(\alpha-r)}-\frac{e^{(\alpha t_1-rT)}}{r}-\frac{e^{(\alpha-r)T}}{(\alpha-r)}]+\frac{L_b e^{-rt_1}\lambda_0(1-B)}{T(\alpha-r)}[e^{(\alpha-r)T}-e^{(\alpha-r)t_1}]$$
$$-\frac{PI_e\lambda_0}{T_2}[(M-t_1)t_1+\{1+(\alpha-r)(M-t_1)\}\frac{t_1^{\,2}}{2}+\frac{t_1^{\,3}(\alpha-r)}{3}]] \tag{32}$$

$$\text{Where } A=\int\theta_0(\delta)p(\delta)d\delta \tag{33}$$

$$<TC_v>= OC_v + HC_v + DC_v - IC_1$$

$$=[C_{vs}(T_1+T_2)+\frac{\beta_2(T_1^2+T_2^{\,2})}{2}]+\frac{1}{T_2}[(K-1)\lambda_0 C_{hv}\{\frac{T_1^2}{2}+\frac{T_1^3}{6}(\alpha-2r)-\frac{T_1^4}{8}r(r-\alpha)$$
$$+\frac{\alpha T_1^5r^2}{20}-\frac{AT_1^4}{12}-\frac{AT_1^5(3\alpha+2r)}{60}\}]+\frac{\alpha_2(K-1)\lambda_0}{T_2}[\frac{T_1^3}{3}+\frac{T_1^4}{8}(\alpha-2r)$$
$$+\frac{T_1^5}{10}r(r-\alpha)-\frac{AT_1^5}{15}]+\frac{C_{bh}\lambda_0 e^{-rT_1}}{T}[\frac{T_2^{\,2}}{2}-\frac{T_2^{\,3}(\alpha-2r)}{6}+\frac{r^2T_2^{\,4}}{24}-\frac{AT_2^{\,4}}{24}$$

$$+\frac{AT_2^{\,5}(4\alpha+3r)}{60}]+\frac{\lambda_0 e^{-rT_1}\alpha_2}{T_2}[\frac{T_2^{\,3}}{6}+\frac{T_2^{\,4}(3\alpha-2r)}{24}]-\frac{I_0}{T}[t_1-\frac{rt_1^{\,2}}{2}$$
$$+(r^2-A)\frac{t_1^{\,3}}{6}]+\frac{\lambda_0}{T}[\frac{t_1^{\,2}}{2}+(\alpha-2r)\frac{t_1^{\,3}}{6}+r(r-\alpha)\frac{t_1^{\,4}}{8}+\frac{\alpha r^2t_1^{\,5}}{20}-\frac{At_1^{\,4}}{12}$$
$$-(3\alpha+2r)\frac{At_1^{\,5}}{30}]+C_v[\frac{(K-1)\lambda_0 A}{T_2}\{\frac{T_1^3}{3}+\frac{T_1^4(\alpha-2r)}{8}+\frac{T_1^5r(r-\alpha)}{10}\}$$
$$+\frac{e^{-rT_1}\lambda_0 A}{T_2}\{\frac{T_2^{\,3}}{6}+\frac{T_2^{\,4}(3\alpha-2r)}{24}\}] \tag{34}$$

To minimize the total cost per unit time, the optimum value of t_1, T_2 is the solution of following equation.

Case II: $M < t_1 < N$

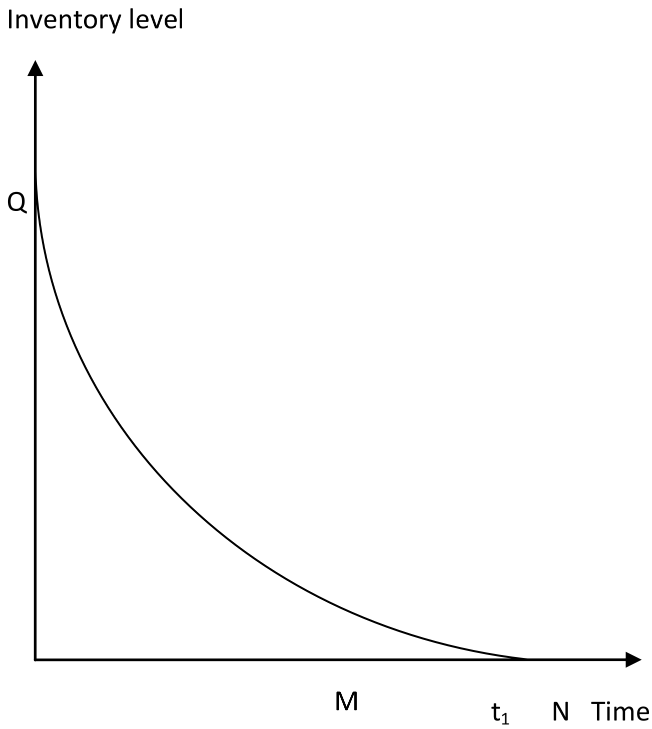

Fig: 2 $M < t_1 < N$

In the second case, supplier charges interest at the rate Ic_1 on unpaid balance.

Interest earned, IE_2 during [0, M] is

$$IE_2 = PI_e\int_0^M e^{-rt}D(t)tdt$$

$$= PI_e\lambda_0[\frac{M^2}{2}+\frac{M^3}{6}(\alpha-r)+\frac{M^4}{8}(\alpha-r)^2] \tag{35}$$

Retailer pay for I_0 units purchased at time t = 0 at the rate of C / $ / unit to the supplier during [0, M]. The retailer sells D (M).M units at selling price P/ unit. So, he has generated revenue of P D(M).M + IE_2. Then two sub cases may arise:

Sub Case: 2.1

Let P D(M).M + $IE_2 \geq CI_0$, i.e. retailer has enough money to settle his account for all I_0 units procured at time t = 0. Then interest charge will be

$$IC_{2.1} = 0 \tag{36}$$

and interest earned

$$IE_{2.1}=\frac{IE_2}{T_2}$$

$$=\frac{PI_e\lambda_0}{T_2}[\frac{M^2}{2}+\frac{M^3}{6}(\alpha-r)+\frac{M^4}{8}(\alpha-r)^2] \quad (37)$$

So, total cost $TC_{2.1}$ per unit time of inventory system is

$<TC_b> = OC_b + HC_b + DC_b + SC_b + LC_b + IC_{2.1} - IE_{2.1}$

$$S=[C_{bs}+\frac{\beta_1 T}{2}-\frac{\lambda_0}{T}C_{bh}[\frac{t_1^2}{2}+\frac{t_1^3}{6}(\alpha-2r)-\frac{t_1^4}{8}r(r-\alpha)+\frac{\alpha t_1^5 r^2}{20}-\frac{At_1^4}{12}$$

$$-\frac{At_1^5(3\alpha+2r)}{60}]-\frac{\alpha_1\lambda_0}{T}[\frac{t_1^3}{3}+\frac{t_1^4}{8}(\alpha-2r)-\frac{t_1^5}{10}r(r-\alpha)-\frac{At_1^5}{15}]$$

$$+\frac{C_{bh}I_0}{T}[t_1-\frac{rt_1^2}{2}+\frac{(r^2-A)t_1^3}{6}]+\frac{\alpha_1 I_0}{T}[\frac{t_1^2}{2}-\frac{rt_1^3}{3}+\frac{(r^2-A)t_1^4}{8}]$$

$$+[\frac{I_0 A}{T}\{\frac{t_1^2}{2}-\frac{rt_1^3}{6}+\frac{r^4t_1^4}{8}\}-\frac{\lambda_0 A}{T}\{\frac{t_1^3}{3}+\frac{t_1^4(\alpha-2r)}{8}-\frac{t_1^5}{10}r(r-\alpha)\}]$$

$$+\frac{S_b\lambda_0 e^{-rt_1}}{T\alpha}[\frac{\alpha e^{(\alpha-r)t_1}}{r(\alpha-r)}-\frac{e^{(\alpha t_1-rT)}}{r}-\frac{e^{(\alpha-r)T}}{(\alpha-r)}]+\frac{L_b e^{-rt_1}\lambda_0(1-B)}{T(\alpha-r)}[e^{(\alpha-r)T}-e^{(\alpha-r)t_1}]$$

$$-\frac{PI_e\lambda_0}{T_2}[\frac{M^2}{2}+\frac{M^3}{6}(\alpha-r)+\frac{M^4}{8}(\alpha-r)^2] \quad (38)$$

$<TC_v> = OC_v + HC_v + DC_v - IC_{2.1}$

$$=[C_{vs}(T_1+T_2)+\frac{\beta_2(T_1^2+T_2^2)}{2}]+\frac{1}{T_2}[(K-1)\lambda_0 C_{hv}\{\frac{T_1^2}{2}+\frac{T_1^3}{6}(\alpha-2r)-\frac{T_1^4}{8}r(r-\alpha)$$

$$+\frac{\alpha T_1^5 r^2}{20}-\frac{AT_1^4}{12}-\frac{AT_1^5(3\alpha+2r)}{60}\}]+\frac{\alpha_2(K-1)\lambda_0}{T_2}[\frac{T_1^3}{3}+\frac{T_1^4}{8}(\alpha-2r)$$

$$+\frac{T_1^5}{10}r(r-\alpha)-\frac{AT_1^5}{15}]+\frac{C_{bh}\lambda_0 e^{-rT_1}}{T}[\frac{T_2^2}{2}-\frac{T_2^3(\alpha-2r)}{6}+\frac{r^2T_2^4}{24}-\frac{AT_2^4}{24}$$

$$+\frac{AT_2^5(4\alpha+3r)}{60}]+\frac{\lambda_0 e^{-rT_1}\alpha_2}{T_2}[\frac{T_2^3}{6}+\frac{T_2^4(3\alpha-2r)}{24}]-\frac{I_0}{T}[t_1-\frac{rt_1^2}{2}$$

$$+(r^2-A)\frac{t_1^3}{6}]+\frac{\lambda_0}{T}[\frac{t_1^2}{2}+(\alpha-2r)\frac{t_1^3}{6}+r(r-\alpha)\frac{t_1^4}{8}+\frac{\alpha r^2 t_1^5}{20}-\frac{At_1^4}{12}$$

$$-(3\alpha+2r)\frac{At_1^5}{30}]+C_v[\frac{(K-1)\lambda_0 A}{T_2}\{\frac{T_1^3}{3}+\frac{T_1^4(\alpha-2r)}{8}+\frac{T_1^5 r(r-\alpha)}{10}\}$$

$$+\frac{e^{-rT_1}\lambda_0 A}{T_2}\{\frac{T_2^3}{6}+\frac{T_2^4(3\alpha-2r)}{24}\}]-$$

$$\frac{PI_e\lambda_0}{T_2}[\frac{M^2}{2}+\frac{M^3}{6}(\alpha-r)+\frac{M^4}{8}(\alpha-r)^2] \quad (39)$$

To minimize the total cost per unit time, the optimum value of t_1, T_2 is the solution of following equation.

Sub Case: 2.2

Let P D(M).M + IE_2 < CI_0. Here, retailer will have to pay interest on unpaid balance $U_1 = CI_0 - (P\,D(M).M + IE_2)$ at the rate of Ic_1 at time M to the supplier. Then interest paid per unit time is given by

$$IC_{2.2}=\frac{U_1^2 Ic_1}{PI_0}\int_M^{t_1} e^{-rt}I(t)dt$$

$$=\frac{U_1^2 Ic_1}{PI_0}[\frac{(t_1^2-M^2)}{2}+\frac{(\alpha-r)(t_1^3-M^3)}{2}+\frac{(\alpha-r)^2(t_1^4-M^4)}{8}] \quad (40)$$

Where,

$U_1 = CI_0 - (P\,D(M).M + IE_2)$

$$=CI_0-P\lambda_0[M+(\alpha+\frac{I_e}{2})M^2+(\frac{\alpha^2}{2}+\frac{I_e(\alpha-r)}{2})M^3+(\frac{\alpha^3}{6}+\frac{I_e(\alpha-r)^2}{8})M^4] \quad (41)$$

And interest earned

$$IE_{2.2}=\frac{IE_2}{T_2}$$

$$=\frac{PI_e\lambda_0}{T_2}[\frac{M^2}{2}+\frac{M^3}{6}(\alpha-r)+\frac{M^4}{8}(\alpha-r)^2] \quad (42)$$

So, total cost $TC_{2.2}$ per unit time of inventory system is

$<TC_b> = OC_b + HC_b + DC_b + SC_b + LC_b + IC_{2.2} - IE_{2.2}$

$$=[C_{bs}+\frac{\beta_1 T}{2}-\frac{\lambda_0}{T}C_{bh}[\frac{t_1^2}{2}+\frac{t_1^3}{6}(\alpha-2r)-\frac{t_1^4}{8}r(r-\alpha)+\frac{\alpha t_1^5 r^2}{20}-\frac{At_1^4}{12}$$

$$-\frac{At_1^5(3\alpha+2r)}{60}]-\frac{\alpha_1\lambda_0}{T}[\frac{t_1^3}{3}+\frac{t_1^4}{8}(\alpha-2r)-\frac{t_1^5}{10}r(r-\alpha)-\frac{At_1^5}{15}]$$

$$+\frac{C_{bh}I_0}{T}[t_1-\frac{rt_1^2}{2}+\frac{(r^2-A)t_1^3}{6}]+\frac{\alpha_1 I_0}{T}[\frac{t_1^2}{2}-\frac{rt_1^3}{3}+\frac{(r^2-A)t_1^4}{8}]$$

$$+[\frac{I_0 A}{T}\{\frac{t_1^2}{2}-\frac{rt_1^3}{6}+\frac{r^4t_1^4}{8}\}-\frac{\lambda_0 A}{T}\{\frac{t_1^3}{3}+\frac{t_1^4(\alpha-2r)}{8}-\frac{t_1^5}{10}r(r-\alpha)\}]$$

$$+\frac{S_b\lambda_0 e^{-rt_1}}{T\alpha}[\frac{\alpha e^{(\alpha-r)t_1}}{r(\alpha-r)}-\frac{e^{(\alpha t_1-rT)}}{r}-\frac{e^{(\alpha-r)T}}{(\alpha-r)}]+\frac{L_b e^{-rt_1}\lambda_0(1-B)}{T(\alpha-r)}[e^{(\alpha-r)T}-e^{(\alpha-r)t_1}]$$

$$-\frac{PI_e\lambda_0}{T_2}[\frac{M^2}{2}+\frac{M^3}{6}(\alpha-r)+\frac{M^4}{8}(\alpha-r)^2] \quad (43)$$

$<TC_v> = OC_v + HC_v + DC_v - IC_{2.2}$

$$=S[C_{vs}(T_1+T_2)+\frac{\beta_2(T_1^2+T_2^2)}{2}]+\frac{1}{T_2}[(K-1)\lambda_0 C_{hv}\{\frac{T_1^2}{2}+\frac{T_1^3}{6}(\alpha-2r)-\frac{T_1^4}{8}r(r-\alpha)$$

$$+\frac{\alpha T_1^5 r^2}{20}-\frac{AT_1^4}{12}-\frac{AT_1^5(3\alpha+2r)}{60}\}]+\frac{\alpha_2(K-1)\lambda_0}{T_2}[\frac{T_1^3}{3}+\frac{T_1^4}{8}(\alpha-2r)$$

$$+\frac{T_1^5}{10}r(r-\alpha)-\frac{AT_1^5}{15}]+\frac{C_{bh}\lambda_0 e^{-rT_1}}{T}[\frac{T_2^{\,2}}{2}-\frac{T_2^{\,3}(\alpha-2r)}{6}+\frac{r^2T_2^{\,4}}{24}-\frac{AT_2^{\,4}}{24}$$

$$+\frac{AT_2^{\,5}(4\alpha+3r)}{60}]+\frac{\lambda_0 e^{-rT_1}\alpha_2}{T_2}[\frac{T_2^{\,3}}{6}+\frac{T_2^{\,4}(3\alpha-2r)}{24}]-\frac{I_0}{T}[t_1-\frac{rt_1^{\,2}}{2}$$

$$+(r^2-A)\frac{t_1^{\,3}}{6}]+\frac{\lambda_0}{T}[\frac{t_1^{\,2}}{2}+(\alpha-2r)\frac{t_1^{\,3}}{6}+r(r-\alpha)\frac{t_1^{\,4}}{8}+\frac{\alpha r^2 t_1^{\,5}}{20}-\frac{At_1^{\,4}}{12}$$

$$-(3\alpha+2r)\frac{At_1^{\,5}}{30}]+C_v[\frac{(K-1)\lambda_0 A}{T_2}\{\frac{T_1^3}{3}+\frac{T_1^4(\alpha-2r)}{8}+\frac{T_1^5 r(r-\alpha)}{10}\}$$

$$+\frac{e^{-rT_1}\lambda_0 A}{T_2}\{\frac{T_2^{\,3}}{6}+\frac{T_2^{\,4}(3\alpha-2r)}{24}\}]-\frac{U_1^2 Ic_1}{PI_0}[\frac{(t_1^{\,2}-M^2)}{2}$$

$$+\frac{(\alpha-r)(t_1^{\,3}-M^3)}{2}+\frac{(\alpha-r)^2(t_1^{\,4}-M^4)}{8}] \qquad (44)$$

To minimize the total cost per unit time, the optimum value of t_1, T_2 is the solution of following equation.

Case III: t_1 □ N

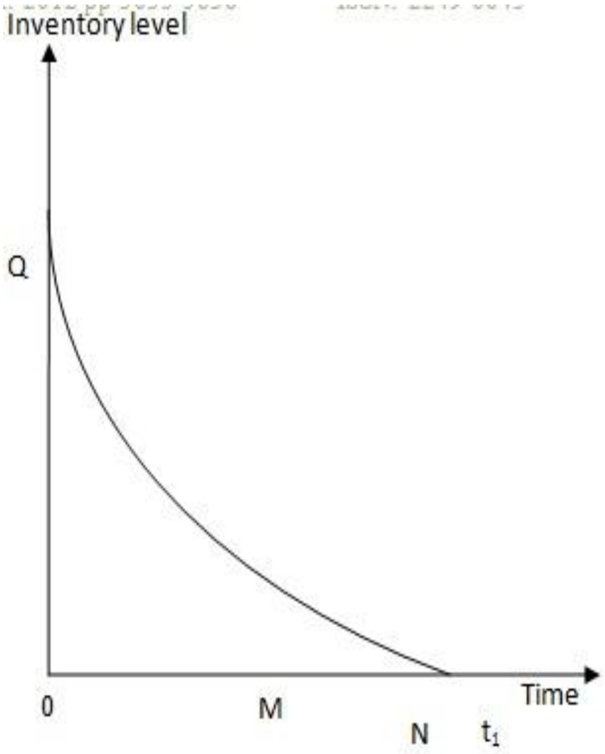

Fig 3: t_1 □ N

In the final case, retailer pays interest at the rate of Ic_2 to the supplier. Based on the total purchased cost, CI0, total money P D(M).M + IE_2

$$=\frac{PI_e\lambda_0}{T_2}[\frac{M^2}{2}+\frac{M^3}{6}(\alpha-r)+\frac{M^4}{8}(\alpha-r)^2]$$

in account at M and total money

PD(N).N=P λ_0 $e^{-\lambda N}$ N at N, there are three sub cases may arise:

Sub Case 3.1 Let P D(M).M + $IE_2 \geq CI_0$

This case is same as sub case 2.1, here 3.1 designate decision variables and objective function.

Sub Case 3.2 Let P D(M).M + $IE_2 < CI_0$ and

$$PD(N-M).(N-M)+PI_e\int_M^N D(t)dt \geq CI_0-(PD(M).M+IE_2)$$

$$P\lambda_0 e^{\alpha(N-M)}(N-M)+PI_e\lambda_0[(N-M)+\frac{\alpha}{2}(N^2-M^2)] \geq CI_0-(P\lambda_0 e^{\alpha M}M+IE_2)$$

This case similar to sub case 2.2.

Sub Case 3.3 Let P D(M).M + $IE_2 < CI_0$ and

$$P\lambda_0 e^{\alpha(N-M)}(N-M)+PI_e\lambda_0[(N-M)+\frac{\alpha}{2}(N^2-M^2)] < CI_0-(P\lambda_0 e^{\alpha M}M+IE_2)$$

Here, retailer does not have enough money to pay off total purchase cost at N. He will not pay money of P D(M).M + IE_2 at M and $PD(N-M).(N-M)+PI_e[(N-M)+\frac{\alpha}{2}(N^2-M^2)]$ at N. That's why he has to pay interest on unpaid balance U_1 = CI_0 – (P D(M).M + IE_2) with Ic_1 interest rate during (M, N) and $U_2=U_1-PD(N-M).(N-M)+PI_e\int_M^N D(t)dt$ with interest rate Ic_2 during (N, t_1).

Therefore, total interest charged on retailer, $IC_{3.3}$ per unit time is

$$IC_{3.3}=\frac{U_1 Ic_1(N-M)}{T_2}+\frac{U_2^2 Ic_1}{PI_0}\int_N^{t_1} e^{-rt}I_b(t)dt$$

$$=\frac{U_1 Ic_1(N-M)}{T_2}+\frac{U_2^2 Ic_1}{PI_0}[\frac{t_1^{\,2}}{2}-N^2+\frac{(t_1^{\,3}-N^3)(\alpha-2r)}{6}+\frac{(t_1^{\,4}-N^4)}{8}r(r-\alpha)$$

$$+\frac{(t_1^{\,5}-N^5)}{20}r^2\alpha-\frac{\theta_0(\delta)(t_1^{\,4}-N^4)}{12}-\frac{\theta_0(\delta)(t_1^{\,5}-N^5)(3\alpha+2r)}{12}]-\frac{U_2^2 Ic_1}{P}[(t_1-N)$$

$$-\frac{r(t_1^{\,2}-N^2)}{2}+\frac{(r^2-\theta_0(\delta))(t_1^{\,3}-N^3)}{6}] \qquad (45)$$

Interest earned per unit time is

$$IE_{3.3}=\frac{IE_2}{T_2}$$

$$=\frac{PI_e\lambda_0}{T_2}[\frac{M^2}{2}+\frac{M^3}{6}(\alpha-r)+\frac{M^4}{8}(\alpha-r)^2] \qquad (46)$$

So, total cost $TC_{3.3}$ per unit time of inventory system is

$$\langle TC_b\rangle = OC_b + HC_b + DC_b + SC_b + LC_b + IC_{3.3} - IE_{3.3}$$

$$=[C_{bs}+\frac{\beta_1 T}{2}-\frac{\lambda_0}{T}C_{bh}[\frac{t_1^2}{2}+\frac{t_1^3}{6}(\alpha-2r)-\frac{t_1^4}{8}r(r-\alpha)+\frac{\alpha t_1^5 r^2}{20}-\frac{At_1^4}{12}$$

$$-\frac{At_1^5(3\alpha+2r)}{60}]-\frac{\alpha_1\lambda_0}{T}[\frac{t_1^3}{3}+\frac{t_1^4}{8}(\alpha-2r)-\frac{t_1^5}{10}r(r-\alpha)-\frac{At_1^5}{15}]$$

$$+\frac{C_{bh}I_0}{T}[t_1-\frac{rt_1^2}{2}+\frac{(r^2-A)t_1^3}{6}]+\frac{\alpha_1 I_0}{T}[\frac{t_1^2}{2}-\frac{rt_1^3}{3}+\frac{(r^2-A)t_1^4}{8}]$$

$$+[\frac{I_0 A}{T}\{\frac{t_1^2}{2}-\frac{rt_1^3}{6}+\frac{r^4 t_1^4}{8}\}-\frac{\lambda_0 A}{T}\{\frac{t_1^3}{3}+\frac{t_1^4(\alpha-2r)}{8}-\frac{t_1^5}{10}r(r-\alpha)\}]$$

$$+\frac{S_b\lambda_0 e^{-rt_1}}{T\alpha}[\frac{\alpha e^{(\alpha-r)t_1}}{r(\alpha-r)}-\frac{e^{(\alpha t_1-rT)}}{r}-\frac{e^{(\alpha-r)T}}{(\alpha-r)}]+\frac{L_b e^{-rt_1}\lambda_0(1-B)}{T(\alpha-r)}[e^{(\alpha-r)T}-e^{(\alpha-r)t_1}]$$

$$+\frac{U_1 Ic_1(N-M)}{T_2}+\frac{U_2^2 Ic_1}{PI_0}[\frac{t_1^2}{2}-N^2+\frac{(t_1^3-N^3)(\alpha-2r)}{6}+\frac{(t_1^4-N^4)}{8}r(r-\alpha)$$

$$+\frac{(t_1^5-N^5)}{20}r^2\alpha-\frac{\theta_0(\delta)(t_1^4-N^4)}{12}-\frac{\theta_0(\delta)(t_1^5-N^5)(3\alpha+2r)}{12}]-\frac{U_2^2 Ic_1}{P}[(t_1-N)$$

$$-\frac{r(t_1^2-N^2)}{2}+\frac{(r^2-\theta_0(\delta))(t_1^3-N^3)}{6}]$$

$$-\frac{PI_e\lambda_0}{T_2}[\frac{M^2}{2}+\frac{M^3}{6}(\alpha-r)+\frac{M^4}{8}(\alpha-r)^2] \quad (47)$$

$TC_v = OC_v + HC_v + DC_v - IC_{3.3}$

$$=[C_{vs}(T_1+T_2)+\frac{\beta_2(T_1^2+T_2^2)}{2}]+\frac{1}{T_2}[(K-1)\lambda_0 C_{hv}\{\frac{T_1^2}{2}+\frac{T_1^3}{6}(\alpha-2r)-\frac{T_1^4}{8}r(r-\alpha)$$

$$+\frac{\alpha T_1^5 r^2}{20}-\frac{AT_1^4}{12}-\frac{AT_1^5(3\alpha+2r)}{60}\}]+\frac{\alpha_2(K-1)\lambda_0}{T_2}[\frac{T_1^3}{3}+\frac{T_1^4}{8}(\alpha-2r)$$

$$+\frac{T_1^5}{10}r(r-\alpha)-\frac{AT_1^5}{15}]+\frac{C_{bh}\lambda_0 e^{-rT_1}}{T}[\frac{T_2^2}{2}-\frac{T_2^3(\alpha-2r)}{6}+\frac{r^2T_2^4}{24}-\frac{AT_2^4}{24}$$

$$+\frac{AT_2^5(4\alpha+3r)}{60}]+\frac{\lambda_0 e^{-rT_1}\alpha_2}{T_2}[\frac{T_2^3}{6}+\frac{T_2^4(3\alpha-2r)}{24}]-\frac{I_0}{T}[t_1-\frac{rt_1^2}{2}$$

$$+(r^2-A)\frac{t_1^3}{6}]+\frac{\lambda_0}{T}[\frac{t_1^2}{2}+(\alpha-2r)\frac{t_1^3}{6}+r(r-\alpha)\frac{t_1^4}{8}+\frac{\alpha r^2 t_1^5}{20}-\frac{At_1^4}{12}$$

$$-(3\alpha+2r)\frac{At_1^5}{30}]+C_v[\frac{(K-1)\lambda_0 A}{T_2}\{\frac{T_1^3}{3}+\frac{T_1^4(\alpha-2r)}{8}+\frac{T_1^5 r(r-\alpha)}{10}\}$$

$$+\frac{e^{-rT_1}\lambda_0 A}{T_2}\{\frac{T_2^3}{6}+\frac{T_2^4(3\alpha-2r)}{24}\}] -$$

$$\frac{PI_e\lambda_0}{T_2}[\frac{M^2}{2}+\frac{M^3}{6}(\alpha-r)+\frac{M^4}{8}(\alpha-r)^2] \quad (48)$$

To minimize the total cost per unit time, the optimum value of t_1, T_2 is the solution of following equation.

NUMERICAL ILLUSTRATION: THE PRECEDING THEORY CAN BE ILLUSTRATED BY THE FOLLOWING NUMERICAL EXAMPLE WHERE THE PARAMETERS ARE GIVEN AS FOLLOWS:

Demand parameters, a = 500, b = 5, c = 2

Selling price, P = 30

Buyer's purchased cost, C_b = 35

Buyer's percentage holding cost per year per dollar,

C_{bh} = 0.2

Buyer's ordering cost per order, C_{bs} = 500

Buyer's shortage cost, S_b = 50

Vendor's unit cost, C_v = 20

Vendor's percentage holding cost per year per dollar,

C_{vh} = 0.2

Vendor's setup cost per order, C_{vs} = 1000

Vendor's production rate per year, K = 5

Deterioration rate, $\theta_0(\delta)$ = 0.01

First delay period, M= 0.2

Second delay period, N= 0.4

The interest earned, Ie = 0.05

The interest charged, Ic1 = 0.10

The interest charged, Ic2 = 0.20 (Ic1 > Ic2)

Backlogging rate, B=0

Table 1:

N	T_2	t_1	VC	BC	TC
1	0.827183	0.800625	1757.09	1405.95	3163.03
2	0.942755	0.456282	2086.02	1517.28	3603.30
3	1.02889	0.331991	2274.14	1790.02	4064.16
4	1.10067	0.266369	2425.17	2083.54	4508.71
5	1.16312	0.225188	2559.84	2374.64	4935.49

Table 2:

N	T_2	t_1	VC	BC	TC
1	0.792393	0.745431	1966.30	1774.98	3741.28
2	0.921355	0.433375	2435.64	1877.95	4313.59
3	1.01214	0.317385	2708.53	1969.85	4678.39
4	1.08612	0.255438	2927.92	2215.75	5143.68
5	1.14978	0.216326	3121.89	2474.02	5595.9

Table 3:

N	T_2	t_1	VC	BC	TC
1	0.792393	0.745431	1780.22	1823.14	3603.36
2	0.921355	0.433375	1934.79	1957.21	3892.00
3	1.01214	0.317385	2265.29	2049.26	4314.55
4	1.08612	0.255438	2315.26	2320.36	4635.62
5	1.14978	0.216326	2497.69	2546.68	5044.37

Table 4:

N	T_2	t_1	VC	BC	TC
1	1.43526	0.40970	1524.28	6918.29	8442.57
2	2.22410	0.41285	1328.68	4504.47	5833.15
3	2.69166	0.416398	1270.01	3076.3	4346.31
4	3.01918	0.420088	1172.2	2283.4	3455.6
5	3.27318	0.423883	1032.2	1856.66	2888.86

V. CONCLUSION

Here we have studied a two echelon supply chain with some very realistic assumptions. We studied our model in a progressive credit period. No doubt, this assumption imparts an economic viability to the whole study. In real world, it is noted that, as a result of progressive permissible delay in settling the replenishment account, the economic replenishment interval and order quantity generally increase marginally, although the annual cost decreases considerably. The saving in cost as a result of permissible delay in settling the replenishment account largely come the ability to delay payment without paying any interest. As a result of increasing order quantity under conditions or permissible delay in payments, we need to order less often. So this EOQ model is applicable when supplier gives the trade credit to the retailer.

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OBSERVATION

The data obtained clearly shows that individual optimal solutions are very different from each other. However, there exists a solution which ultimately provides the minimum operating cost to the whole supply chain. All the observations can be summed up as follows:

1. An increase in the interest charged, increases the buyer cost BC and decrease the vendor cost VC of the commodity.

2. Optimal solution for the buyer is n=1 in table first while for the vendor, it is n=5 in table 4. The overall optimal solution which ultimately minimizes the cost across the whole supply chain is n=5 in table 4

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