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Effect of Cracked Section on Lateral Response of Reinforced Concrete Flanged Beams

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Abstract: *In an analysis of reinforced concrete structure the flexural stiffness of section is an important parameter and the change in the value of flexural stiffness may result in significant change in analysis result. The current study aims to estimate the reduction in flexural stiffness of reinforced concrete flanged beam sections subjected to lateral loading and take the cracking effect of reinforced concrete section into account. The reduction in flexural stiffness due to earthquake shaking may increase the lateral deflection and it can be significantly greater as compared to deflection estimated using gross flexural stiffness. To take this effect into consideration, the design code of some countries suggest reduction factors or equations to reduce the gross flexural stiffness to effective flexural stiffness but they have some drawbacks because not consider all important parameters in there equation. However in Indian seismic code IS 1893 (2002) and many countries there are no provisions to account for reduction in stiffness due to concrete cracking. Analytical work in present study identified the parameters and influences of these parameters on effective stiffness are determined and suggest the simplified but reasonable accurate expression for computation of effective stiffness of reinforced concrete flanged beam section. Proposed equations can be easily use by designer to estimate the effective stiffness of reinforced concrete flanged beam sections accurately.*

Keywords: *Concrete cracking, Deflections, effective stiffness, flanged beam, regression analysis.*

I. Introduction

While analyzing the reinforced concrete structures under vertical and lateral loads, the designers are consider the assumed value of the flexural stiffness, but under the combined action of vertical and lateral loads, some sections within critical member will reach near yield point resulting in cracking of the member on bending tensile side, the flexural stiffness *(EI)* of member starts decreasing or reducing. There will be considerable reduction in flexural stiffness due to cracking. Because of reduction in flexural stiffness value, the lateral deflection of reinforced concrete members increases and it can be significantly greater as compared to deflection estimated using gross flexural stiffness. Also the natural time period, deflection, internal force distribution and dynamic response all changes due to change in stiffness *(EI)* i.e. whole analysis changes therefore it is essential to use the reduced or effective stiffness of reinforced concrete structure. To take these effects into consideration the design code of many countries suggests some reduction factor or equations to reduce the gross stiffness to effective stiffness. However in Indian seismic code IS 1893 (2002) and many countries there are no provisions to account for reduction in stiffness due to concrete cracking.

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II. Methodology

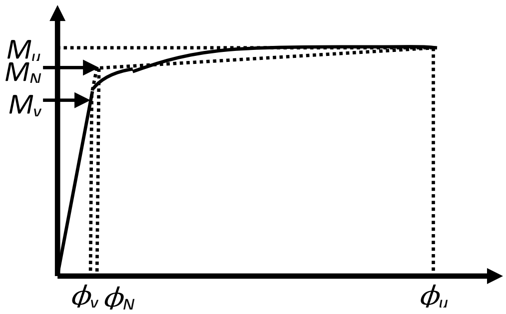


Fig. 1: Typical moment curvature relationship and its bilinear approximation.

To find out the reduction in stiffness of reinforced concrete section due to concrete cracking, beam sections have been analyzed. Moment curvature relationships have been obtained by using confined Mander's stress strain curve for concrete and simple stress strain curve for steel reinforcement for different sections. Figure 1 show typical moment curvature relationship and its bilinear approximation. It is a well known fact that the most approximate linearization of moment curvature relation is by an initial elastic segment passing through first yield and extrapolated to the nominal flexural strength M_N, and a post yield segment connected to the ultimate strength. Then the flexural stiffness at yield level (EI_y) has been calculated using Eqn. (1), when section first attains the reinforcement tensile yield strain of $\varepsilon_y = \frac{f_y}{E_S}$, or concrete extreme compression fiber attains a strain of 0.002, whichever occurs fir

$$EI_y = \frac{M_y}{\phi_y} \quad (1)$$

Where, EI_y is flexural stiffness at yield level, M_y is moment capacity at yield level and $\emptyset_y$ is yield curvature. The nominal flexural strength M_N develops when the extreme compression fiber strain reaches 0.004 or the reinforcement tension strain reaches 0.015, whichever occurs first (Priestley 2003). So reduce or effective flexural stiffness at this level is given by Eqn. (2)

$$EI_{eff} = \frac{M_N}{\phi_N} \quad (2)$$

Where, EI_{eff} is the reduced flexural stiffness, M_N is the nominal flexural strength and $\emptyset_N$ is the curvature corresponding to M_N.

Finally EI_{eff} *and* EI_y are normalized with EI_{gross} i.e. flexural stiffness of gross cross section, where $E = 5000\sqrt{f_{ck}}$ as per IS 456 (2000) and $I_{gross} = bD^3/12$. Here f_{ck} is the characteristic compressive strength of concrete cubes of size 150 mm at 28 days. The reinforced concrete flanged beam sections varying in b_w/b_f ratio from 0.135 to 0.255 and in D_f/D ratio from 0.192 to 0.277 were analyzed for percentage of steel varying from 0.4 to 2.5. Finally all these flanged beam sections were analyzed for two different concrete grade having f_{ck} equal to 20 Mpa and 25 Mpa) and reinforcement grade having f_y equal to 415 Mpa and 500 MPa .To know the variation of EI_{eff} of beam section with one parameter all other parameters were kept constant, e.g., to know the variation of EI_{eff} with percentage of steel, the aspect ratio, grade of concrete and reinforcement, confinement reinforcement were kept same and only percentage of steel was changed.

2.1 Typical Moment Curvature Relationship

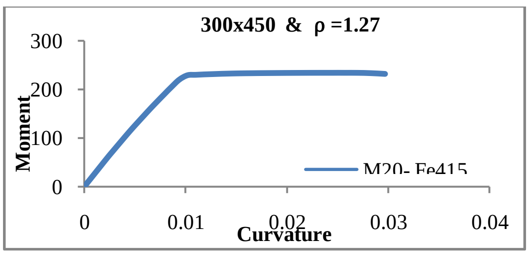


Fig. 2: Typical moment curvature relationship

Approximately 400 flanged beam sections were analyzed in section designer of computer program SAP 2000 V14 (2009). Moment curvature relationships have been obtained in section designer by using confined Mender's stress strain curve for concrete and simple stress strain curve for steel reinforcement for different sections. Then the EI_{eff} was found out by using Eqn. (2). Figure 2 shows typical moment curvature relationship obtained from section designer for rectangular beam section.

2.2 Influence of Different Type of Stress-Strain Curve of Concrete on Young's Modulus (E)

Different researchers have used the different stress strain curve of concrete for developing the moment curvature relationship for reinforced concrete sections. In the current study confined Mander's stress strain curve for concrete is used. To notice the difference between E values in different stress strain curves for concrete, Figure 3 is plotted.

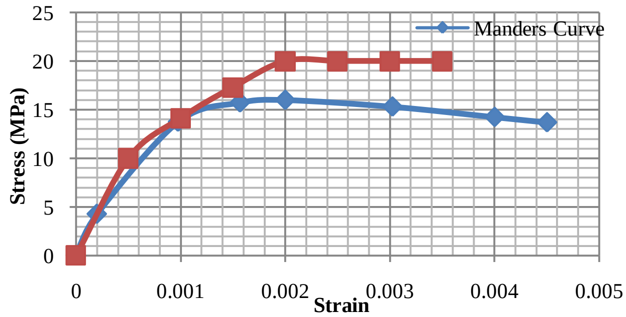


Fig. 3: Stress strain curves for concrete under compression

Figure 3 shows stress strain curve for M25 grade of concrete proposed by Mander and parabolic stress strain curve given in IS 456 (2000). From above figure it was clear that the difference in value of E is very insignificant up to strain level of 0.0015, but on further increase in strain level of concrete the difference in E value increased significantly. The value of E obtained by using parabolic stress strain curve was approximately 1.33 and 1.5 times the value obtained by using Mander's stress strain curve, when strain value in concrete was 0.003 and 0.004, respectively. In the current study equations were proposed to estimate the effective stiffness (EI_{eff}) value of reinforced concrete rectangular and flanged beam sections by using Mander's stress strain curve [12]. To obtain the effective stiffness (EI_{eff}) value corresponding to parabolic or design stress strain curve it is recommended that effective stiffness (EI_{eff}) value should be multiplied by factor of 1.33 when strain in concrete is 0.003 and by a factor of 1.5 when strain in concrete is 0.004, respectively.

III. Analysis of Reinforced Concrete (RC) Flanged Beam Sections

There were up to 400 flanged beam sections analyzed here. In this analysis case observed here is the estimation of $EI_{eff}/EI_{gross\ (Rect.)}$ i.e. in these case the gross stiffness calculate for rectangular section of flanged beam. In each case ten different reinforced concrete flanged beam section varying with different parameter such as percentage of steel , b_w/b_f and D_f/D ratio of section and grade of concrete and steel. out of this, in five different flanged beam section keep D_f/D ratio constant and b_w/b_f ratio varying from 0.135 to 0.255 and in other five different flanged beam section keep b_w/b_f ratio constant and D_f/D ratio varying from 0.192 to 0.277. Figure 4 show flanged beam section.

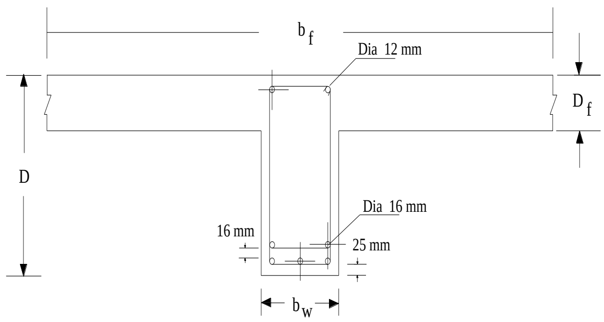


Fig. 4: Flanged beam diagram

3.1 Effective Stiffness of Reinforced Concrete (RC) Flanged Beam Sections ($EI_{eff}/EI_{gross\ (Rect.)}$)

Determine the effective stiffness of reinforced concrete flanged beam section, in these case the gross moment of inertia ($b_wD^3/12$) was calculated for rectangle section of flanged beam. $EI_{eff}/EI_{gross\ (Rect.)}$ is varying with the parameter same like above such as percentage of steel, D_f/D ratio, b_w/b_f ratio, grade of concrete and steel.

3.1.1 Keeping Df /D Constant and b_w/b_f Varying

Keep D_f/D constant and b_w/b_f varying from 0.135 to 0.255

a) Variation of EI with Percentage of Steel

In the current study the percentage of steel considered in bottom of section was from nearly 0.4 (just above the minimum) to maximum of up to 2.5 percent. The minimum of two bars were provided at top, because of need for provision of confinement reinforcement (2 legs). The section was developed in section designer of SAP2000 V14. Figure 5 shows variation of EI_{eff}/EI_{gross} of reinforced concrete flanged beam sections with percentage of steel. .EI_{eff}/EI_{gross} is directly proportional to percentage of steel

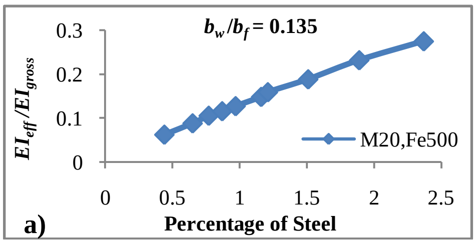
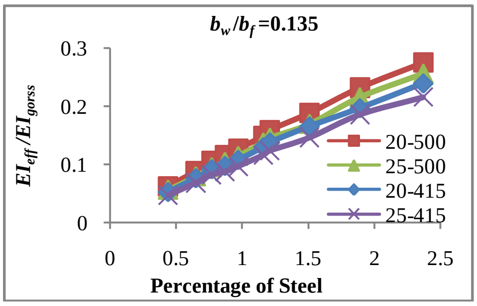


Fig. 5: Variation of *EI* with percentage of steel for different b_w/b_f ratio

b) Variation of EI with Grade of Concrete and Steel

From Figure 6 it may be observed that EI_{eff}/EI_{gross} of flanged beam sections increases with increase in grade of steel and decreases with increase in concrete grade steel for b_w/b_f ratio 0.135 and 0.255 it also same for b_w/b_f ratio 0.153, 0.177 and 0.209. Hence it may be said that EI_{eff}/EI_{gross} of flanged beam sections was directly proportional to grade of steel, while it was inversely proportional to grade of concrete.



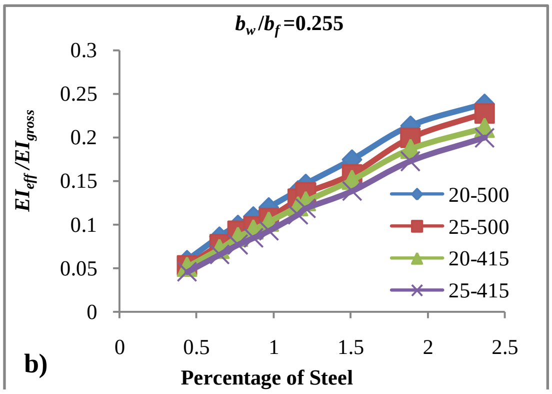


Fig. 6: Variation of *EI* with grade of concrete and steel for different b_w/b_f ratio

c) Variation of EI with b_w/b_f Ratio of Reinforced Concrete (RC) Flanged Beam Section

From Figure 7 it may be clearly observed that EI_{eff}/EI_{gross} of flanged beam sections increases with decrease in b_w/b_f ratio for same percentage of steel in section. i.e.; EI_{eff}/EI_{gross} of flanged beam was inversely proportional to b_w/b_f ratio. This observation was valid for percentage of steel ranging from 1 to 2.5. The EI_{eff}/EI_{gross} of flanged beam sections for very low level of steel percentage (upto1) was almost constant.

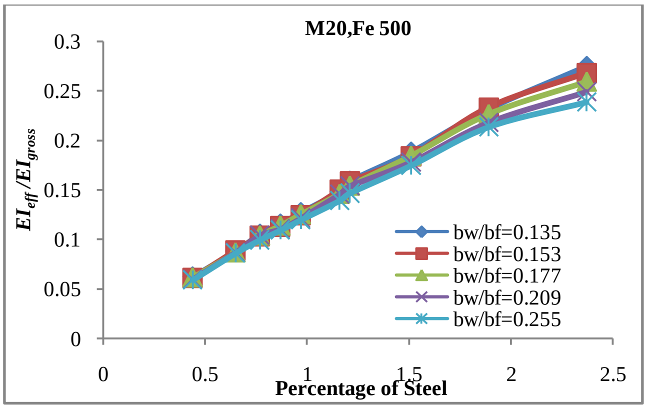
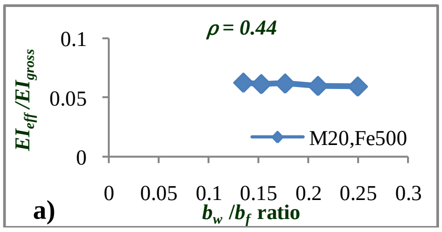


Fig. 7: Variation of *EI* with b_w/b_f ratio

d) Variation of EI with b_w/b_f Ratio for same Percentage of Steel

From Figure 8 it clearly observed that EI_{eff}/EI_{gross} of flanged beam sections is almost constant in case of percentage of steel 0.44 for different b_w/b_f *ratio*. And EI_{eff}/EI_{gross} increases with decrease in b_w/b_f ratio having percentage of steel 2.37, the difference in EI_{eff}/EI_{gross} due to same percentage of steel 2.37 is relatively significant.



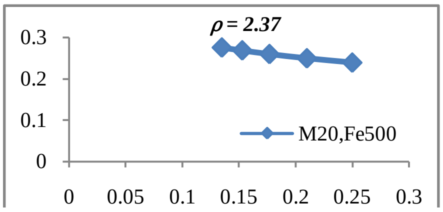


Fig. 8: Variation of *EI* with b_w/b_f ratio for same Percentage of Steel

3.1.2 Keep b_w/b_f Constant and D_f/D Varying

Keep b_w/b_f constant and D_f/D varying from 0.192 to 0.277

a) Variation of EI with Percentage of Steel

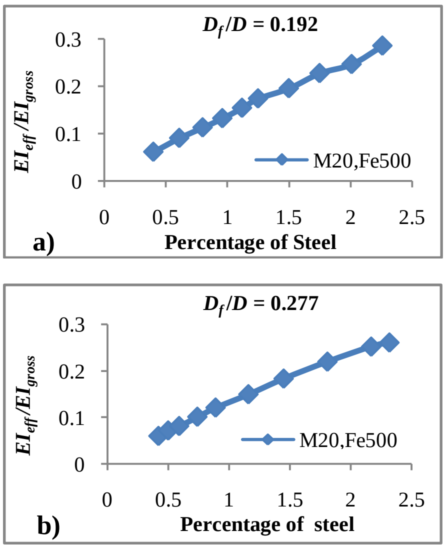
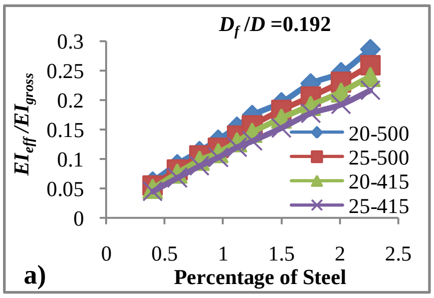


Fig. 9: Variation of *EI* with percentage of steel for different D_f/D ratio

From Figure 9, it may be observed that the EI_{eff}/EI_{gross} of flanged beam sections increases with increase in percentage of steel for b_w/b_f ratio 0.192 and 0.277 i.e; EI_{eff}/EI_{gross} is directly proportional to percentage of steel.

b) Variation of EI with Grade of Concrete and Steel

From Figure 10, observed that EI_{eff}/EI_{gross} of flanged beam sections was directly proportional to grade of steel, while it was inversely proportional to grade of concrete.



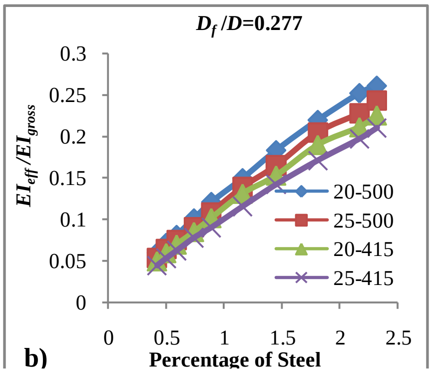


Fig. 10: Variation of *EI* with grade of concrete and steel for different D_f/D ratio

c) Variation of EI with D_f/D Ratio of RC Flanged Beam Section

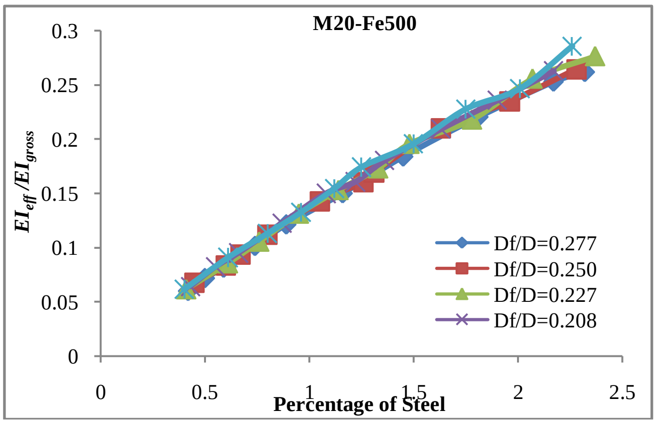
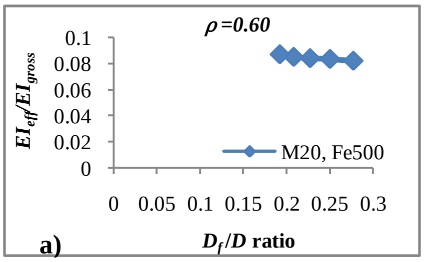


Fig. 11: Variation of *EI* with D_f/D ratio

From Figure 11, observed that EI_{eff}/EI_{gross} of flanged beam sections increases with decrease in D_f /D ratio for same percentage of steel in section. i.e.; EI_{eff}/EI_{gross} of flanged beam was inversely proportional to D_f/D ratio. This observation was valid for percentage of steel ranging from 1 to 2.5. The EI_{eff}/EI_{gross} of flanged beam sections for very low level of steel percentage (upto1) was almost constant.

d) Variation of EI with D_f/D Ratio for same Percentage of Steel

From Figure 12 it observed that EI_{eff}/EI_{gross} of RC flanged beam sections is almost constant in percentage of steel 0.6 for different D_f /D ratio. And EI_{eff}/EI_{gross} increases with decrease in D_f /D ratio having percentage of steel above the 1 percent



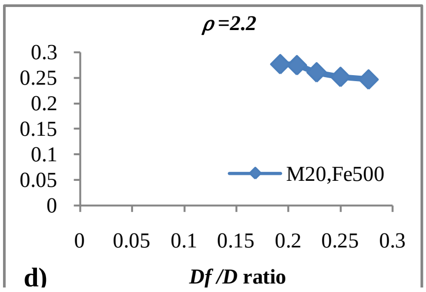


Fig. 12: Variation of *EI* with D_f/D ratio for same percentage of steel

3.1.3 Effective stiffness of RC flanged beam section (EI_{eff} / $EI_{gross(Rect.)}$)

Linear unconstrained regression analysis was carried out on data obtained from analysis of nearly 400 reinforced concrete flanged beam sections with different section parameters. The Eqn. (3) was proposed to estimate the EI_{eff}/EI_{gross} of RC flanged beam sections. The equation estimates the EI_{eff}/EI_{gross} at concrete strain of 0.004 in extreme compression fiber.

$$\frac{EI_{eff}}{EI_{gross\ (Rect.)}} = 0.08\left(\frac{{f_y}^{0.87} \quad {\rho_B}^{0.88}}{{f_{ck}}^{0.42} \quad (D_f/D)^{0.18} \quad (b_w/b_f)^{0.09}}\right) \qquad (3)$$

Where, f_{ck} = grade of concrete in MPa, Df/D = ratio of flanged depth to total depth of section, b_w/b_f = ratio of web width to flanged width, ρ_B = Steel area at bottom expressed as fraction of cross sectional area of section (in decimal), $I_{gross\ (Rect.)}$ = gross moment of inertia of rectangle section in flanged beam.

Coefficient of correlation between analytically obtained values of EI_{eff}/EI_{gross} and values obtained using developed equations for flanged beam sections was found out to be 0.99. If parabolic or design stress strain curve is used it is recommended that to account for the difference in Young's Modulus (*E*) with different stress strain curve the value of EI_{eff}/EI_{gross} obtained by Eqn. (3) should be multiplied by factor of 1.5 if strain in concrete at extreme fiber is 0.004.

$$\left[\frac{EI_{eff}}{EI_{gross\ (Rect.)}}\right]_{(P.S.S)} = 1.5\left[\frac{EI_{eff}}{EI_{gross\ (Rect.)}}\right]_{(M.S.S)} \qquad (4)$$

At concrete strain of 0.004

Where, P.S.S = Parabolic or design stress strain curve and M.S.S = Stress strain curve proposed by Mander

IV. Verification of Proposed Equation

To verify the proposed equations, the values obtained from proposed equations were compared with the equations developed by other researchers and from experimental data published in various literatures. The equations developed for estimating effective stiffness of flanged beams were verified by comparison with equations or values proposed by khuntia and Ghosh (2004) and Priestley (2003). It was noticed that the equations proposed by other researchers were holding good only up to particular limits, also some of important parameters were not considered by other researchers in there equations and not estimating the value of effective stiffness of reinforced concrete flanged beam sections accurately. Following are equations proposed by Khuntia and Ghosh (2004) for effective stiffness determination of reinforced concrete rectangular and flanged beam sections.

For rectangular beam sections-

$$EI_{eff} = E_c I_g (0.10 + 25\rho)\left(1 - 0.2\frac{b}{d}\right) \leq 0.6 E_c I_g \qquad (5)$$

For flanged beam sections-

$$\frac{EI_{eff\ (T)}}{EI_{eff}}\left(1 + 2\frac{t_f}{h}\right) \leq 1.4 \qquad (6)$$

Where: b/d= aspect ratio or ratio of width to depth of beam; ρ = steel area express as fraction of cross sectional area of section (in decimal); t_f / h = ratio of flanged thickness to total depth. $EI_{eff\ (T)}$ = effective stiffness of flanged beam section. EI_{eff} = effective stiffness of rectangular beam section from Eqn. (5)

Proposed Eqn. (3) in terms of Parabolic or design stress strain curve is that Eqn. (4) were compared with values given by Priestley (2003) and values obtained from Eqn. (6) proposed by Khuntia and Ghosh (2004). Table 1, shows the values of effective stiffness estimated by using Khuntia and Ghosh Eqn. (6), proposed Eqn. in current study Eqn. (3) and values proposed by Priestley (2003).

It may be clearly observed from Table 1 that the difference in values of estimated effective stiffness by using Eqn. (6) proposed by Khuntia and Ghosh (2004) and that proposed in current study was significant; though the difference in values of estimated effective stiffness by using Eqn. (3) proposed in current study and that proposed by Priestley (2003) was also significant. The reason for difference was some limitations of equation proposed by Khuntia and Ghosh (2004) and Priestley (2003), which are discussed later in next point.

4.1 Limitation of Existing Equations

In case of RC flanged beam sections, From Khuntia &Ghosh Eqn. (6) it observed that there was no parameter to incorporate the effect of b_w/b_f ratio and grade of reinforcement which are important parameter in effective stiffness determination of flanged beam sections. Because of this reason the difference estimated between Khuntia& Ghosh (2004) Eqn. value and that obtained from Eqn. (3)

Table I. Comparison of EI_{eff} values of flanged beam section proposed by different researcher

b_w (mm)	b_f (mm)	D_f (mm)	D (mm)	f_{ck} MPa	f_y MPa	% ρ_B Bottom	Ghosh Eqn. (6)	Priestley Eqn. (2003)	Prop. Eqn. (3)	Prop. Eqn x 1.5
350	1550	150	600	30	400	0.82	**0.17**	**0.17**	0.076	**0.11**
350	1550	150	600	30	400	1.54	**0.27**	**0.36**	0.132	**0.20**
350	1550	150	600	30	400	2.2	**0.37**	**0.49**	0.180	**0.27**
350	1550	150	600	30	400	2.2	**0.37**	**0.27**	0.180	**0.27**
350	1550	150	600	30	300	0.82	**0.17**	**0.19**	0.059	**0.09**
350	1550	150	600	30	300	1.54	**0.27**	**0.34**	0.103	**0.15**
350	1550	150	600	30	300	2.2	**0.37**	**0.49**	0.140	**0.21**
350	1550	150	600	30	300	2.2	**0.37**	**0.27**	0.140	**0.21**

proposed in current study and it is significant difference. which is shown in Table 1. Also the difference between values proposed Priestley (2003) and that obtained from using Eqn. (3) was also significant, because the values proposed by Priestley (2003) not consider the parameter to incorporate the effect of b_w /b_f ratio.To summaries this chapter it may be said that the equations proposed in current study for effective stiffness determination for RC flanged beam sections were verified by comparing with equations developed by other researchers and some of experimental data obtained from various literatures. Some of problems or drawbacks were found in equation proposed by Khuntia and Ghosh (2004) and Priestley (2003) for effective stiffness determination of RC flanged beam sections are discussed in detailed and mentioned. The equations developed in current study estimates the values of effective stiffness of reinforced concrete flanged beam section correctly and hence are valid.

V. Conclusions

The main points of conclusion can be summarized as follows:

1. The present work analyzed the reinforced concrete flanged beam section subjected to lateral loading with taking into account a cracking effect of reinforced concrete structure and proposed the expression to estimate the effective stiffness accurately.
2. Due to cracking, the main dominating factor affecting the nonlinear dependence of structure deformation from lateral load is stiffness reduction of reinforced concrete members.
3. Because of reduction in stiffness *(EI)* of structure, the lateral deflections of structure may exceed the limit given in seismic code and also the reduction in stiffness *(EI)* it directly affect on base shear, natural frequency, natural time period of structure, force distribution within members of structure and dynamic response of structure.
4. The stiffness *(EI)* of reinforced concrete flanged beam sections primarily depends on percentage of steel in section, D_f/D ratio, b_w/b_f ratio and grade of concrete and steel, which has significant influence on effective stiffness flanged beam sections.
5. The expression of the current work is also simplified method of the estimation of structure deflection will allow a quick evaluation of the behaviour of cracked structure under lateral loads.

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