

## Quantum Computer (Information) and Quantum Mechanical Behaviour- A Quid Pro Quo Model

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**Abstract:** Perception may not be what you think it is. Perception is not just a collection of inputs from our sensory system. Instead, it is the brain's interpretation (positive, negative or neutral-no signature case) of stimuli which is based on an individual's genetics and past experiences. Perception is therefore produced by (e) brain's interpretation of stimuli. The universe actually a giant quantum computer? According to Seth Lloyd--professor of quantum-mechanical engineering at MIT and originator of the first technologically feasible design for a working quantum computer--the answer is yes. Interactions between particles in the universe, Lloyd explains, convey not only (- to+) energy but also information-- In brief, a quantum is the smallest unit of a physical quantity expressing (anagrammatic expression and representation) a property of matter having both a particle and wave nature. On the scale of atoms and molecules, matter (e&eb) behaves in a quantum manner. The idea that computation might be performed more efficiently by making clever use (e) of the fascinating properties of quantum mechanics is nothing other than the quantum computer. In actuality, everything that happens (either positive or negative e&eb) in our daily lives conforms (one that does not break the rules) (e (e)) to the principles of quantum mechanics if we were to observe them on a microscopic scale, that is, on the scale of atoms and molecules. But because a great many degrees of freedom (such as a huge number of atomic movements) contribute to phenomena that we as human beings can perceive, this quantum mechanical behavior is normally hidden (not perceived-e(e)) by us. That it is not in the visible field; We cannot see it let alone decipher the progressive movements and dynamics of the system. Yet, if we were to look into the world of individual atoms, we would find that an electron (+) moving about the atomic nucleus can only take on (e) energy having specific (discrete) values. In other words, an electron may enter only a fixed number of set states. It is like I can build house in my site; not on somebody's site or on corporation designated area for public utility; This resembles the way in which strings on a guitar can only resonate at set frequencies, and(e some light&eb some light ) reflects the wave nature of quantum states. This electron, moreover, may take on a "**superposition state**" that combines (e&eb) different energy states simultaneously. Superposition state is important concept in quantum computing. Applying a strong electric field to an atom can also (eb) make the electrons circulating around it tunnel through a wall created (eb) by strong nuclear binding energy and (eb or=) become unbound. Although the tunneling of say a soccer ball through a wall does not occur in reality, this kind of phenomenon can occur in the microscopic world. Such quantum mechanical behavior must be artificially (e&eb) controlled and measured to achieve (eb) a quantum computer. Quantum computer thus utilizes (e) Quantum mechanical behaviour that is artificially controlled. Quantum mechanical behavior in state one controls (e&eb) Quantum Mechanical behaviour in state two. R. Schilling, M. Selecky, W. Baltensperger studied the influence (e&eb) of the hyperfine interaction  $\sim S_n$ . In between the ionic and the nuclear spins at the site n on the Eigenvalues of a 2-domain Heisenberg ferromagnetic with a 180°-domain wall. A level splitting is obtained even when  $\langle S_n \rangle = 0$  due (e) to quantum fluctuations The idea quantum states used for a computer first came about in the 1980s. In 1985, David Tsche, a professor at Oxford University and a proponent of quantum computers, wrote a paper titled "Quantum theory, the Church-Turing principle, and the universal quantum computer" that touched upon the possibility of quantum computers. Frank Verstraete, Michael M. Wolf & J. Ignacio Cirac STUDIED THE EFFECTS OF QUANTUM MECHANICAL STATES ON QUANTUM

**KEYWORDS:** Quantum mechanical states, Quantum computation, Decoherence, Quantum cryptography, Quantum simulation, Tunneling, nonadiabatic multiphonon process in the strong vibronic coupling limit, Schrodinger's Hamiltonian, Claude Shannon Theories of Redundancy and Noise, Kraus operators, a non-zero energy state,\*

### I. INFORMATON

The strongest adversary in quantum information science is decoherence, which (eb) arises owing to the coupling (e&eb) of a system with its environment. The induced dissipation tends to *destroy and (e) wash out* the interesting quantum effects that give (eb) rise to the power of quantum computation, cryptography and simulation. Whereas such a statement is true for many forms of dissipation, they showed that dissipation can also have exactly the opposite effect: it can be a fully fledged *resource* (eb) for universal quantum computation without any coherent dynamics needed to complement it. Universal Quantum Computation utilizes (e) decoherence. The coupling (e&eb) to the environment drives (eb) the system to a steady state where the outcome (eb) of the computation is encoded. In a similar vein, they showed that dissipation *can be (e) used* to engineer a large variety of strongly correlated states in steady state, including all stabilizer codes, matrix product states, and their generalization to higher dimensions. Words "e" and "eb" are used for better comprehension of the paper. They represent 'encompasses' and 'encompassed by'. There are no other attributions or ascriptions for the same. For the system of Quantum entanglement and Quantum Information we discuss the stability analysis, solution behaviour and asymptotic stability in detail. Asymptotic stability is proved for the system in accord with the extant obtention of appositve factors in the system.

## II. Constitution, Composition And Outlay Of The Paper

### 1: Review Of the Literature:

Under this head we take an intimate and hawk's look at the various aspectualities and attributions in the literature available. Quantum Information and Quantum Mechanical behaviour is a subject which is not a rarefied and moribund field. Quantum Information and Quantum Mechanical behaviour and consummation, consolidation, concretization, consubstantiation is a field which is many a time attempted to. Piece de resistance of the work is to put the study the concatenated formulated equations which has not been done earlier on terra firma. Under this head, in consideration to the fact that there are some Gordian Knots, we point out the extant and existential problems thereof. This helps develop a two pronged strategy: one it helps the reader and author and academician alike to appreciate the generalized strain of rationalized consistency and cumulative choice of variables for the development of the model and second, provides the cognitive orientation towards the model built itself in the sense that aspects like Theory of Classification, Dissipation Coefficient formulation, Accentuation Coefficient induction which are cat hectic evaluative integrational necessary mechanisms so that qubits act as individual components and componential clusterings with a certain predefined set of specification. In case of randomness, the constraints under which Model holds well are stated. Evaluative motivational orientation for the development of a Quantum Computer, with variable integration and role differentiation is obtained so that it suits our model. It is important that this factor is to be borne in mind. Introduction is not just a brush up on the topic but also a breeding ground to fulfill and render conceptual soundness and system orientation and process orientation to the model. While engaging the attention, on the introductory aspects, we also point to the integrative function, model adequacy, instrumental applicable orientations, and the implications of functional imperatives of Technological change are also perceived and given a decent guess. In Quantum Computers, structural relational context of qubits like quantum entanglement and when such an entanglement would become disentangles; its functional exigencies and contingencies are also stated. They are done in order to see that the model is developed further so that it acts as a better tool for vindication of certain objectives for which it is applied. Gritty narrative brings out the subtleties and nuances of the subject in addition to the fact how it would help the formulation of the Model. Additionally, expatiation, enucleation, elucidation and exposition of the points that are necessary for the formulation of the present problem are also notified. Here we study the aging process, dissipatory mechanism, obliteration, obfuscation and abjuration of the Vacuum energy and Quantum Field, with thrust on the problem solving capacity and state sytemal and processual thinking on the subject matter.

### 2. Work Suggested/Done:

Under this appellation, we enumerate the work done, namely the sole aim, primary objective and sum mum bonuum of the work done. In the extant case we give the formulation of the problem. Statement of governing equations for both Quantum Information and Quantum Mechanical behaviour, write down in unmistakable terms the conceptual jurisprudence, phenomenological methodology, formal characterization, programmatic and anagrammatic concatenation of the equations. We discuss in detail for the system Vacuum Energy and Quantum Field, the stability analysis, solution behaviour, asymptotic analysis, the three formidable but very important tools for the system to remain as sangfroid like salamander under various conditionalities or undergo transformation with environmental decoherence. This aspect throws light on hither to untouched regions of Quark similarity, Schwarzschild radius, Zero Point Energy, Quantum Chromo Dynamics, GTR, STR, Quarks, Gluons,, and the concomitant and corresponding accentuatory, corroboratory ,augmentatory or dissipatory relationship. As is stated in the foregoing, these factors are very important for the model to be put as a promethaleon, primogeniture and proponent for further study which the author intends to do. These constituent structures, transformational minimal conditions, structural morphology, dependent variability, normative aspect of expectations from the model are discussed. Integrative structure of Quantum Mechanical System of process sytemal orientation, with entanglement patterns on the relational level of entanglement, decoherence, redundancy, presumptuousness, is also studied taking in to consideration the overall collectivity. For instance, *Quantum Tunneling*, as well as the twelve order of magnitude *increase* of the low-temperature tunnelling rate constant on going from a spin-crossover compound with a small zero-point energy difference to a low-spin compound with a substantially larger one, *can be understood on the(e) basis* of a *nonadiabatic multiphonon process in the strong vibronic coupling limit is mentioned to drive home the importance of the Quantum Tunneling in the formulation of Quantum Computation and Quantum Computers*. In the case of QCD it is increased energy states and Dominant Asymptotic freedom that is responsible for the diffusion of parton momentum and diffusion scattering which form important role in the transference of Quantum Information. Many erudite studies are quoted as a fleeting mention so that the researchers have shown proactive approach to the encumbrances that have come their way in providing enriching contribution and mind boggling logistics of lack of misnomerliness in the accentuation of the production of the Quantum Computation.

## III. Conclusions

Under this category, we summarize the work done, namely the study of formulation of the Governing Equations, necessary sine qua non attributions like accentuation and dissipation which are essential functional prerequisite for the consummation and success of the model. It is to be stated that primary focus and locus is of homologues nature and differentially instrumentally activities of the model performance such as stability analysis, asymptotic analysis, solution behaviour, and the sententious and pithy prognostications under which the systems become functional or for that matter dysfunctional. We do not write a separate conclusive note. Herein itself is mentioned the holistic and generalizational view

of the work done, as has been done in the foregoing. Imperative compatibilities and structural variabilities of a *nonadiabatic multiphonon process in the strong vibronic coupling limit*, Vacuum Energy and Quantum Field vis a vis Quantum Computation are stated in unequivocal terms. Common patterns of phenomenological methodology, essential predications, suspensional neutralities, rational representations, conferential extrinsicness, interfacial interference and syncopated justice the model does for the generalized goal to be consummated is dealt with. Solutional behavior, stability analyses, asymptotic analysis bear ample testimony, infallible observatory, and impeccable demonstration to the predicational anteriority, character constitution, ontological consonance, primordial exactitude to the accolytish representation and apocryphal aneurism and associated asseveration of the Governing Equations and the obtention of Stability analysis which is dealt in detail ,stating what happens to the singularities, antigeneralities or event at contracted points, and normal performance of the model in normal conditions. This helps in the optimum development of the system of governing equations and dynamical improvement of the conditionalities for the instrumental efficaciousness of the stability, or reduction of asymptotic stability as the practical applications demand. No conjecture however is made because of ignorance of such possibilities and possibilities. Concatenated governing equations thus provide a qualitative gradient of internal structural differentiation, and diffuse solidarity abstraction. No separate conclusion statement is made in consideration to the fact that motivational orientation and institutionalization of pattern variables that are used have already been stated here. *At Planck's scale there might be energetic frantiness or ensorcelled frenzy of the Vacuum Energy and Quantum Field*, and these extrapolations have to be explored in detail by more eminent, erudite, and esteemed researchers. Vacuum energy is one type of energy which could be highly belligerently tempestuous and temerarily reckless when put to different uses. While a tendentious testament is not provided, a first step of a progenitor is taken for the intimate comprehension of the system. Further papers build on this framework towards the consummation of higher theories envisioned. This on the other hand provides a rich receptacle, reliquarium repository to other researchers to study practical applications right from the simple appliances like piston to highly sophisticated ones like in CERN. That any contribution that helps towards achievement and consummation of the power house performance of Quantum Computers is a fair accompli desideratum. It is in this direction, we have directed our thoughts and expositions for the better presentation of the subject matter. One doth hear portentous voice of doomsday Sayers, but it is better to listen to optimists and stick to the subterranean realm of spatio temporal actualization, in which Quantum Mechanism is non pariel and par excellent.

#### IV. INTRODUCTION

##### Quantum information

In quantum mechanics, quantum information is physical information that is held in the "state" of a quantum system. The most popular unit of quantum information is the qubit, a two-level quantum system. However, unlike classical digital states (which are discrete), a two-state quantum system can actually be in a superposition of the two states at any given time. Quantum information differs from classical information in several respects, among which we note the following:

- An arbitrary state cannot be cloned,
- The state may be in a *superposition* of basis values.

However, despite this, the amount of information that *can be retrieved* in a single qubit is equal to one bit. It is in the *processing* of information (quantum computation) that the differentiation *occurs*. The ability to *manipulate* quantum information enables us to perform tasks that would be unachievable in a classical context, such as unconditionally secure transmission of information. Quantum information processing is the most general field that is concerned with quantum information. There are certain tasks which classical computers cannot perform "efficiently" (that is, in polynomial time) according to any known algorithm. However, a quantum computer can compute the answer to some of these problems in polynomial time; one well-known example of this is Shor's factoring algorithm. Other algorithms can speed up a task less dramatically—for example, Grover's search algorithm which gives a quadratic speed-up over the best possible classical algorithm.

Quantum information, and changes in quantum information, can be quantitatively measured by using an analogue of Shannon entropy, called the von Neumann entropy. Given statistical of quantum mechanical systems with the density matrix  $\rho$ , it is given by

$$S(\rho) = -\text{Tr}(\rho \ln \rho).$$

Many of the same entropy measures in classical information theory can also be generalized to the quantum case, such as Holevo entropy and the conditional quantum entropy.

##### Quantum information theory

The theory of quantum information is a result of the effort to generalize classical information theory to the quantum world. Quantum information theory aims to answer the following question: What happens if information is stored in a state of a quantum system?

One of the strengths of classical information theory is that physical representation of information can be disregarded: There is no need for an 'ink-on-paper' information theory or a 'DVD information' theory. This is because it is always possible to efficiently transform information from one representation to another. However, this is not the case for

quantum information: it is not possible, for example, to write down on paper the previously unknown information contained in *the polarization* of a photon.

In general, quantum mechanics does not allow us to read out the state of a quantum system with arbitrary precision. The existence of Bell correlations between quantum systems cannot be converted into classical information. It is only possible to transform quantum information between quantum systems of sufficient information capacity. The information content of a message  $\mathcal{M}$  can, for this reason, be measured in terms of the minimum number  $n$  of two-level systems which are needed to store the message:  $\mathcal{M}$  consists of  $n$  qubits. In its original theoretical sense, the term qubit is thus a measure for the amount of information. A two-level quantum system can carry at most one qubit, in the same sense a classical binary digit can carry at most one classical bit. As a consequence of the noisy-channel coding theorem, noise limits the information content of an analog information carrier to be finite. It is very difficult to protect the remaining finite information content of analog information carriers against noise. The example of classical analog information shows that quantum information processing schemes must necessarily be tolerant against noise, otherwise there would not be a chance for them to be useful. It was a big breakthrough for the theory of quantum information, when quantum error correction codes and fault-tolerant quantum computation schemes were discovered.

THE precise manner in which quantum-mechanical behaviour at the microscopic level underlies classical behaviour at the macroscopic level remains unclear, despite seventy years of theoretical investigation. Experimentally, the crossover between these regimes can be explored by looking for *signatures* of quantum-mechanical behaviour—such as tunneling—in macroscopic systems. Magnetic systems (such as small grains, spin glasses and thin films) are often investigated in this way *because (e)* transitions between different magnetic states can be closely monitored. But transitions between states *can be (e) induced* by thermal fluctuations, as well as by *tunnelling*, and definitive *identification* of macroscopic tunnelling events in these complex systems is therefore difficult. In an applied magnetic field, the magnetization *shows (eb)* hysteresis loops *with (e&eb)* a distinct 'staircase' structure: the steps *occur (eb)* at values of the applied field where the energies of different collective spin states of the manganese clusters coincide. At these special values of the field, *relaxation* from one spin state to another is *enhanced* above the thermally activated rate by the *action (e)* of resonant quantum-mechanical tunnelling. These observations corroborate the results of similar experiments performed recently on a *system of oriented* crystallites made from a powdered sample

Intersystem crossing is the crucial first step *determining (eb)* the quantum efficiency of very many photochemical and photo physical processes. Spin-crossover compounds of first-row transition metal ions, in particular of Fe (II), provide model systems for studying it in detail. Because in these compounds there are no competing relaxation processes, intersystem crossing rate constants can be *determined (eb)* over a large temperature interval. The characteristic features are tunnelling at temperatures below  $\sim 80$  K and a thermally activated process above  $\sim 100$  K.

This *Quantum Tunneling*, as well as the twelve order of magnitude *increase* of the low-temperature tunnelling rate constant on going from a spin-crossover compound with a small zero-point energy difference to a low-spin compound with a substantially larger one, *can be understood on the(e) basis* of a *nonadiabatic multiphonon process in the strong vibronic coupling limit*.

### Quantum mechanics is Quantum Information:

Quantum information theory deals with four main topics:

(1) Transmission of classical information over quantum channels. (2) The *tradeoff between* acquisition of information about a quantum state *and (e&eb)* disturbance of the state (3) Quantifying quantum entanglement (4) *Transmission of* quantum information over quantum channels. As a precursor, promethaleon and primogeniture to the comprehension of Von Neumann entropy and its relevance to quantum information, calls for Shannon entropy and its relevance to classical information. Claude Shannon established the two core results of classical information theory in his landmark 1948 paper. The two central problems that he solved were: (1) How much can a message be *compressed*; i.e., how redundant is the information? (The "noiseless coding theorem.") (2) At what rate can we communicate reliably over a noisy channel; i.e., how much redundancy must be incorporated into a message to *protect (e) against* errors? I.e./ redundancy doth reduce errors. (The "noisy channel coding theorem.") Both questions concern redundancy – how unexpected is the next letter of the message, on the average. One of Shannon's key insights was that entropy *provides a(eb) suitable way* to quantify redundancy. Or, redundancy helps reduce (e) entropy Quantum mechanics (QM - also known as quantum physics, or quantum theory) is a branch of physics dealing with physical phenomena where the action is on the order of the Planck constant. Quantum mechanics *departs from* classical mechanics primarily at the *quantum realm* of atomic and subatomic length scales. *QM provides* a mathematical description of much of the dual *particle-like* and *wave-like* behavior *and interactions* of energy and matter. All objects *exhibit* wave/particle duality to some extent, but the larger the object the harder it is to observe. Observation is proportional to largeness of the objects. Even individual molecules are often too large to show the quantum mechanical behavior. Now physicists at the Université de Paris have demonstrated wave/particle duality with a droplet made of trillions of molecules. The experiment involved an oil droplet *bouncing* on the surface of an agitated layer of oil. The droplet *created* waves on the surface, which in turn *affected* the motion of the droplet. As a result, the droplet and waves *formed* a single entity that *consisted* of a hybrid of wave-like and particle-like characteristics. When the wave/droplet bounced its way through a slit, the waves allowed it to *interfere with* its own motion, much as a single photon *can interfere* with itself via quantum mechanics. Although the wave/droplet is clearly a denizen of the classical world, the experiment *provides a clever analogue* of quantum weirdness at a scale that is much easier to study and visualize than is typical of many true quantum experiments. In advanced topics of quantum mechanics, some of these behaviors are macroscopic and only *emerge* at extreme (i.e., very low or very high) energies or temperatures. The name *quantum mechanics* derives from the

observation that some physical quantities can *change* only *indiscrete* amounts (Latin *quanta*), and not in a continuous (*cf.* analog) way. For example, the angular momentum of an electron *bound to* an atom or molecule is quantized. In the context of quantum mechanics, the wave-particle duality of energy and matter and the uncertainty principle provide a *unified view* of the behavior of photons, electrons, and other atomic-scale objects.

The mathematical formulations of quantum mechanics are abstract. A mathematical function called the wavefunction *provides* information about the probability amplitude of position, momentum, and other physical properties of a particle. Mathematical *manipulations* of the wavefunction usually *involve the* bra-ket notation, which requires an understanding of complex numbers and linear functional. The wavefunction *treats the object as a* quantum harmonic oscillator, and the mathematics is akin to that *describing* acoustic resonance. Many of the results of quantum mechanics are not easily visualized in terms of classical mechanics - for instance, the ground state in a quantum mechanical model *is a non-zero energy state* that is the lowest permitted energy state of a system, as opposed a more "traditional" system that is thought of *as simply being at rest*, with zero kinetic energy. Instead of a traditional static, *unchanging zero state*, quantum mechanics allows for far more *dynamic, chaotic possibilities*, according to John Wheeler.

The earliest versions of quantum mechanics were formulated in the first decade of the 20th century. At around the same time, the atomic theory and the corpuscular theory of light (as updated by Einstein) first came to be widely accepted as scientific fact; these latter theories *can be viewed as* quantum theories of matter and electromagnetic radiation, respectively. Early quantum theory was significantly reformulated in the mid-1920s by Werner Heisenberg, Max Born, Wolfgang Pauli and their collaborators, and the Copenhagen interpretation of Niels Bohr became widely accepted. By 1930, quantum mechanics had been further *unified* and formalized by the work of Paul Dirac and John von Neumann, with a greater emphasis placed on measurement in quantum mechanics, the statistical nature of our knowledge of reality, and philosophical speculation about the role of the observer. Quantum mechanics has since branched out into almost every aspect of 20th century physics and other disciplines, such as *quantum chemistry, quantum electronics, quantum optics, and quantum information science*. Much 19th century physics has been re-evaluated as the "classical limit" of quantum mechanics, and its more advanced developments in terms of *quantum field theory, string theory, and speculative quantum gravity theories*.

#### A HISTORICAL NITTY GRITTY PERSPECTIVE AND FUTURISTIC PROGNOSTICATION:

The history of quantum mechanics dates back to the 1838 discovery of cathode rays by Michael Faraday. This was followed by the 1859 statement of the black body radiation problem by Gustav Kirchhoff, the 1877 suggestion by Ludwig Boltzmann that the *energy states of a physical system can be discrete*, and the 1900 quantum hypothesis of Max Planck. Planck's hypothesis that *energy is radiated and absorbed* in discrete "quanta" (or "energy elements") precisely matched the *observed patterns of blackbody radiation*. According to Planck, each energy element  $E$  is proportional to its frequency  $\nu$ :

$$E = h\nu$$

Where  $h$  is Planck's constant. Planck (cautiously) insisted that this was simply an aspect of the *processes of absorption and emission of radiation* and had nothing to do with the *physical reality* of the radiation itself. However, in 1905 Albert Einstein interpreted Planck's quantum hypothesis realistically and used it to explain the photoelectric effect, in which shining light on certain materials *can eject* electrons from the material. The foundations of quantum mechanics were established during the first half of the 20th century by Niels Bohr, Werner Heisenberg, Max Planck, Louis de Broglie, Albert Einstein, Erwin Schrödinger, Max Born, John von Neumann, Paul Dirac, Wolfgang Pauli, David Hilbert, and others. In the mid-1920s, developments in quantum mechanics *led to its* becoming the standard formulation for atomic physics. In the summer of 1925, Bohr and Heisenberg published results that closed the "Old Quantum Theory". Out of deference to their particle-like behavior in certain processes and measurements, light quanta came to be called photons (1926). From Einstein's simple postulation was born a flurry of debating, theorizing, and testing. Thus the entire field of quantum physics *emerged, leading to its wider acceptance* at the Fifth Solvay Conference in 1927. The other exemplar that led to quantum mechanics was the study of electromagnetic waves, such as visible light. When it was found in 1900 by Max Planck that the energy of waves could be described as consisting of small packets or "quanta", Albert Einstein further developed this idea to show that an electromagnetic wave such as light could be described as a particle (later called the photon) with a discrete quantum of energy that was *dependent on its frequency*. This led to a theory of *unity* between subatomic particles and electromagnetic waves, called wave-particle duality, in which particles and waves were neither one nor the other, but had certain properties of both. While quantum mechanics traditionally described the world of the very small, it is also needed to explain certain recently investigated macroscopic systems such as superconductors and superfluids.

The word *quantum* derives from the Latin, meaning "how great" or "how much". In quantum mechanics, it refers to a discrete unit that quantum theory *assigns to* certain physical quantities, such as the energy of an atom at rest. The discovery that particles are discrete packets of energy with wave-like properties *led to the* branch of physics dealing with atomic and sub-atomic systems which is today called quantum mechanics. It is the underlying mathematical framework of many fields of physics and chemistry, including condensed matter physics, solid-state physics, atomic physics, molecular physics, computational physics, computational chemistry, quantum chemistry, particle physics, nuclear chemistry, and nuclear physics. Some fundamental aspects of the theory are still actively studied. Quantum mechanics is essential to understanding the behavior of systems at atomic length scales and smaller. For example, if classical mechanics *truly governed* the workings of an atom, electrons *would rapidly travel toward, and collide with*, the nucleus, *making* stable atoms impossible. *However, in the natural world electrons normally remain in an uncertain, non-deterministic,*

"smeared", probabilistic wave-particle wavefunction orbital path around (or through) the nucleus, defying classical electromagnetism. Atlas, the fulminating avenger, crackling debutante with seething intensity has taken its pride of place.

Quantum mechanics was initially developed to provide a better explanation of the atom, especially the differences in the spectra of light emitted by different isotopes of the same element. The quantum theory of the atom was developed as an explanation for the electron remaining in its orbit, which could not be explained by Newton's laws of motion and Maxwell's of (classical) electromagnetism.

Broadly speaking, quantum mechanics incorporates four classes of phenomena for which classical physics cannot account: The quantization of certain physical properties; Wave; The Uncertainty principle; Quantum. Gravity

### Mathematical formulations OF Quantum Mechanics:

In the mathematically rigorous formulation of quantum mechanics developed by Paul Dirac and John von Neumann, the possible states of a quantum mechanical system are represented by unit vectors (called "state vectors"). Formally, these reside in a complex separable Hilbert space - variously called the "state space" or the "associated Hilbert space" of the system - that is well defined up to a complex number of norm 1 (the phase factor). In other words, the possible states are points in the projective space of a Hilbert space, usually called the *complex projective space*. The exact nature of this Hilbert space is dependent on the system - for example, the state space for position and momentum states is the space of square-integrable functions, while the state space for the spin of a single proton is just the product of two complex planes. Each observable is represented by a maximally Hermitian (precisely: by a self-adjoint) linear operator acting on the state space. Each eigenstate of an observable corresponds to an eigenvector of the operator, and the associated eigenvalue corresponds to the value of the observable in that eigenstate. If the operator's spectrum is discrete, the observable can only attain those discrete eigenvalues.

In the formalism of quantum mechanics, the state of a system at a given time is described by a complex wave function, also referred to as state vector in a complex vector space. This abstract mathematical object allows for the calculation of probabilities of outcomes of concrete experiments. For example, it allows one to compute the probability of finding an electron in a particular region around the nucleus at a particular time. Contrary to classical mechanics, one can never make simultaneous predictions of conjugate variables, such as position and momentum, with accuracy. For instance, electrons may be considered (to a certain probability) to be located somewhere within a given region of space, but with their exact positions unknown. Contours of constant probability, often referred to as "clouds", may be drawn around the nucleus of an atom to conceptualize where the electron might be located with the most probability. Heisenberg's uncertainty principle quantifies the inability to precisely locate the particle given its conjugate momentum. According to one interpretation, as the result of a measurement the wave function containing the probability information for a system collapses from a given initial state to a particular eigenstate. The possible results of a measurement are the eigenvalues of the operator representing the observable — which explains the choice of Hermitian operators, for which all the eigenvalues are real. The probability distribution of an observable in a given state can be found by computing the spectral decomposition of the corresponding operator. Heisenberg's uncertainty principle is represented by the statement that the operators corresponding to certain observables do not commute. That is they cannot be converted, go back and forth, cannot be transformed. There is a lot of discussion and deliberation at the level of being polemical. Many people including the author aver that consciousness or the presence of consciousness makes the Truth explicit.

The probabilistic nature of quantum mechanics thus stems from the act of measurement. This is one of the most difficult aspects of quantum systems to understand. It was the central topic in the famous Bohr-Einstein debates, in which the two scientists attempted to clarify these fundamental principles by way of thought experiments. In the decades after the formulation of quantum mechanics, the question of what constitutes a "measurement" has been extensively studied. Newer interpretations of quantum mechanics have been formulated that do away with the concept of "wavefunction collapse" (see, for example, the relative state interpretation). The basic idea is that when a quantum system interacts with a measuring apparatus, their respective wave functions become entangled, so that the original quantum system ceases to exist as an independent entity. Generally, quantum mechanics does not assign definite values. Instead, it makes a prediction using a probability distribution; that is, it describes the probability of obtaining the possible outcomes from measuring an observable. Often these results are skewed by many causes, such as dense probability clouds. Probability clouds are approximate, but better than the Bohr model, whereby electron location is given by a probability function, the wave function eigenvalued, such that the probability is the squared modulus of the complex amplitude, or quantum state nuclear attraction. Naturally, these probabilities will depend on the quantum state at the "instant" of the measurement. Hence, uncertainty is involved in the value. There are, however, certain states that are associated with a definite value of a particular observable. These are known as eigenstates of the observable ("eigen" can be translated from German as meaning "inherent" or "characteristic"). In the everyday world, it is natural and intuitive to think of everything (every observable) as being in an eigenstate. Everything appears to have a definite position, a definite momentum, a definite energy, and a definite time of occurrence. However, quantum mechanics does not pinpoint the exact values of a particle's position and momentum (since they are conjugate pairs) or its energy and time (since they too are conjugate pairs); rather, it only provides a range of probabilities of where that particle might be given its momentum and momentum probability. Therefore, it is helpful to use different words to describe states having uncertain values and states having definite values (eigenstates). Usually, a system will not be in an eigenstate of the observable (particle) we are interested in. However, if one measures the observable, the wavefunction will instantaneously be an eigenstate (or "generalized" eigenstate) of that observable. This process is known as wavefunction collapse, a controversial and much-debated process that involves expanding the system under study to include the measurement device. If one knows the corresponding wave function at the instant before the measurement, one

will be able to compute the probability of the wavefunction collapsing into each of the possible eigenstates. For example, the free particle in the previous example will usually have a wavefunction that is a packet centered on some mean position  $x_0$  (neither an eigenstate of position nor of momentum). When one measures the position of the particle, it is impossible to predict with certainty the result. It is probable, but not certain, that it will be near  $x_0$ , where the amplitude of the wave function is large. After the measurement is performed, having obtained some result  $x$ , the wave function collapses into a position eigenstate centered at  $x$ .

#### STATEDESCRIPTION:

All physical states of a quantum system are described mathematically by a set at most countable of positive numbers  $p_k$ ,  $\sum_k p_k = 1$  and unit norm vectors  $\psi_k$  in a complex separable Hilbert space  $H$ .

#### QUANTIZATION:

- The physical observables of the quantum theory are described through linear self-adjoint operators on the Hilbert space of states.
- For classical systems with Hamiltonians at most quadratic in momenta, the classical observables  $p, q$  are described by the closures (in the Hilbert space topology) of the following operators obeying the Born-Jordan commutation relations:  $[q, p] = i\hbar 1_H$  on the common dense everywhere domain of  $p$  and  $q$ .

#### Hamiltonian

In quantum mechanics, the Hamiltonian is the operator corresponding to the total energy of the system. It is usually denoted by  $H$ , also  $\hat{H}$  or  $\hat{H}$ . Its spectrum is the set of possible outcomes when one measures the total energy of a system. Because of its close relation to the time-evolution of a system, it is of fundamental importance in most formulations of quantum theory. The Hamiltonian is the sum of the kinetic energies of all the particles, plus the potential energy of the particles associated with the system. For different situations and/or number of particles, the Hamiltonian is different since it includes the sum of kinetic energies of the particles, and the potential energy function corresponding to the situation.]

#### The Schrödinger Hamiltonian AND Quantum Decoherence:

Decoherence in quantum-computer memory due to the inevitable coupling to the external environmental quantum bits (qubits) interact with the same environment rather than the assumption of separate environments for different qubits. It is found that the qubits decohere collectively. For some kinds of entangled input states, no decoherence occurs at all in the memory, even if the qubits are interacting with the environment. Based on this phenomenon, SOME METHODOLOGIES AND MODALITIES ARE PROPOSED for reducing the collective decoherence. Decoherence model has implications for quantum measurements.

Quantum computation. Suppose we are given a quantum system with a Hamiltonian of the form  $E|w\rangle\langle w|$  where  $|w\rangle$  is an unknown (normalized) state. The problem is to produce  $|w\rangle$  by adding a Hamiltonian (independent of  $|w\rangle$ ) and evolving the system. If  $|w\rangle$  is chosen uniformly at random we can (with high probability) produce  $|w\rangle$  in a time proportional to  $N^{1/2}/E$ . If  $|w\rangle$  is instead chosen from a fixed, known orthonormal basis we can also produce  $|w\rangle$  in a time proportional to  $N^{1/2}/E$  and we show that this time is optimally short. This restricted problem is an analog analogue to Grover's algorithm, a computation on a conventional (!) quantum computer that locates a marked item from an unsorted list of  $N$  items in a number of steps proportional to  $N^{1/2}$ .

#### One particle

By analogy with classical mechanics, the Hamiltonian is commonly expressed as the sum of operators corresponding to the kinetic and potential energies of a system, in the form

$$\hat{H} = \hat{T} + \hat{V}$$

where

$$\hat{V} = V = V(\mathbf{r}, t)$$

is the potential energy operator;

$$\hat{T} = \frac{\hat{\mathbf{p}} \cdot \hat{\mathbf{p}}}{2m} = \frac{\hat{p}^2}{2m} = -\frac{\hbar^2}{2m} \nabla^2$$

is the kinetic energy operator, where  $m$  is the mass of the particle, the dot denotes the dot product of vectors, and;

$$\hat{p} = -i\hbar \nabla$$

is the momentum operator, wherein  $\nabla$  is the gradient operator. The dot product of  $\nabla$  with itself is the laplacian  $\nabla^2$ , in three dimensions using Cartesian coordinates the Laplace operator is

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

Although this is not the technical definition of the Hamiltonian in classical mechanics, it is the form it most commonly takes. Combining these together yields the familiar form used in the Schrödinger equation:

$$\begin{aligned}\hat{H} &= \hat{T} + V \\ &= \frac{\hat{\mathbf{p}} \cdot \hat{\mathbf{p}}}{2m} + V(\mathbf{r}, t) \\ &= -\frac{\hbar^2}{2m} \nabla^2 + V(\mathbf{r}, t)\end{aligned}$$

Which allows one to apply the Hamiltonian to systems described by a wave function  $\Psi(\mathbf{r}, t)$ . This is the approach commonly taken in introductory treatments of quantum mechanics, using the formalism of Schrödinger's wave mechanics.

### BILLIONS OF ENTANGLED PARTICLES ADVANCE QUANTUM COMPUTING:

John Markoff filed in New York Times the report that in a step toward a generation of ultrafast computers, physicists have used bursts of radio waves to *briefly create* 10 billion quantum-entangled pairs of subatomic particles in silicon. The research offers a glimpse of a future computing world in which individual atomic nuclei store and retrieve data and single electrons *shuttle it back and forth*. In a paper in the journal Nature, a team led by the physicists John Morton of Oxford University and Kohei Itoh of Keio University describes bombarding a three-dimensional crystal with microwave and radio frequency pulses *to create* the entangled pairs. This is one of a range of competing approaches to making qubits, the quantum computing *equivalent of today's* transistors.

Transistors *store* information on the basis of whether they are *on or off*. In the experiment, qubits *store* information in the form of the *orientation, or spin, of an atomic nucleus or an electron*. The storage ability is *dependent on entanglement*, in which a change in one particle *instantaneously affects another* particle even if they are widely separated. The new approach has significant potential, because it might permit quantum computer designers *to(e) exploit* low-cost and easily manufacturable components and technologies now widely used in the consumer electronics industry. As at present there are only a few qubits, albeit an ambitious programme has been chalked out for the production of millions of such qubits. In today's binary computers, transistors can be in either an "on" or an "off" state, but quantum computing *exploits(e) the* notion of superposition, in which a qubit can be *constructed to* represent both a 1 and a zero state simultaneously. The potential power of quantum computing comes from the possibility of performing a *mathematical(+xetc.,) operation on* both states simultaneously. *In a two-qubit system it would be possible to compute on four values at once, in a three-qubit system on eight, in a four-qubit system on 16, and so on. As the number of qubits grows, potential (e&eb)processing power increases exponentially.*

There is, of course, a catch. The mere act of measuring or observing a qubit can *strip(e) it* of its computing potential. So researchers have used quantum entanglement — in which particles *are(e&eb) linked so* that measuring a property of one *instantly reveals(eb)* information about the other, no matter how far apart the two particles are — to extract information. But creating and maintaining qubits in entangled states has been tremendously challenging. The new approach is based on a purified silicon isotope doped with phosphorus atoms. The research group was able *to create and measure* vast numbers of quantum-entangled pairs of atomic nuclei and electrons when the crystal was cooled to about 3 kelvin. Scientists *to produce the basis* for a quantum computing system by moving the entangled electrons to simultaneously *entangle(e&eb) them with* a second nucleus.

"We would move the electron from the nuclear spin it is on to the neighboring nuclear spin," says Dr. Morton. Electrons thus gain (+) the nuclear spin of neighboring electron but loses (e) its own spin. "That shifting step is what we really now need to show works while preserving entanglement." One of the principal advantages of the new silicon-based approach is that the group believes that it will be *able(eb) to maintain* the entangled state needed to preserve quantum information as long as several seconds, far longer than competing technologies which currently measure the persistence of entanglement for billionths of a second.

For quantum information, the *lifetime* of a second is very exciting, *because* there are ways to refresh data. The advance indicates there is an impending convergence between the subatomic world of quantum computers and today's classical microelectronic systems, which are reaching a level of miniaturization in which wires and devices are composed of just dozens or hundreds of atoms. This is on a single-nucleus scale, but it isn't that far away from what is being used today," said Stephanie Simmons, a graduate physics researcher at Oxford and the lead author of the paper. One is its power, but the other is that the size of silicon transistors is shrinking to the point where quantum effects are becoming important.

### Quantum Formalism Extended To N Particles Like In Classical Computing Portentious Voice Of Quantum Computing::

$$\hat{H} = \sum_{n=1}^N \hat{T}_n + V$$

where

$$V = V(\mathbf{r}_1, \mathbf{r}_2 \cdots \mathbf{r}_N, t)$$

is the potential energy function, now a function of the spatial configuration of the system and time (*a particular set of spatial positions at some instant of time defines a configuration*) and;

$$\hat{T}_n = \frac{\mathbf{p}_n \cdot \mathbf{p}_n}{2m_n}$$

is the kinetic energy operator of particle  $n$ , and  $\nabla_n$  is the gradient for particle  $n$ ,  $\nabla_n^2$  is the Laplacian for particle using the coordinates:



$$\nabla_n^2 = \frac{\partial^2}{\partial x_n^2} + \frac{\partial^2}{\partial y_n^2} + \frac{\partial^2}{\partial z_n^2}$$

Combining these together yields the Schrödinger Hamilton for the  $N$ -particle case:

$$\begin{aligned}\hat{H} &= \sum_{n=1}^N \hat{T}_n + V \\ &= \sum_{n=1}^N \frac{\hat{\mathbf{p}}_n \cdot \hat{\mathbf{p}}_n}{2m_n} + V(\mathbf{r}_1, \mathbf{r}_2 \cdots \mathbf{r}_N, t) \\ &= -\frac{\hbar^2}{2} \sum_{n=1}^N \frac{1}{m_n} \nabla_n^2 + V(\mathbf{r}_1, \mathbf{r}_2 \cdots \mathbf{r}_N, t)\end{aligned}$$

However, complications can arise in the *many-body problem*. Since the potential energy *depends on* the spatial arrangement of the particles, the kinetic energy *will also depend* on the spatial configuration to conserve energy. The motion due to any one particle *will vary due to* the motion of all the other particles in the system. For this reason cross terms for kinetic energy may appear in the Hamiltonian; a mix of the gradients for two particles:

$$-\frac{\hbar^2}{2M} \nabla_i \cdot \nabla_j$$

Where  $M$  denotes the mass of the collection of particles *resulting in* this extra kinetic energy. Terms of this form are known as *mass polarization terms*, and appear in the Hamiltonian of many electron atoms. For  $N$  interacting particles, i.e. particles *which interact mutually and constitute* a many-body situation, the potential energy function  $V$  is *not* simply a sum of the separate potentials (and certainly not a product, as this is dimensionally incorrect). The potential energy function can only be written as above: *a function of all the spatial positions of each particle*.

For non-interacting particles, i.e. particles *which do not interact* mutually and move independently, the potential of the *system is the* sum of the separate potential energy for each particle, that is

$$V = \sum_{i=1}^N V(\mathbf{r}_i, t) = V(\mathbf{r}_1, t) + V(\mathbf{r}_2, t) + \cdots + V(\mathbf{r}_N, t)$$

The general form of the Hamiltonian in this case is:

$$\begin{aligned}\hat{H} &= -\frac{\hbar^2}{2} \sum_{i=1}^N \frac{1}{m_i} \nabla_i^2 + \sum_{i=1}^N V_i \\ &= \sum_{i=1}^N \left( -\frac{\hbar^2}{2m_i} \nabla_i^2 + V_i \right) \\ &= \sum_{i=1}^N \hat{H}_i\end{aligned}$$

Where *the sum is taken over all particles and their corresponding potentials*; the result is that the *Hamiltonian of the system is the sum of the separate Hamiltonians for each particle*. This is an idealized situation - in practice the particles are usually *always influenced* by some potential, and there are many-body *interactions*. One illustrative example of a two-body interaction where this form would not apply is for electrostatic potentials due to charged particles, because they certainly *do interact with each other* by the coulomb interaction.

## SCHRODINGER EQUATION AND QUANTUM FORMALISM:

Most computer and information scientists believe that the next big leap forward in computing will be the invention of a quantum computer. Actually, there are people already at work on such a device and very basic prototypes are under scrutiny. However, there's a problem with quantum computing and it has to do with a certain cat. Erwin Schrödinger, an Austrian physicist, one proposed a thought experiment. Take a cat and put it in a box with a deadly poison. Hook the poison up to a Geiger counter which will detect radiation from a substance that decays at the rate of one atom per hour. If the counter detects a radioactive effect, the poison is released and the cat dies. If not, then the cat lives. Now, seal the box and protect it from outside influence. At that point we don't know the fate of the cat. The radioactive substance might lose an atom, it might not. Because of this, the cat can be seen as being alive and dead at the same time.

Only when we open the box and observe the cat do we collapse the probabilities into a single reality.

This, in a nutshell, is how a quantum computer works. We take quantum superpositions in atoms or particles and change them to represent data. So instead of a transistor's power state (on or off) representing a 1 or 0, the spin of an electron indicates a 1 or 0. However, quantum physics indicates that things like spin and superpositions can exist in multiple states at the same time, just like the cat in the box. Only when we observe them do the probabilities fall into reality. This is called wave function collapse. Quantum mechanics says that some particles exist in multiple states simultaneously, kind of like how light behaves as both a particle and as a wave. As long as nothing observes the particle, it remains in multiple states and perhaps even in multiple places. But, as soon as something or someone observes the particle, it snaps into one state. In other words, a quantum computer must first protect the atoms manipulating the data from direct observation. A mere glance makes the whole thing fall apart. So while progress is being made on the quantum computer, there's a long way to go.

**HAMILTONIAN AND QUANTUM INFORMATION:**

The Hamiltonian generates the time evolution of quantum states. If  $|\psi(t)\rangle$  is the state of the system at time  $t$ , then

$$H |\psi(t)\rangle = i\hbar \frac{\partial}{\partial t} |\psi(t)\rangle .$$

This equation is the Schrödinger equation. (It takes the same form as the Hamilton–Jacobi equation, which is one of the reasons  $H$  is also called the Hamiltonian). Given the state at some initial time ( $t = 0$ ), we can solve it to obtain the state at any subsequent time. In particular, if  $H$  is independent of time, then

$$|\psi(t)\rangle = e^{-iHt/\hbar} |\psi(0)\rangle .$$

The exponential operator on the right hand side of the Schrödinger equation is usually defined by the corresponding power series in  $H$ . One might notice that taking polynomials or power series of unbounded operators that are not defined everywhere may not make mathematical sense. Rigorously, to take functions of unbounded operators, a functional calculus is required. In the case of the exponential function, the continuous, or just the holomorphic functional calculus suffices. We note again, however, that for common calculations the physicists' formulation is quite sufficient.

**Adiabatic quantum computation** (AQC) relies on the adiabatic theorem to do calculations. First, a complex Hamiltonian is found whose ground state describes the solution to the problem of interest. Next, a system with a simple Hamiltonian is prepared and initialized to the ground state. Finally, the simple Hamiltonian is adiabatically evolved to the complex Hamiltonian. By the adiabatic theorem, the system remains in the ground state, so at the end the state of the system describes the solution to the problem.

AQC is a possible method to get around the problem of energy relaxation. Since the quantum system is in the ground state, interference with the outside world cannot make it move to a lower state. If the energy of the outside world (that is, the "temperature of the bath") is kept lower than the energy gap between the ground state and the next higher energy state, the system has a proportionally lower probability of going to a higher energy state. Thus the system can stay in a single system eigenstate as long as needed.

Universality results in the adiabatic model are tied to quantum complexity and QMA-hard problems. The  $k$ -local Hamiltonian is QMA-complete for  $k \geq 2$ . QMA-hardness results

$$H = \sum_i h_i Z_i + \sum_{i < j} J^{ij} Z_i Z_j + \sum_{i < j} K^{ij} X_i X_j$$

are known for physically realistic lattice models of qubits such as

where  $Z, X$  represent the Pauli matrices  $\sigma_z, \sigma_x$ . Such models are used for universal adiabatic quantum computation. The Hamiltonians for the QMA-complete problem can also be restricted to act on a two dimensional grid of qubits or a line of quantum particles with 12 states per particle and if such models were found to be physically realizable, they too could be used to form the building blocks of a universal adiabatic quantum computer.

In practice, there are problems during a computation. As the Hamiltonian is gradually changed, the interesting parts (quantum behaviour as opposed to classical) occur when multiple qubits are close to a tipping point. It is exactly at this point when the ground state (one set of qubit orientations) gets very close to a first energy state (a different arrangement of orientations). Adding a slight amount of energy (from the external bath, or as a result of slowly changing the Hamiltonian) could take the system out of the ground state, and ruin the calculation. Trying to perform the calculation more quickly increases the external energy; scaling the number of qubits makes the energy gap at the tipping points smaller

By the \*-homomorphism property of the functional calculus, the operator

$$U = e^{-iHt/\hbar}$$

is a unitary operator. It is the *time evolution operator*, or *propagator*, of a closed quantum system. If the Hamiltonian is time-independent,  $\{U(t)\}$  form a one parameter unitary group (more than a semi group); this gives rise to the physical principle of detailed balance

**DIRAC FORMALISM AND RAMIFICATIONS IN QUANTUM INFORMATION:**

Despite many common concepts with classical computer science, quantum computing is still widely considered as a special discipline within the broad field of theoretical physics. One reason for the slow adoption of QC by the computer science community is the confusing variety of formalisms (Dirac notation, matrices, gates, operators, etc.), none of which has any similarity with classical programming languages, as well as the rather "physical" terminology in most of the available literature. QCL (Quantum Computation Language) tries to fill this gap: QCL is a high level, architecture independent programming language for quantum computers, with a syntax derived from classical procedural languages like C or Pascal. This allows for the complete implementation and simulation of quantum algorithms (including classical components) in one consistent formalism. However, in the more general formalism of Dirac, the Hamiltonian is typically implemented as an operator on a Hilbert space in the following way: The eigenkets (eigenvectors)  $H$  provide

an **orthonormal basis** for the Hilbert space. The spectrum of allowed energy levels of The system is given by the set of

eigenvalues, denoted  $\{E_a\}$ , solving the equation:

Since  $H$  is a Hermitian operator, the energy is always a real number.

From a mathematically rigorous point of view, care must be taken with the above assumptions. Operators on infinite-dimensional Hilbert spaces **need not have** eigenvalues (the set of eigenvalues does not necessarily coincide with the spectrum of an operator). However, all routine quantum mechanical calculations can be done **using** the physical formulation.

Following are expressions for the Hamiltonian in a number of situations. **Typical ways to classify the expressions are the number of particles, number of dimensions, and the nature of the potential energy function - importantly space and time dependence. Masses are denoted by  $m$ , and charges by  $q$ .**

### General forms for one particle

#### Free particle

The particle is not bound by any potential energy, so the potential is zero and this Hamiltonian is the simplest. For

#### one dimension:

and in three dimensions:

$$\hat{H} = -\frac{\hbar^2}{2m}\nabla^2$$

### CONSTANT POTENTIAL WELL AND QUANTUM INFORMATION:

#### Potential Well and Quantum Computer

In physics, a bounded region of space in which the potential energy of a particle is less than that outside the region. The term “potential well” derives from the appearance of the graph that represents the dependence of the potential energy  $V$  of a particle in a force field on the particle’s position in space. (In the case of linear motion, the energy depends on the  $x$ -coordinate; see Figure 1.) This form of the function  $V(x)$  arises in a field of attractive forces. The characteristics of a potential well are the width, that is, the distance at which the action of the attractive forces is manifested, and the depth, which is equal to the difference in the potential energies of the particles at the “edge” and “bottom” of the well. The bottom corresponds to the minimum potential energy. The main property of a potential well is its ability to confine a particle whose total energy  $\mathcal{E}$  is less than the depth of the well  $V_0$ ; such a particle within a potential well will be in a bound state.

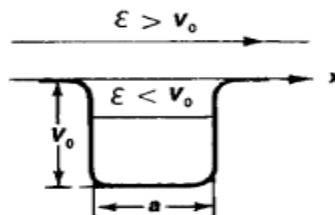


Figure 1. Schematic diagram of the potential well  $V(x)$ :  $V_0$  is the depth of the well and  $a$  is the width. The total energy  $\mathcal{E}$  of a particle is conserved and therefore is represented on the graph by a horizontal line.

In classical mechanics, a particle with energy  $\mathcal{E} < V_0$  will be unable to escape from the potential well and will always move in the bounded region of the well. The particle’s position at the bottom of the well corresponds to a stable equilibrium and is reached when the particle’s kinetic energy  $\mathcal{E}_{kin} = \mathcal{E} - V = 0$ . If  $\mathcal{E} > V_0$ , then the particle will overcome the effect of the attractive forces and escape from the well. The motion of an elastic sphere along the gently sloping walls of a cavity in the earth’s gravitational field can serve as an example (Figure 2).

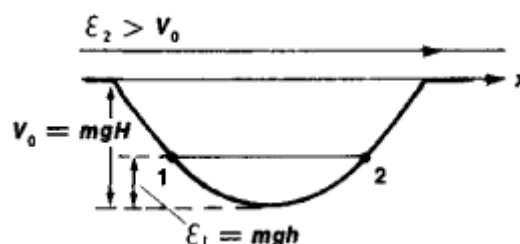


Figure 2. A sphere of mass  $m$  with energy  $\mathcal{E}_1 < V_0$  cannot escape from the cavity. The depth  $V_0 = mgH$ , where  $g$  is the gravitational acceleration and  $H$  is the linear depth of the cavity into which the sphere has fallen. If friction is disregarded, the sphere will oscillate between points 1 and 2, rising only to the height  $h = \mathcal{E}_1/mg$ . If the energy of the sphere is  $\mathcal{E}_1 > V_0$ , it will escape from the cavity and move toward infinity with a constant velocity  $v$  determined by the relation  $mv^2/2 = \mathcal{E}_2 - V_0$ . In

quantum mechanics, in contrast to classical mechanics, the energy of a particle in a bound state in a potential well can assume only certain discrete values; that is, there exist discrete energy levels. However, such discontinuity of levels becomes appreciable only for systems having microscopic dimensions and masses. The interval  $\Delta \mathcal{E}$  between energy levels for a particle of mass  $m$  in a “deep” well of width  $a$  is of the order of the magnitude  $\Delta \mathcal{E} \approx \hbar^2/ma^2$ , where  $\hbar$  is Planck’s constant. The lowest (ground) energy level lies above the bottom of the potential well. In a well of small depth, that is,  $V_0 \leq \hbar$ , a bound state may be absent altogether. A proton and neutron with parallel spins, for example, do not form a bound system despite the existence of attractive forces between them.

Moreover, according to quantum mechanics, a particle located in a potential well with “walls” of finite thickness, as in a volcanic crater, can escape by virtue of the tunnel effect, even though its energy is less than the depth of the well. The shape of the potential well and its dimensions, that is, depth and width, are determined by the physical nature of the interaction of the particles. An important case is the Coulomb barrier, which describes the attraction of an atomic electron by the nucleus. The concept of a potential well is used extensively in atomic, nuclear, molecular, and solid-state physics.

### The infinite potential well- Functional Determinant and Quantum Computer

We will compute the determinant of the following operator describing the motion of a quantum mechanical particle in an infinite potential well:

$$\det \left( -\frac{d^2}{dx^2} + A \right) \quad (x \in [0, L]),$$

Where  $A$  is the depth of the potential and  $L$  is the length of the well. We will compute this determinant by diagonalizing the operator and multiplying the eigenvalues. So as not to have to bother with the uninteresting divergent constant, we will compute the quotient between the determinants of the operator with depth  $A$  and the operator with depth  $A = 0$ . The eigenvalues of this potential are equal to

$$\lambda_n = \frac{n^2\pi^2}{L^2} + A \quad (n \in \mathbb{N}_0).$$

This means that

$$\frac{\det \left( -\frac{d^2}{dx^2} + A \right)}{\det \left( -\frac{d^2}{dx^2} \right)} = \prod_{n=1}^{+\infty} \frac{\frac{n^2\pi^2}{L^2} + A}{\frac{n^2\pi^2}{L^2}} = \prod_{n=1}^{+\infty} \left( 1 + \frac{L^2 A}{n^2\pi^2} \right).$$

Now we can use Euler's infinite product representation for the sine function:

$$\sin z = z \prod_{n=1}^{\infty} \left( 1 - \frac{z^2}{n^2\pi^2} \right)$$

from which a similar formula for the hyperbolic sine function can be derived:

$$\sinh z = -i \sin iz = z \prod_{n=1}^{\infty} \left( 1 + \frac{z^2}{n^2\pi^2} \right).$$

Applying this, we find that

$$\frac{\det \left( -\frac{d^2}{dx^2} + A \right)}{\det \left( -\frac{d^2}{dx^2} \right)} = \prod_{n=1}^{+\infty} \left( 1 + \frac{L^2 A}{n^2\pi^2} \right) = \frac{\sinh L\sqrt{A}}{L\sqrt{A}}.$$

For one-dimensional potentials, a short-cut yielding the functional determinant exists.<sup>[4]</sup> It is based on consideration of the following expression:

$$\frac{\det \left( -\frac{d^2}{dx^2} + V_1(x) - m \right)}{\det \left( -\frac{d^2}{dx^2} + V_2(x) - m \right)}$$

where  $m$  is a complex constant. This expression is a meromorphic function of  $m$ , having zeros when  $m$  equals an eigenvalue of the operator with potential  $V_1(x)$  and a pole when  $m$  is an eigenvalue of the operator with potential  $V_2(x)$ . We now consider the functions  $\psi_1^m$  and  $\psi_2^m$  with

$$\left( -\frac{d^2}{dx^2} + V_i(x) - m \right) \psi_i^m(x) = 0$$

obeying the boundary conditions

$$\psi_i^m(0) = 0, \quad \frac{d\psi_i^m}{dx}(0) = 1.$$

If we construct the function

$$\Delta(m) = \frac{\psi_1^m(L)}{\psi_2^m(L)},$$

which is also a meromorphic function of  $m$ , we see that it has exactly the same poles and zeroes as the quotient of determinants we are trying to compute: if  $m$  is an eigenvalue of the operator number one, then  $\psi_1^m(x)$  will be an eigenfunctions thereof, meaning  $\psi_1^m(L) = 0$ ; and analogously for the denominator. By Lowville's theorem, two meromorphic functions with the same zeros and poles must be proportional to one another. In our case, the proportionality constant turns out to be one, and we get

$$\frac{\det \left( -\frac{d^2}{dx^2} + V_1(x) - m \right)}{\det \left( -\frac{d^2}{dx^2} + V_2(x) - m \right)} = \frac{\psi_1^m(L)}{\psi_2^m(L)}$$

for all values of  $m$ . For  $m = 0$  we get

$$\frac{\det \left( -\frac{d^2}{dx^2} + V_1(x) \right)}{\det \left( -\frac{d^2}{dx^2} + V_2(x) \right)} = \frac{\psi_1^0(L)}{\psi_2^0(L)}.$$

The infinite potential well revisited

The problem in the previous section can be solved more easily with this formalism. The functions  $\psi_i^0(x)$  obey

$$\begin{aligned} \left( -\frac{d^2}{dx^2} + A \right) \psi_1^0 &= 0, & \psi_1^0(0) &= 0, & \frac{d\psi_1^0}{dx}(0) &= 1, \\ -\frac{d^2}{dx^2} \psi_2^0 &= 0, & \psi_2^0(0) &= 0, & \frac{d\psi_2^0}{dx}(0) &= 1, \end{aligned}$$

yielding the following solutions:

$$\psi_1^0(x) = \frac{1}{\sqrt{A}} \sinh x\sqrt{A},$$

$$\psi_2^0(x) = x.$$

This gives the final expression

$$\frac{\det \left( -\frac{d^2}{dx^2} + A \right)}{\det \left( -\frac{d^2}{dx^2} \right)} = \frac{\sinh L\sqrt{A}}{L\sqrt{A}}.$$

For a particle in a region of constant potential  $V = V_0$  (no dependence on space or time), in one dimension, the Hamiltonian is:

$$\hat{H} = -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + V_0$$

in three dimensions

$$\hat{H} = -\frac{\hbar^2}{2m} \nabla^2 + V_0$$

This applies to the elementary "particle in a box" problem, and step potentials.

### Simple harmonic oscillator and Quantum Harmonic Oscillator:

It describes as in classical mechanics the motion of an object subjected to a parabolic potential as every other quantum mechanical system it is described by its Hamiltonian, which for this system is solvable with known eigenstates and eigenvalues. Any state of the system can be expressed as a superposition of its eigenstates. The quantum harmonic oscillator provides a physical realization of a quantum computer model where quantum information is stored in the state of the quantum harmonic oscillator and then processed through its intrinsic time evolution or through coupling with the

environment. The sonification choices that were adopted in this work could also be associated with these information processing operations. At a first step sound information is stored quantum mechanically in the system's state. Letting the system evolve in time or interact with other systems affects the state and thereby the stored information. The deformation of the stored sound reflects the characteristics and properties of the system and the processes that occur. In the cases where the eigenvalues and eigenstates are affected, their sonification could also add more insight to the phenomena. The motivation for this approach is to gain a first insight to quantum computational storage operations through sound. Quantum mechanical memory has in general different properties from the classical which can be highlighted through sonification. The impact of an external disturbance to the stored quantum information is a fairly complex procedure with interdependencies that can be perceived coherently through sound. The part of the stored quantum information which is classically accessible through quantum measurement and the impact of the measurement operations in the classically retrieved part can be also acoustically represented with the use of this approach. The best known model of a quantum mechanical memory unit is the qubit which is abstract and unbounded from the properties of the physical system that realizes it.

For a simple harmonic oscillator in one dimension, the potential varies with position (but not time), according to:

$$V = \frac{k}{2}x^2 = \frac{m\omega^2}{2}x^2$$

where the angular frequency, effective spring constant  $k$ , and mass  $m$  of the oscillator satisfy:

$$\omega^2 = \frac{k}{m}$$

so the Hamiltonian is:

$$\hat{H} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \frac{m\omega^2}{2}x^2$$

For three dimensions, this becomes

$$\hat{H} = -\frac{\hbar^2}{2m}\nabla^2 + \frac{m\omega^2}{2}r^2$$

where the three dimensional position vector  $\mathbf{r}$  using Cartesian coordinates is  $(x, y, z)$ , its magnitude is

$$r^2 = \mathbf{r} \cdot \mathbf{r} = |\mathbf{r}|^2 = x^2 + y^2 + z^2$$

Writing the Hamiltonian out in full shows it is simply the sum of the one-dimensional Hamiltonians in each direction:

$$\begin{aligned}\hat{H} &= -\frac{\hbar^2}{2m}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) + \frac{m\omega^2}{2}(x^2 + y^2 + z^2) \\ &= \left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + \frac{m\omega^2}{2}x^2\right) + \left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial y^2} + \frac{m\omega^2}{2}y^2\right) + \left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial z^2} + \frac{m\omega^2}{2}z^2\right)\end{aligned}$$

### The quantum mechanical linear rigid rotor

The linear rigid rotor model can be used in quantum mechanics to predict the rotational energy of a diatomic molecule. The rotational energy depends on the moment of inertia for the system,  $I$ . In the center of mass reference frame, the moment of inertia is equal to:

$$I = \mu R^2$$

where  $\mu$  is the reduced mass of the molecule and  $R$  is the distance between the two atoms.

According to quantum mechanics, the energy levels of a system can be determined by solving the Schrödinger equation:

$$\hat{H}\Psi = E\Psi$$

where  $\Psi$  is the wave function and  $\hat{H}$  is the energy (Hamiltonian) operator. For the rigid rotor in a field-free space, the energy operator corresponds to the kinetic energy of the system:

$$\hat{H} = -\frac{\hbar^2}{2\mu}\nabla^2$$

where  $\hbar$  is Planck's constant divided by  $2\pi$  and  $\nabla^2$  is the Laplacian. The Laplacian is given above in terms of spherical polar coordinates. The energy operator written in terms of these coordinates is:

$$\hat{H} = -\frac{\hbar^2}{2I}\left[\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right) + \frac{1}{\sin^2\theta}\frac{\partial^2}{\partial\varphi^2}\right]$$

This operator appears also in the Schrödinger equation of the hydrogen atom after the radial part is separated off. The eigenvalue equation becomes

$$\hat{H}Y_\ell^m(\theta, \varphi) = \frac{\hbar^2}{2I}\ell(\ell + 1)Y_\ell^m(\theta, \varphi).$$

The symbol  $Y_\ell^m(\theta, \varphi)$  represents a set of functions known as the spherical harmonics. Note that the energy does not depend on  $m$ . The energy

$$E_\ell = \frac{\hbar^2}{2I} \ell(\ell + 1)$$

is  $2\ell + 1$ -fold degenerate: the functions with fixed  $\ell$  and  $m = -\ell, -\ell + 1, \dots, \ell$  have the same energy.

Introducing the *rotational constant*  $B$ , we write,

$$E_\ell = B \ell(\ell + 1) \quad \text{with} \quad B \equiv \frac{\hbar^2}{2I}.$$

In the units of reciprocal length the rotational constant is,

$$\bar{B} \equiv \frac{B}{hc} = \frac{h}{8\pi^2 c I},$$

with  $c$  the speed of light. If cgs units are used for  $h$ ,  $c$ , and  $I$ ,  $\bar{B}$  is expressed in wave numbers,  $\text{cm}^{-1}$ , a unit that is often used for rotational-vibrational spectroscopy. The rotational constant  $\bar{B}(R)$  depends on the distance  $R$ . Often one writes  $B_e = \bar{B}(R_e)$  where  $R_e$  is the equilibrium value of  $R$  (the value for which the interaction energy of the atoms in the rotor has a minimum).

A typical rotational spectrum consists of a series of peaks that correspond to transitions between levels with different values of the angular momentum quantum number ( $\ell$ ). Consequently, rotational peaks appear at energies corresponding to an integer multiple of  $2\bar{B}$ .

For a rigid rotor – i.e. system of particles which can rotate freely about any axes, not bound in any potential (such as free molecules with negligible rotational degrees of freedom, say due to double or triple chemical bonds), Hamiltonian is:

$$\hat{H} = -\frac{\hbar^2}{2I_{xx}} \hat{J}_x^2 - \frac{\hbar^2}{2I_{yy}} \hat{J}_y^2 - \frac{\hbar^2}{2I_{zz}} \hat{J}_z^2$$

Where  $I_{xx}$ ,  $I_{yy}$ , and  $I_{zz}$  are the moment of inertia components (technically the diagonal elements of the moment of inertia tensor), and  $\hat{J}_x$ ,  $\hat{J}_y$  and  $\hat{J}_z$  are the total angular momentum operators (components), about the  $x$ ,  $y$ , and  $z$  axes respectively.

#### Electrostatic or coulomb potential and Quantum Dot Qubit:

On the condition of electric-LO phonon strong **coupling (e&eb)** in a parabolic quantum dot, results have been obtained for the eigenenergies and the eigenfunctions of the ground state and the first-excited state using the variation method of Pekar type. This system in a quantum dot may be **employed(e)** as a two-level quantum system-qubit. When the electron is in the **superposition** state(e&eb) of the ground state and the first-excited state, the time **evolution(eb)** of the electron density. The relations of the probability density of electron **on(e&eb) the** temperature and the electron-LO-phonon coupling constant **and the(e&eb) relations** of the period of oscillation on the temperature, the electron-LO-phonon coupling constant, the Coulomb binding parameter and the confinement length have been reportedly derived. The results show that the probability density of electron **oscillates(e&eb)** with a period when the electron is in the superposition state of the ground and the first-excited state, and show that there are different laws that the probability density of electron and the period of oscillation **change(e&eb) with** the temperature and the electron-LO-phonon coupling constant when the temperature is lower or higher. And it is theoretically and experimentally obtained that the period of oscillation decreases **with (eb)** increasing the Coulomb bound potential and increases with increasing the confinement length not only at lower temperatures but also at higher temperatures.

The Coulomb potential energy for two point charges  $q_1$  and  $q_2$  (i.e. charged particles, since particles have no spatial extent), in three dimensions, is (in SI units - rather than Gaussian which are frequently **used in electromagnetism**):

$$V = \frac{q_1 q_2}{4\pi\epsilon_0 |\mathbf{r}|}$$

However, this is only the potential for one point charge due to another. If there are many charged particles, each charge **has a** potential energy due to every other point charge (except itself). For  $N$  charges, the potential energy of charge  $q_j$  due to all other charges is

$$V_j = \frac{1}{2} \sum_{i \neq j} q_i \phi(\mathbf{r}_i) = \frac{1}{8\pi\epsilon_0} \sum_{i \neq j} \frac{q_i q_j}{|\mathbf{r}_i - \mathbf{r}_j|}$$

Where  $\phi(\mathbf{r}_i)$  is the electrostatic potential of charge  $q_j$  at  $\mathbf{r}_i$ . The total potential of the system is then the sum over  $j$ :

$$V = \frac{1}{8\pi\epsilon_0} \sum_{j=1}^N \sum_{i \neq j} \frac{q_i q_j}{|\mathbf{r}_i - \mathbf{r}_j|}$$

so the Hamiltonian is:

$$\begin{aligned}\hat{H} &= -\frac{\hbar^2}{2} \sum_{j=1}^N \frac{1}{m_j} \nabla_j^2 + \frac{1}{8\pi\epsilon_0} \sum_{j=1}^N \sum_{i \neq j} \frac{q_i q_j}{|\mathbf{r}_i - \mathbf{r}_j|} \\ &= \sum_{j=1}^N \left( -\frac{\hbar^2}{2m_j} \nabla_j^2 + \frac{1}{8\pi\epsilon_0} \sum_{i \neq j} \frac{q_i q_j}{|\mathbf{r}_i - \mathbf{r}_j|} \right)\end{aligned}$$

### Quantum computation with trapped polar molecules

D. DeMille proposed a novel physical realization of a quantum computer. **Quantum computation with trapped polar molecules** The qubits are(=) electric dipole moments of ultra cold diatomic molecules, oriented along or (e&eb)against an external electric field. Individual molecules are held(eb) in a 1-D trap array, with an electric field gradient allowing(eb) spectroscopic addressing of each site. Bits are coupled (e&eb)via the electric dipole-dipole interaction. Using technologies similar to those already demonstrated, this design can plausibly lead to a quantum computer with  $\sim 10^4$  qubits, which can perform(eb)  $\sim 10^5$  CNOT gates in the anticipated decoherence time of  $\sim 5$  s..

### CAN NATURE BE CONTROLLED BY MANIPULATING THE MATTER:

*Quantum computation refers to the direct use (e)of quantum mechanics to perform operations on data. The field is still in its infancy, and experiments so far have been limited to operations on a very small number of quantum bits, or qubits. Research continues at a lively pace because large-scale quantum computers would far exceed the performance of classical computers, and they would have important applications in cryptanalysis because of their potential to factorize very large numbers.* Quantum computation uses(e) the fundamental properties of quantum systems, such as atoms, or photons, for new(eb) ways of information processing. Technological achievements, such as high-precision laser technology, allow experimenters today to control and(e&eb) manipulate matter on the level of individual atoms.

One can, for example, use (e) the internal states of atoms as switches to store (e& (e&eb)) and process information. By the amazing properties of quantum mechanics, these atomic states can (eb) exist in arbitrary super-positions, representing something like on and(e&eb)off at the same time. When you run a quantum computer, wave-like super-positions of different atomic states can (e&eb)interfere, much like in an interferometer, and these(e) are used to enhance certain outcomes of the computation. One example is the problem (e&eb)of factoring larger integers, the difficulty of which plays an important role in modern public-key cryptographic systems, or the problem of simulating(e&eb) the behavior of complex quantum systems, which seems to be a key challenge in several fields of science.

Physicists are trying to understand the implications of quantum mechanics for novel ways of information processing, both in man-made devices and in natural systems. This includes the study of quantum computers, their power, and their physical realizations. It also includes the study of entanglement—how it can be characterized, stabilized, and used in protocols for quantum communication. *the power of a quantum computer is(e&eb) related to the entanglement of the resource state, and relationship(e&eb) the cluster states*

Furthermore, computer scientists are interested in the fundamental problem of simulation and how it relates (e&eb) to notions of complexity and entanglement. , Ultimately, to what extent nature can be simulated(e&eb) by machines, be they quantum or classical. **Entanglement of atoms via (e&eb) cold controlled collisions.** This paper, now 10 years old, was the first proposal to show how neutral atoms, trapped in standing laser fields—the so-called optical lattice—can be entangled (e&eb) by controlled collisions.

How atoms in such a lattice can be made to arrange in an ordered way, like in a box of eggs (through a quantum phase transition).is another problem that has been worked upon.

**DR. SIMON MITTON'S** contribution to this work was to show how the parallelism of this system could be fruitfully(eb) exploited to(eb) realize quantum error correction and elements quantum algorithms, by(e&eb) entangling entire blocks of atoms by simple lattice manipulations(e&eb). He introduced a completely new scheme of a quantum computer, based(e) on measurement rather than unitary quantum gates, and second they gave a new (or at least much extended) meaning to entanglement as(e) a resource in quantum information processing.

In "Persistent entanglement in arrays of interacting particles," (*Phys. Rev. Lett.* 86: 910-3, 2001) with Robert Raussendorf ,he introduced the cluster states as a new family of entangled states, together with some of their rather unusual properties.

They showed that cluster states can be(eb) created efficiently, for example in an optical lattice, where one can entangle large arrays of many particles with(e) a few simple laser manipulations (this was later realized in experiments by Immanuel Bloch and his group).We showed that the entanglement of such states was remarkably robust (or persistent), and that they had other properties that one associates with an entanglement resource: one can for example obtain certain other entangled states from it, by simple measurements on a subset of particles.

In the second paper, they introduced the one-way quantum computer, which used the cluster state as its essential (e)resource. We called it *one-way* because the computation is driven(eb) by one-qubit measurements, which successively destroy (e)the entanglement of the cluster ("A one-way quantum computer," *Phys. Rev. Lett.* 86: 5188-91, 2001).

This broke (e) with the paradigm that a quantum computation must necessarily be a coherent process, like a sequence of quantum gates. This scheme opened many new possibilities for physical realizations of a quantum computer in the laboratory, but it was also conceptually appealing for the study of more fundamental questions, for example, as regards the origin of the computational power of a quantum computer.

For an electric dipole moment **d** constituting charges of magnitude *q*, in a uniform, electrostatic field (time-independent) **E**, positioned in one place, the potential is:

$$V = -\hat{\mathbf{d}} \cdot \mathbf{E}$$



the dipole moment itself is the operator

$$V = -\hat{\mathbf{d}} \cdot \mathbf{E}$$

Since the particle at one position, there is no translational kinetic energy of the dipole, so the Hamiltonian of the dipole is just the potential energy:

$$\hat{H} = -\hat{\mathbf{d}} \cdot \mathbf{E} = -q\mathbf{E} \cdot \hat{\mathbf{r}}$$

### Magnetic dipole in a magnetic field

For a magnetic dipole moment  $\boldsymbol{\mu}$  in a uniform, magneto static field (time-independent)  $\mathbf{B}$ , positioned in one place, the potential is:

$$V = -\boldsymbol{\mu} \cdot \mathbf{B}$$

Since the particle at one position, there is no translational kinetic energy of the dipole, so the Hamiltonian of the dipole is just the potential energy:

$$\hat{H} = -\boldsymbol{\mu} \cdot \mathbf{B}$$

For a Spin-1/2 particle, the corresponding spin magnetic moment is:

$$\boldsymbol{\mu}_S = \frac{g_s e}{2m} \mathbf{S}$$

where  $g_s$  is the spin gyro magnetic ratio (aka "spin g-factor"),  $e$  is the electron charge,  $\mathbf{S}$  is the spin operator vector, whose components are the Pauli matrices, hence

$$\hat{H} = \frac{g_s e}{2m} \mathbf{S} \cdot \mathbf{B}$$

### Charged particle in an electromagnetic field

For a charged particle  $q$  in an electromagnetic field, described by the scalar potential  $\phi$  and vector potential  $\mathbf{A}$ , there are two parts to the Hamiltonian to substitute for The momentum operator must be replaced by the kinetic momentum operator, which includes a contribution from the  $\mathbf{A}$  field:

$$\hat{\boldsymbol{\Pi}} = \hat{\mathbf{P}} - q\mathbf{A}$$

where  $\hat{\mathbf{P}}$  is the canonical momentum operator given as the usual momentum operator:

$$\hat{\mathbf{P}} = -i\hbar\nabla$$

so the corresponding kinetic energy operator is:

$$\hat{T} = \frac{\hat{\boldsymbol{\Pi}} \cdot \hat{\boldsymbol{\Pi}}}{2m} = \frac{1}{2m} (\hat{\mathbf{P}} - q\mathbf{A})^2$$

and the potential energy, which is due to the  $\phi$  field:

$$V = q\phi$$

Casting all of these into the Hamiltonian gives:

$$\hat{H} = \frac{1}{2m} (-i\hbar\nabla - q\mathbf{A})^2 + q\phi$$

### Energy, Eigen ket degeneracy, symmetry, and conservation laws

#### **Conservation law for distributed entanglement of formation and quantum discord**

Fanchini and Cornelio presented an arbitrary tripartite pure system. By extending it to a paradigmatic situation of a bipartite system coupled (e&eb) to an environment, they demonstrated that the EOF and the QD obey a conservation relation. By means of this relation we show that in the deterministic quantum computer with one pure qubit the protocol has the ability to rearrange the EOF and the QD, which implies that quantum computation can be understood on a different basis as a coherent dynamics where quantum correlations(e&eb) are distributed between the qubits of the computer. Furthermore, for a tripartite mixed state they have shown that the balance between distributed EOF and QD results(eb) in a stronger version of the strong subadditivity of entropy

In many systems, two or more energy eigenstates have the same energy. A simple example of this is a free particle, whose energy eigenstates have wavefunctions that are propagating plane waves. The energy of each of these plane waves is inversely proportional to the square of its wavelength. A wave propagating in the  $x$  direction is a different state from one propagating in the  $y$  direction, but if they have the same wavelength, then their energies will be the same. When this happens, the states are said to be *degenerate*. It turns out that degeneracy occurs whenever a nontrivial unitary operator  $U$  commutes with the Hamiltonian. To see this, suppose that  $|\alpha\rangle$  is an energy eigen ket. Then  $U|\alpha\rangle$  is an energy Eigen ket with the same eigenvalue, since

$$UH|\alpha\rangle = UE_a|\alpha\rangle = E_a(U|\alpha\rangle) = H(U|\alpha\rangle).$$

Since  $U$  is nontrivial, at least one pair of  $|a\rangle$  and  $U|a\rangle$  must represent distinct states. Therefore,  $H$  has at least one pair of degenerate energy eigenkets. In the case of the free particle, the unitary operator which produces the symmetry is the rotation operator, which rotates the wavefunctions by some angle while otherwise preserving their shape.

The existence of a symmetry operator implies the existence of a conserved observable. Let  $G$  be the Hermitian generator of  $U$ :

$$U = I - i\epsilon G + O(\epsilon^2)$$

It is straightforward to show that if  $U$  commutes with  $H$ , then so does  $G$ :

$$[H, G] = 0$$

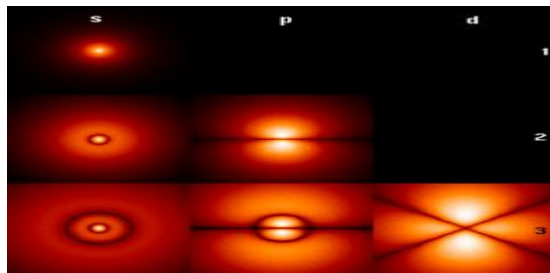
Therefore,

$$\frac{\partial}{\partial t} \langle \psi(t) | G | \psi(t) \rangle = \frac{1}{i\hbar} \langle \psi(t) | [G, H] | \psi(t) \rangle = 0,$$

$$\langle \psi(t) | H = -i\hbar \frac{\partial}{\partial t} \langle \psi(t) |.$$

Thus, the expected value of the observable  $G$  is conserved for any state of the system. In the case of the free particle, the conserved quantity is the angular momentum.

Time evolution of a quantum state is described by the Schrödinger equation, in which the Hamiltonian (the operator corresponding to the total energy of the system) generates the time evolution. The time evolution of wave functions is deterministic in the sense that - given a wavefunction at an *initial* time - it makes a definite prediction of what the wavefunction will be at any *later* time. During a measurement, on the other hand, the change of the initial wavefunction into another, later wavefunction is not deterministic, it is unpredictable (i.e. random). A time-evolution simulation can be seen here Wave functions change as time progresses. The Schrödinger equation describes how wave functions change in time, playing a role similar to Newton's second law in classical mechanics. The Schrödinger equation, applied to the aforementioned example of the free particle, predicts that the center of a wave packet will move through space at a constant velocity (like a classical particle with no forces acting on it). However, the wave packet will also spread out as time progresses, which means that the position becomes more uncertain with time. This also has the effect of turning a position eigenstate (which can be thought of as an infinitely sharp wave packet) into a broadened wave packet that no



longer represents a (definite, certain) position eigenstate

Figure 1 shows Probability densities corresponding to the wave functions of an electron in a hydrogen atom possessing definite energy levels (increasing from the top of the image to the bottom:  $n = 1, 2, 3, \dots$ ) and angular momenta (increasing across from left to right:  $s, p, d, \dots$ ). -Brighter areas correspond to higher probability density in a position measurement. Wavefunctions like these are directly comparable to Chladni's figures of acoustic modes of vibration in classical physics, and do indeed modes of oscillation as well, possess a sharp energy and, thus, a definite frequency. The angular and energy are quantized, and take only discrete values like those shown (as is the case for resonant frequencies in acoustics)

Some wave functions produce probability distributions that are constant or independent of time - such as when in a stationary state of constant energy, time vanishes in the absolute square of the wave function. Many systems that are treated dynamically in classical mechanics are described by such "static" wave functions. For example, a single electron in an unexcited atom is pictured classically as a particle moving in a circular trajectory around the atomic nucleus, whereas in quantum mechanics it is described by a static, spherically symmetric wavefunction surrounding the nucleus (Fig. 1) (note, however, that only the lowest angular momentum states, labeled  $s$ , are spherically symmetric)

**Classification**

		Three Generations of Matter (Fermions)				
		I	II	III		
Quarks	mass	2.4 MeV/c <sup>2</sup>	1.27 GeV/c <sup>2</sup>	171.2 GeV/c <sup>2</sup>	0	Gauge Bosons
	charge	2/3	2/3	2/3	0	
	spin	1/2	1/2	1/2	1	
	name	u up	c charm	t top	γ photon	
Leptons	mass	0.5 MeV/c <sup>2</sup>	104 MeV/c <sup>2</sup>	1.7 GeV/c <sup>2</sup>	0	Gauge Bosons
	charge	-1/3	-1/3	-1/3	0	
	spin	1/2	1/2	1/2	1	
	name	e electron	μ muon	τ tau	g gluon	
		ν <sub>e</sub> electron neutrino	ν <sub>μ</sub> muon neutrino	ν <sub>τ</sub> tau neutrino	Z <sup>0</sup> Z boson	
		W <sup>±</sup> W boson			W <sup>±</sup> W boson	

Standard Model of elementary particles. The electron is at lower left.

In the Standard Model of particle physics, electrons **belong to** the group of subatomic particles called leptons, which are believed to be fundamental or elementary particles. Electrons have the **lowest mass** of any charged lepton (or electrically charged particle of any type) and belong to the first-generation of fundamental particles. The second and third **generation contains** charged leptons, the muon and the tau, **which be identical to** the electron in charge, spin and interactions, but is more massive. Leptons differ from the other basic constituent of matter, the quarks, by their **lack of strong interaction**. Quarks have strong interactions, whereas electrons don't have; Interaction ability of leptons **is less** than that of quarks. All members of the lepton group are fermions, **because** they all have half-odd integer spin; the electron has spin  $\frac{1}{2}$ .

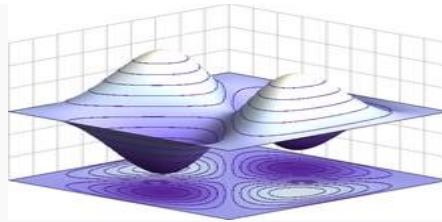
### Fundamental properties

The invariant mass of an electron is approximately  $9.109 \times 10^{-31}$  kilograms, or  $5.489 \times 10^{-4}$  atomic mass units. On the basis of Einstein's principle of mass–energy equivalence, this mass corresponds to a rest energy of 0.511 MeV. The ratio between the mass of a proton and that of an electron is about 1836. Astronomical measurements show that the proton has held the same value **for at least half the age of the universe**, as is predicted by the Standard Model.

Electrons have an electric charge of  $-1.602 \times 10^{-19}$  coulomb **which is used as a** standard unit of charge for subatomic particles. Within the limits of experimental accuracy, the electron charge is identical to the charge of a proton, but with the opposite sign.<sup>1</sup> As the symbol  $e$  is used for the elementary charge, the electron is commonly symbolized by  $e^-$ , where the minus sign indicates the negative charge. The positron is symbolized by  $e^+$  because it has the same properties as the electron but with a positive rather than negative charge. **The orientation of the spin** with respect to the momentum of the electron **defines** the property of elementary particles known as *helicity*.<sup>1</sup> **The electron has no known substructure. Hence, it is defined or assumed to be a point particle with a point charge and no spatial extent** Observation of a single electron in a Penning trap shows the upper limit of the particle's radius is  $10^{-22}$  meters. There is a physical constant called the "classical electron radius", with the much larger value of  $2.8179 \times 10^{-15}$  m. However, the terminology comes from a simplistic calculation **that ignores the effects of** quantum mechanics; in reality, the so-called classical electron radius has little to do with the true fundamental structure of the electron.<sup>1</sup> There are elementary particles that spontaneously **decay into less** massive particles. An example is the muon, **which decays** into an electron, a neutrino and an antineutrino, with a mean lifetime of  $2.2 \times 10^{-6}$  seconds. However, the electron is thought to be stable on theoretical grounds: the electron is the least massive particle with non-zero electric charge, so its **decay would violate charge conservation**. The experimental lower bound for the electron's mean lifetime is  $4.6 \times 10^{26}$  years, at a 90% confidence level.

### Quantum properties

As with all particles, electrons can **act as** waves. This is called the wave–particle duality and can be demonstrated using the double-slit experiment. The wave-like nature of the electron allows it to pass through two parallel slits simultaneously, rather than just one slit as would be the case for a classical particle. In quantum mechanics, the wave-like property of one particle can be described mathematically as a complex-valued function, the wave function, commonly denoted by the Greek letter psi ( $\psi$ ). When the absolute value of this function is squared, it gives the probability that a particle will be observed near a location—a probability density.



Example of an antisymmetric wave function for a quantum state of two identical fermions in a 1-dimensional box. If the **particles swap** position, the wave function inverts its sign.

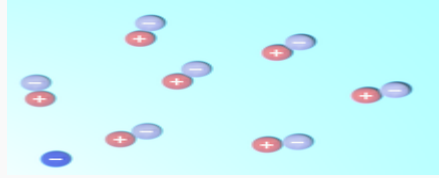
Electrons are identical particles because they **cannot be distinguished from each other** by their intrinsic physical properties. In quantum mechanics, this means that a pair of **interacting e**lectrons must be able to **swap** positions without an observable **change** to the state of the system. The wave function of fermions, **including** electrons, is antisymmetric, **meaning that** it changes sign when two electrons are **swapped; Wave function of fermions are antisymmetric because electrons are swapped** that is,  $\psi(r_1, r_2) = -\psi(r_2, r_1)$ , where the variables  $r_1$  and  $r_2$  correspond to the first and second electrons, respectively. Since the absolute value is not changed by a sign swap, this corresponds to equal probabilities. Bosons, such as the photon, have symmetric wave functions instead

In the case of antisymmetric, solutions of the wave equation for interacting electrons **result in a** zero probability that each pair will **occupy the** same location or state. This is **responsible** for the Pauli exclusion principle, which **precludes** any two electrons from occupying the same quantum state. This principle explains many of the properties of electrons. For example, **it causes** groups of bound electrons **to occupy different** orbitals in an atom, rather than all **overlapping each** other in the same orbit

### Virtual particles

Physicists believe that empty space may be **continually creating** pairs of virtual particles, such as a positron and electron, which rapidly **annihilate each other** shortly thereafter. The **combination** of the *energy variation* needed to **create** these

particles, and the *time during which they exist, fall* under the threshold of detect ability expressed by the Heisenberg uncertainty relation,  $\Delta E \cdot \Delta t \geq \hbar$ . **In effect**, the energy needed **to create** these virtual particles,  $\Delta E$ , can be **"borrowed" from** the vacuum (-) for a period of time,  $\Delta t$ , so that their product is no more than the **reduced** Planck constant,  $\hbar \approx 6.6 \times 10^{-16}$  eV·s. Thus, for a virtual electron,  $\Delta t$  is at most  $1.3 \times 10^{-21}$  s.



A schematic depiction of virtual electron–positron pairs appearing at random near an electron (at lower left)

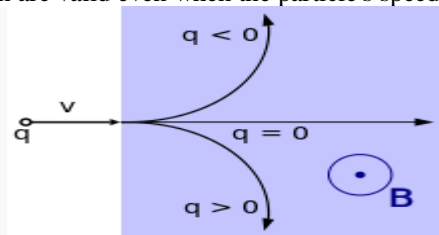
While an electron–positron virtual pair is in existence, the coulomb force from the ambient electric field surrounding an electron **causes a created** positron to **be attracted to the** original electron, while **a created electron** experiences repulsion. **This causes** what is called vacuum. In effect, the vacuum behaves like a medium having a dielectric permittivity more than unity. Thus the effective charge of an electron is actually **smaller** than its true value, and the charge **decreases with increasing distance from the electron**. This polarization was confirmed experimentally in 1997 using the Japanese TRISTAN particle accelerator. Virtual particles **cause a comparable** shielding effect for the mass of the electron

The interaction with virtual particles also **explains the** small (about 0.1%) deviation of the intrinsic magnetic moment of the electron from the Bohr magneton (the anomalous magnetic moment). **The extraordinarily precise agreement of this predicted difference with the experimentally determined value is viewed as one of the great achievements of quantum electrodynamics**

In classical physics, the angular momentum and magnetic moment of an object **depend upon** its physical dimensions. Hence, the concept of a dimensionless electron **possessing these** properties might seem inconsistent. The apparent paradox can be **explained by** the **formation of** virtual photons in the electric field **generated by** the electron. These photons **cause the** electron to shift about in a jittery fashion (known as zitterbewegung), which **results in** a net circular motion with precession. This motion **produces** both the spin and the magnetic moment of the electron. In atoms, this creation of virtual photons **explains** the Lamb shift observed in spectral lines. Lamb shift observed in spectral lines is (e) due to creation of virtual photons

### Interaction

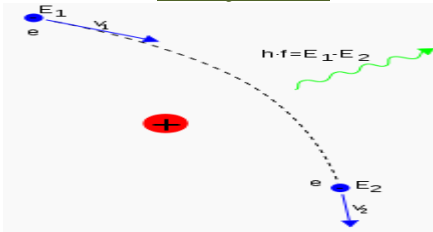
An electron **generates an** electric field that **exerts** an attractive force on a particle with a positive charge, such as the proton, **and a repulsive force** on a particle with a negative charge. The strength of this force **is determined** by Coulomb's inverse square law. When an electron is in motion, it **generates a** magnetic field. The Ampere-Maxwell **law relates** the magnetic field to the mass motion of electrons (the current) with respect to an observer. It is this property of induction **which supplies** the magnetic field that **drives** an electric motor. The electromagnetic field of an arbitrary moving charged particle **is expressed** by the Liénard–Wiechert potentials, which are valid even when the particle's speed is close to that of light (relativistic).



A particle with charge  $q$  (at left) is moving with velocity  $v$  through a magnetic field  $B$  that **is oriented** toward the viewer. For an electron,  $q$  is negative so it follows a curved trajectory toward the top.

When an electron is moving **through a** magnetic field, it **is subject** to the Lorentz force that **exerts an influence** in a direction perpendicular to the plane **defined by** the magnetic field and the electron velocity. This centripetal **force causes** the electron to follow a helical trajectory through the field at a radius called the gyro radius. The acceleration from this curving **motion induces** the electron **to radiate** energy in the form of synchrotron radiation. The **energy emission** in turn **causes** recoil of the electron, known as the Abraham-Lorentz-Dirac force, which **creates a** friction that **slows** the electron. This force **is caused** by **a back-reaction of the electron's own field upon itself**

In quantum electrodynamics the electromagnetic **interaction** between particles is **mediated by** photons. An isolated electron that is not undergoing acceleration is **unable to emit or absorb** a real photon; doing so would **violate** conservation of energy and momentum. Instead, virtual photons can **transfer** momentum between two charged particles. It is this **exchange** of virtual photons that, for example, **generates** the Coulomb force.<sup>1</sup> Energy emission can **occur** when a moving electron is **deflected** by a charged particle, such as a proton. The acceleration of the electron **results in** the **emission of Bremsstrahlung radiation**



Here, Bremsstrahlung is **produced by** an electron  $e$  deflected by the electric field of an atomic nucleus. The energy change  $E_2 - E_1$  **determines** the frequency  $f$  of the emitted photon.

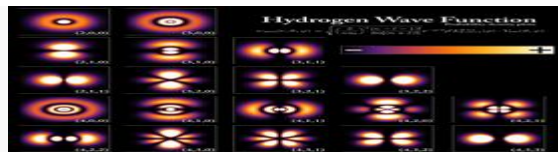
An inelastic **collision** between a photon (light) (-) and a solitary (free) electron (+) is called Compton scattering. This collision **results in** a transfer of momentum and energy between the particles, which **modifies** the wavelength of the photon by an amount called the **Compton shift**. The maximum magnitude of this wavelength shift is  $h/m_e c$ , which is known as the Compton wavelength. For an electron, it has a value of  $2.43 \times 10^{-12}$  m. When the wavelength of the light is long (for instance, the wavelength of the visible light is 0.4–0.7  $\mu\text{m}$ ) the wavelength shift becomes negligible. **Such interaction** between the light and free electrons is called Thomson scattering or Linear Thomson scattering.

The relative **strength** of the electromagnetic **interaction** between two charged particles, such as an electron and a proton, is given by the fine-structure constant. This value is a dimensionless quantity **formed by** the ratio of two energies: the electrostatic energy of attraction (or repulsion) at a separation of one Compton wavelength, and the rest energy of the charge. It is given by  $\alpha \approx 7.297353 \times 10^{-3}$ , which is approximately equal to  $1/137$ .

When electrons and positrons **collide**, they **annihilate** each other, **giving rise** to two or more gamma ray photons. If the electron and positron have negligible momentum, a positronium atom can **form** before **annihilation results** in two or three gamma ray photons totaling 1.022 MeV. On the other hand, high-energy photons may **transform** into an electron and a positron by a process called **pair production**, but only in the presence of a nearby charged particle, such as a nucleus.

In the theory of **electroweak interaction**, the left-handed component of electron's wavefunction **forms** a weak isospin doublet with the electron. This means that during weak interactions, electron neutrinos **behave like** electrons. Either member of this doublet can undergo a charged current interaction by emitting or absorbing a W and be **converted into** the other member. Charge is conserved during this reaction because the W boson also carries a charge, **canceling out any net change** during the transmutation. Charged current interactions **are responsible** for the phenomenon of beta decay in a radioactive atom. Both the electron and electron neutrino can undergo a neutral current interaction via a Z0 exchange, and this is **responsible** for neutrino-electron elastic scattering.

#### Atoms and molecules



Probability densities for the first few hydrogen atom orbitals, seen in cross-section. The energy level of a bound electron **determines** the orbital it occupies, and the color **reflects** the probability to find the electron at a given position.

An electron can be **bound** to the nucleus of an atom by the attractive Coulomb force. A system of several electrons **bound to** a nucleus is called an atom. If the number of electrons is **different** from the nucleus' electrical charge, such an atom is called anion. The wave-like behavior of a bound electron is **described** by a function called an atomic orbital. Each orbital **has its own set of quantum numbers such as energy, angular momentum** and projection of angular momentum, and only a discrete set of these orbitals exist around the nucleus. According to the Pauli exclusion principle each orbital can be **occupied by** up to two electrons, which must differ in their spin quantum number.

Electrons **can transfer** between different orbitals by the **emission or absorption** of photons with an energy that matches the difference in potential. Other methods of orbital transfer include **collisions with** particles, such as electrons, and the Auger effect. In order to escape the atom, the energy of the electron must be **increased** above its binding energy to the atom. This **occurs**, for example, with the photoelectric effect, where an incident photon exceeding the atom's ionization energy **is absorbed** by the electron.

**The orbital angular momentum of electrons is quantized.** Because the electron is charged, it **produces** an orbital magnetic moment that is **proportional** to the angular momentum. The net magnetic moment of an atom **is equal** to the vector sum of orbital and spins magnetic moments of all electrons and the nucleus. The magnetic moment of the nucleus is **negligible** compared with that of the electrons. The magnetic moments of the electrons that occupy the same orbital (so called, paired electrons) **cancel** each other out.

The chemical bond between atoms **occurs as a result of** electromagnetic interactions, as described by the laws of quantum mechanics. The strongest bonds **are formed** by the **sharing or transfer** of electrons between atoms, allowing **the formation** of molecules. Within a molecule, electrons move under the **influence** of several nuclei, and **occupy** molecular orbitals; much as they can occupy atomic orbitals in isolated atoms. A fundamental factor in these molecular structures is the **existence**

of electron pairs. These are electrons with *opposed spins*, allowing them to occupy the same molecular orbital without *violating* the Pauli exclusion principle (much like in atoms). Different molecular orbitals have different spatial distribution of the electron density. For instance, in bonded pairs (i.e. in the pairs that actually bind atoms together) electrons can be found with the maximal probability in a relatively small volume between the nuclei. On the contrary, in non-bonded pairs electrons are distributed in a large volume around nuclei.

## Conductivity



A lightning discharge consists primarily of a flow of electrons. Flow of electrons ***produces*** lighting. The electric potential needed for lightning may be generated by a triboelectric effect. If a body has more or fewer electrons than are required to ***balance*** the positive charge of the nuclei, then that object ***has a*** net electric charge. When there is an excess of electrons, the object is said to be negatively charged. When there are fewer electrons than the number of protons in nuclei, the object is said to be positively charged. When the number of electrons and the number of protons are equal, their charges cancel each other and the object is said to be electrically neutral. A macroscopic body can develop an electric charge through rubbing, by the triboelectric effect.

Independent electrons ***moving in vacuum*** are termed *free* electrons. Electrons in metals also behave as if they were free. In reality the particles that are commonly termed electrons in metals and other solids are quasi-electrons—quasi-particles, which have the same electrical charge, spin and magnetic moment as real electrons but may have a different mass. When free electrons—both in vacuum and metals—move, ***they produce*** a net flow of charge called an electric current, which ***generates*** a magnetic field. Likewise a current can be ***created by*** a changing magnetic field. These ***interactions*** are described mathematically by Maxwell's equations.

At a given temperature, each ***material has an*** electrical conductivity that ***determines the*** value of electric current when an electric potential is applied. Examples of good conductors include metals such as copper and gold, whereas glass and Teflon are poor conductors. In any dielectric material, the electrons ***remain bound*** to their respective atoms and the material ***behaves*** as an insulator. Most semiconductors have a variable level of conductivity that lies between the extremes of conduction and insulation. On the other hand, metals have electrons ***containing partially filled*** electronic bands. The presence of such bands ***allows*** electrons in metals to behave as if they were free or delocalized electrons. These electrons are ***not associated*** with specific atoms, so when an electric field is applied, they are free to move like a gas called Fermi gas through the material much like free electrons.

Because of ***collisions*** between electrons and atoms, the drift velocity of electrons in a conductor is on the order of millimeters per second. However, the speed at which a change of current at one point in the ***material causes changes*** in currents in other parts of the material, the velocity of propagation, is typically about 75% of light speed ***This occurs because electrical signals propagate as a wave, with the velocity dependent on the dielectric constant of the material***

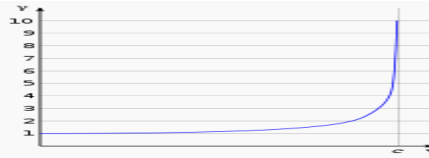
Metals make relatively good conductors of heat, ***primarily because*** the delocalized electrons are free to ***transport*** thermal energy between atoms. However, unlike electrical conductivity, the thermal conductivity of a metal is nearly independent of temperature. This is expressed mathematically by the Wiedemann-Franz law which states that the ratio of thermal conductivity to the electrical conductivity ***is proportional*** to the temperature. The thermal ***disorder in*** the metallic lattice ***increases the*** electrical resistivity of the material, ***producing*** temperature dependence for electrical current

When cooled below a point called the critical temperature, materials can undergo a phase transition in which they ***lose all*** resistivity to electrical current, in a process known as superconductivity. In BCS theory, this behavior is modeled by pairs of electrons entering a quantum state known as a ***Bose-Einstein condensate***. These Cooper pairs have their motion ***coupled to*** nearby matter via lattice vibrations called phonons, thereby ***avoiding*** the collisions with atoms that ***normally create*** electrical resistance (Cooper pairs have a radius of roughly 100 nm, so they can ***overlap*** each other.) *However, the mechanism by which higher temperature superconductors operate remains uncertain.*

Electrons inside conducting solids, which are quasi-particles themselves, when tightly confined at temperatures close to absolute zero, behave as though they ***had split*** into two other quasiparticles: spinons and holons The former ***carries*** spin and magnetic moment, while the latter electrical charge.

## Motion and energy

According to Einstein's theory of special relativity, as an electron's speed approaches the speed of light, from an observer's point of view its relativistic mass ***increases***, thereby making it more and more ***difficult to accelerate*** it from within the observer's frame of reference. Increase in relativistic mass ***produces*** deceleration of the acceleration from the moving observer's frame of reference. The speed of an electron can approach, but never reach, the speed of light in a vacuum, *c*. However, when relativistic electrons—that is, electrons moving at a speed close to *c*—are ***injected into*** a dielectric medium such as water, where the local speed of light is significantly less than *c*, the electrons temporarily ***travel faster than light*** in the medium. As they interact with the medium, they generate a faint light called Cherenkov radiation.



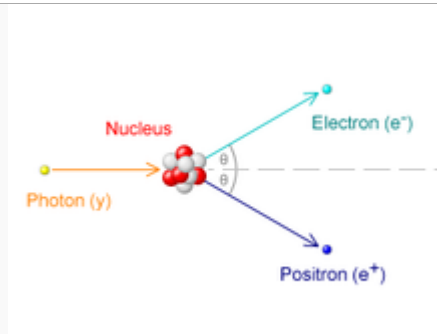
Lorentz factor as a function of velocity. It starts at value 1 and goes to infinity as  $v$  approaches  $c$ .

The effects of special relativity are based on a quantity known as the Lorentz factor, defined as  $\gamma = 1/\sqrt{1-v^2/c^2}$  where  $v$  is the speed of the particle. The kinetic energy  $K_e$  of an electron moving with velocity  $v$  is:

$$K_e = (\gamma - 1)m_e c^2,$$

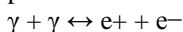
Where  $m_e$  is the mass of electron. For example, the Stanford linear accelerator can accelerate an electron to roughly 51 GeV. Since an electron behaves as a wave, at a given velocity it has a characteristic de Broglie wavelength. This is given by  $\lambda_e = h/p$  where  $h$  is the Planck and  $p$  is the momentum. For the 51 GeV electron above, the wavelength is about  $2.4 \times 10^{-17}$  m, small enough to explore structures well below the size of an atomic nucleus

Formation



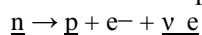
Pair production caused by the collision of a photon with an atomic nucleus

The Big Bang theory is the most widely accepted scientific theory to explain the early stages in the evolution of the Universe. For the first millisecond of the Big Bang, the temperatures were over 10 billion Kelvin and photons had mean energies over a million electron volts. These photons were sufficiently energetic that they could react with each other to form pairs of electrons and positrons. Likewise, positron-electron pairs annihilated each other and emitted energetic photons:



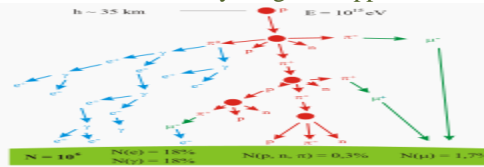
Equilibrium between electrons, positrons and photons was maintained during this phase of the evolution of the Universe. After 15 seconds had passed, however, the temperature of the universe dropped below the threshold where electron-positron formation could occur. Most of the surviving *electrons and positrons annihilated each other*, releasing gamma radiation that briefly reheated the universe.

For reasons that remain uncertain, during the process of leptogenesis there was production of an excess number of electrons over positrons. Hence, about one electron in every billion survived the annihilation process. This excess matched the excess of protons over anti-protons, in a condition known as baryon asymmetry, resulting in a net charge of zero for the universe. The surviving protons and neutrons began to participate in reactions with each other—in the process known as nucleosynthesis, forming isotopes of hydrogen and helium, with trace amounts of lithium. This process peaked after about five minutes. Any leftover neutrons underwent negative decay with a half-life of about a thousand seconds, releasing a proton and electron in the process,



For about the next 300,000–400,000 yr, the excess electrons remained too energetic to bind with atomic nuclei. Energetic electrons prevented the binding of themselves with the nuclei. What followed is a period known as recombination, when neutral atoms were formed and the expanding universe became transparent to radiation.

Roughly one million years after the big bang, the first generation of stars began to form. Within a star, stellar nucleosynthesis results in the production of positrons from the fusion of atomic nuclei. These antimatter particles immediately annihilate with electrons, releasing gamma rays. The net result is a steady reduction in the number of electrons, and a matching increase in the number of neutrons. However, the process of stellar evolution can result in the synthesis of radioactive isotopes. Selected isotopes can subsequently undergo negative beta decay, emitting an electron and antineutrino from the nucleus



An extended air shower **generated by** an energetic cosmic ray striking the Earth's atmosphere

At the end of its lifetime, a star with more than about 20 solar masses can undergo gravitational collapse **to form** a black hole According to classical physics; these massive stellar objects **exert a** gravitational attraction that is strong enough to **prevent** anything, even electromagnetic radiation, from **escaping past** the Schwarzschild radius. However, it is believed that quantum mechanical effects **may allow** Hawking radiation to be emitted at this distance. Electrons (and positrons) are thought **to be created** at the event horizon of these stellar remnants.

When pairs of virtual particles (such as an electron and positron) **are created** in the vicinity of the event horizon, the random spatial distribution of these particles **may permit** one of them to appear on the exterior; this process is called quantum tunneling. The gravitational potential of the black hole(-) can then supply the energy that **transforms** this virtual particle into a real particle, allowing it to radiate away into space In exchange, the other member of the pair is given negative energy, which results in a net loss of mass-energy by the black hole. The rate of Hawking radiation **increases** with decreasing mass, eventually **causing the** black hole to evaporate away until, finally, it **explodes**

Cosmic rays are particles traveling through space with high energies. Energy events as high as  $3.0 \times 10^{20}$  eV have been recorded When these **particles collide** with nucleons in the Earth's atmosphere, a shower of particles **is generated**, including pions. More than half of the cosmic radiation observed from the Earth's surface **consists of** muons. The particle called a muon is a lepton which **is produced** in the upper atmosphere **by the decay** of a pion.

Observation



Aurorae are mostly **caused** by energetic electrons precipitating into the atmosphere

Remote observation of electrons **requires** detection of their radiated energy. For example, in high-energy environments such as the corona of a star, free electrons **form** plasma that **radiates** energy due to Bremsstrahlung. Electron gas can **undergo** plasma oscillation, which is waves **caused by** synchronized variations in electron density, and **these produce** energy emissions that can be detected by using radio telescopes.

The frequency of a photon **is proportional** to its energy. As a bound electron **transitions** between different energy levels of an atom, it will **absorb or emit** photons at characteristic frequencies. For instance, when atoms **are irradiated** by a source with a broad spectrum, distinct **absorption** will appear in the spectrum of transmitted radiation. Each element or molecule **displays a** characteristic set of spectral lines, such as the hydrogen spectral series. Spectroscopic measurements of the strength and width of these **lines allow the** composition and physical properties of a substance to be determined. In laboratory conditions, **the interactions** of individual electrons can be observed by means of particle detectors, which allow measurement of specific properties such as energy, spin and charge. The development of the Paul trap and Penning trap allows charged particles to be **contained within a** small region for long durations. This enables precise measurements of the particle properties. For example, in one instance a Penning trap **was used** to contain a single electron for a period of 10 months The magnetic moment of the electron was measured to a precision of eleven digits, which, in 1980, was a greater accuracy than for any other physical constant.

The first video images of an electron's energy distribution were captured by a team at Lund University in Sweden, February 2008. The scientists used extremely short flashes of light, called attosecond pulses, which allowed an electron's motion to be observed for the first time

The distribution of the electrons in solid materials can be visualized by angle resolved photoemission spectroscopy (ARPES). This technique employs the photoelectric effect to measure the reciprocal space—a mathematical representation of periodic structures that is used to infer the original structure. ARPES can be used to **determine** the direction, speed and scattering of electrons within the material

Plasma applications

## Particle beams



During a NASA wind tunnel test, a model of the Space Shuttle is targeted by a beam of electrons, simulating the effect of ionizing gases during re-entry Electron beams are used in welding, which allows energy densities up to  $10^7$  W·cm<sup>-2</sup> across a narrow focus diameter of 0.1–1.3 usually does not require a filler material. This welding technique must be performed in a



vacuum, so that the electron beam does *not interact* with the gas prior to reaching the target, and *it can be used* to join conductive materials that would otherwise be considered unsuitable for welding.

Particle accelerators use electric fields *to propel* electrons and their antiparticles to high energies. As these particles *pass through* magnetic fields, *they emit* synchrotron radiation. The intensity of this radiation is spin *dependent*, which causes polarization of the electron beam—a process known as the Sokolov–Ternov effect. The polarized electron beams can be useful for various experiments. Synchrotron radiation *can also be used* for *cooling the* electron beams, *which reduces* the momentum spread of the particles. Once the particles have accelerated to the required energies, particles have acquired required energy levels) separate electron and positron beams are brought into collision. The *resulting energy* emissions are observed with particle detectors and are studied in particle physics

## Imaging

Low-energy electron diffraction (LEED) is a method of *bombarding a* crystalline material with a collimated beam of electrons, then observing the *resulting* diffraction patterns to *determine* the structure of the material. The required energy of the electrons is typically in the range 20–200 eV The reflection high energy electron diffraction (RHEED) technique *uses the* reflection of a beam of electrons fired at various low angles to characterize the surface of crystalline materials. The beam energy is typically in the range 8–20 keV and the angle of incidence is 1–4°

The electron microscope directs a focused beam of electrons at a specimen. As the beam *interacts* with the material, some electrons *change their* properties, *such as movement direction, angle, relative phase and energy*. By recording these changes in the electron beam, microscopists can *produce, fabricate and generate* atomically resolved image of the material. In blue light, conventional optical microscopes have a diffraction-limited resolution of about 200 nm. By comparison, electron microscopes are limited by the de Broglie wavelength of the electron. This wavelength, for example, is equal to 0.0037 nm for electrons accelerated across a 100,000-volt potential The Transmission Electron Aberration-corrected Microscope is capable of sub-0.05 nm resolution, which is more than enough to resolve individual atoms. This capability makes the electron microscope a useful laboratory instrument for high resolution imaging. However, electron microscopes are expensive instruments that are costly to maintain. There are two main types of electron microscopes: *transmission and scanning*. Transmission electron microscopes function in a manner similar to overhead *projector*, with a beam of electrons *passing through* a slice of material then *being projected* by lenses on a photographic slide or a charge-coupled device. In scanning electron microscopes, the image is produced by restoring a finely focused electron beam, as in a TV set, across the studied sample. The magnifications range from 100× to 1,000,000× or higher for both microscope types. The scanning tunneling microscope uses quantum tunneling of electrons from a sharp metal tip into the studied material and *can produce* atomically resolved images of its surface.

## Other Quantum Mechanical applications

In the free electron laser (FEL), a relativistic electron beam is *passed through a* pair of undulators containing arrays of dipole magnets, whose fields are *oriented in* alternating directions. The electrons *emit synchrotron* radiation, which, in turn, *coherently interacts* with the same electrons. This *leads to* the strong *amplification of* the radiation field at the resonance frequency. FEL can *emit a* coherent high-brilliance electromagnetic radiation with a wide range of frequencies, from microwaves to soft X-rays. These devices can be used in the future for manufacturing, communication and various medical applications, such as soft tissue surgery.

Electrons are at the heart of cathode ray tubes, *which have been used* extensively as display devices in laboratory instruments, computer monitors and television sets In a photomultiplier tube, every photon striking the photocathode *initiates an* avalanche of electrons that *produces* a detectable current pulse. Vacuum *tubes use* the flow of electrons to *manipulate* electrical signals, and they played a critical role in the development of electronics technology. However, they have been largely *supplanted* by solid-state devices such as the transistor.

## SCRODINGER'S EXPONENTIAL JURISPRUDENCE:

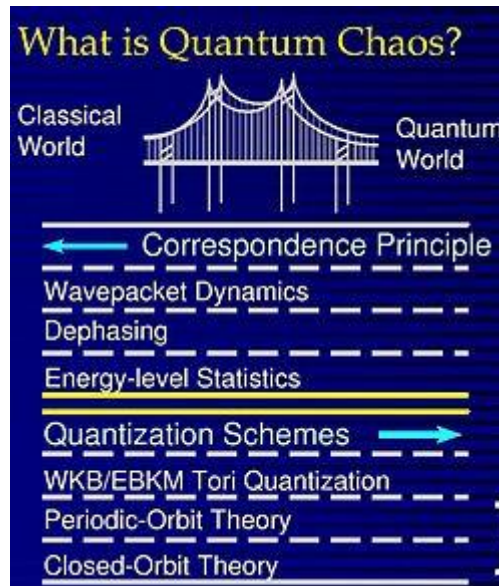
The Schrödinger equation *acts on* the *entire probability amplitude*, not merely its absolute value. Whereas the absolute value of the probability amplitude *encodes* information about probabilities, its phase *encodes* information about the *interference* between quantum states. *This gives rise to the "wave-like" behavior of quantum states*. As it turns out, analytic solutions of the Schrödinger equation are only available for a very small number of relatively simple model Hamiltonians, of which the quantum harmonic oscillator, the particle in a box, the hydrogen molecular ion, and the hydrogen atom are the most important representatives. *Even the helium atom - which contains just one more electron than does the hydrogen atom - has defied all attempts at a fully analytic treatment.*

## GENERATION OF APPROXIMATE SOLUTIONS FOR QUANTUM INFORMATION AND QUANTUM CHAOS:

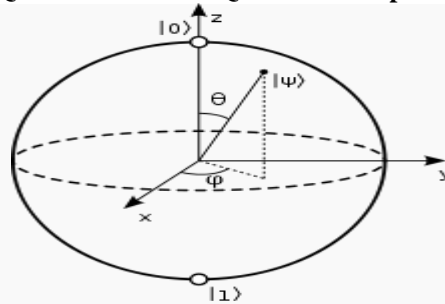
There exist several techniques *for generating* approximate solutions, however. In the important method known as *perturbation theory*, one *uses* the analytic *result for* a simple quantum mechanical model *to generate a result for* a more complicated model *that is related* to the simpler model by (for one example) the addition of a weak potential energy. Another method is the "*semi-classical equation of motion*" approach, which applies to systems for which quantum mechanics *produces only* weak (small) deviations from classical behavior. These *deviations can* then be computed based on the classical motion. This approach is particularly important in the field of quantum chaos.

Quantum chaos is a branch of physics which studies how chaotic classical dynamical systems can **be described in terms** of quantum theory. The primary question that quantum chaos seeks to answer is, "What is the **relationship** between quantum mechanics and classical chaos?" The correspondence principle states that classical mechanics is the classical limit of quantum mechanics. If this is true, then there must be quantum mechanisms **underlying (classical)** chaos; although this may not be a fruitful way of examining classical chaos. If quantum mechanics does not demonstrate an exponential sensitivity to initial conditions, how can exponential sensitivity to initial conditions **arise in** classical chaos, which must be the correspondence principle limit of quantum mechanics? In seeking to address the basic question of quantum chaos, several approaches have been employed:

Development of methods for solving quantum problems where the perturbation cannot be considered small in perturbation theory and where quantum numbers are large. **Correlating** statistical descriptions of eigenvalues (energy levels) with the classical behavior of the same Hamiltonian (system) Semi classical methods such as periodic-orbit theory **connecting the** classical trajectories of the dynamical system **with** quantum features.



### Quantum Chaos & Quantum Computers



**The Bloch sphere is a representation of a qubit, the fundamental building block of quantum computers.**

A **quantum computer** is a device for computation that makes **direct use of** quantum mechanical phenomena, *such as superposition and entanglement*, to perform operations on data. Quantum computers are different from digital computers **based on** transistors. Whereas digital computers require data to be encoded into binary digits (bits), quantum computation **utilizes** quantum properties to represent data and perform **operations on** these data. A theoretical model is the quantum Turing machine, also known as the universal quantum computer. Quantum computers share theoretical similarities *with non-deterministic and probabilistic computers*, like the ability **to be in more than one state** simultaneously. The field of quantum computing was first introduced by Richard Feynman in 1982.

Large-scale quantum computers could be able to solve certain problems much faster than any classical computer **by using the best currently known** algorithms, like *integer factorization using Shor's algorithm or the simulation of quantum many-body systems*. There exist quantum algorithms, such as Simon's algorithm, which run faster than any possible probabilistic classical algorithm. Given unlimited resources, a classical computer can **simulate an** arbitrary quantum algorithm so quantum computation does not **violate the** Church-Turing thesis. However, in practice infinite resources are never available and the computational basis of 500 qubits, for example, would already be too large to be represented on a classical **computer because** it would require  $2^{500}$  complex values to be stored. (For comparison, a terabyte of digital information stores only  $2^{43}$  discrete on/off values) Nielsen and Chuang point out that "Trying to store all these complex numbers would not be possible on any conceivable classical computer."

The standard generic quantum computer model has been studied analytically and numerically and the border for emergence of quantum chaos, ***induced by imperfections*** and residual inter-qubits couplings, is determined. This phenomenon of Quantum chaos ***appears in an*** isolated quantum computer ***without any*** external decoherence. The onset of quantum chaos ***leads to quantum computer hardware melting***, strong quantum entropy growth and ***destruction of computer operability***. The time scales for development of quantum chaos and Ergodicity ***are determined*** in some recent studies. In spite the fact that this phenomenon is rather dangerous for quantum computing it is demonstrated that the quantum chaos border for inter-qubits coupling is exponentially larger ***than (e) the*** energy level spacing between quantum computer eigenstates ***and drops*** only linearly with the number of qubits  $n$ . ***As a result*** the ideal multi-qubits structure of the computer remains rather robust against imperfections. This opens a broad parameter region for a possible realization of quantum computer. The obtained results are related to the recent studies of quantum chaos in such many-body systems as nuclei, complex atoms and molecules, finite Fermi systems and quantum spin glass shards

### **Quantum Computing of Quantum Chaos in the Kicked Rotator Model (See for details B. Levi, B. Georgeot, D.L. Shepelyansky)**

A quantum algorithm which ***simulates efficiently*** the quantum kicked rotator model, a system which ***displays*** rich physical properties, ***and enables*** to study problems of quantum chaos, atomic physics and localization of electrons in solids, was being searched for some time. . The ***effects of*** errors in gate operations are tested on this algorithm in numerical simulations with up to 20 qubits. In this way various physical quantities are investigated. Some of them, such as second moment of probability distribution and tunneling transitions through invariant curves have been shown to be particularly sensitive to errors, in some recent studies.. This study is related to our understanding of the usage of the Quantum Mechanical behaviour for Quantum information. However, investigations of the fidelity and Wigner and Husimi distributions show that these physical quantities are ***robust in presence*** of imperfections. This implies that the algorithm can ***simulate*** the dynamics of quantum chaos in presence of a moderate amount of noise. And part of that noise might come from Quantum chaos. Recent low-temperature scanning-tunneling microscopy experiments (T. Kumara et al., Phys. Rev. B 79,)

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) observed the possible quantum tunneling of hydroxyl groups between two equivalent adsorption configurations on Cu110. Quantum nuclear tunneling dynamics of hydroxyl on Cu110 using density-functional theory based techniques have been studied. Researchers classical, semi classical, and quantum mechanical transition rates for the flipping of OH between two degenerate energy minima. The classical transition rate is essentially zero at the temperatures used in experiment and the tunneling rate along the minimum-energy path is also ***much too low*** compared to experimental observations. When tunneling is taken into account along a direct path connecting the initial and final states with only a minimum amount of the oxygen movement the transition rate obtained is in much better agreement with experiment, suggesting quantum tunneling ***effects because*** a deviation of the reaction coordinates from the classical transition path. Quantum computation is an emerging interdisciplinary field, which takes advantage of concepts from both information theory and quantum mechanics. During the last decade great progress has been made in the understanding of how quantum computing and quantum communications can be performed and efficient algorithms and communication protocols have been developed. Due to the massive parallelism of quantum evolution certain quantum algorithms demonstrate an exponential gain compared to algorithms based on classical dynamics. In fact, a quantum computer represents a complex system of many ***coupled*** qubits, which in general can be ***viewed as*** a many-body ***interacting*** quantum system. On the other hand, qubits or ***spin interactions*** have also been studied extensively in the field of quantum chaos, in which typical ***problems (the ones that dissipate the ability of dissemination of Quantum information or Information processing by Quantum Compute which is our field of study) are*** decoherence and the quantum-to-classical transition, subjects that are also essential for any realistic implementation of a quantum computer. . Theory of quantum-computer algorithms, error-***correcting codes***, decoherence and quantum chaos ***effects*** in the exponentially large ***Hilbert space*** of quantum computers are vital for the ultimate technological success of quantum computation.

### **Quantum Chaos and Quantum Computer/Computing**

#### **Quantum chaos**

Classical chaos refers to the ***sensitive dependence*** on initial condition which is commonly found in nonlinear systems. In quantum mechanics, the ***trajectory loses*** its significance completely. Moreover the Schrodinger equation is a linear equation leaving no room for chaos. The ***correspondence principle, on the other hand, demand(e)s the utilization of the principle*** that in the semi-classical regime, namely, at length scale large compared with the de Broglie wavelength, ***quantum mechanics continuously develops into classical mechanics***. Therefore, at first glance, the name of ***quantum chaos seems self-contradictory***. After several years debate, now people commonly accept that ***quantum chaos*** refers to the study of quantum mechanical behavior of the systems whose ***classical counterparts*** are chaotic. This field has been very active in last two decades. Three manifestation of chaos in quantum systems have been studied so far.

#### **Quantum Manifestation of Classical Chaos : Energy level spacing statistics**

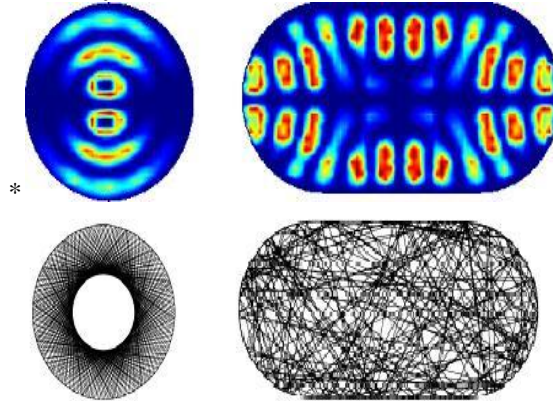
Energy level statistics has some universal features in the semi classical limit. It has been ***conjectured*** that level fluctuations ***depend only on*** general space-time symmetry and they are as predicted by the Random Matrix Theory [R]. For instance, the energy level spacing statistics in circular billiard (representing integrable systems) is Poisson distribution, whereas that in

stadium (or Sinai) billiard (representing chaotic systems with time reversal symmetry) has Wigner distribution. For a generic (mixed) system -neither complete chaotic nor complete integrable the energy level.

### Quantum Manifestation of Classical Chaos II: statistical properties of stationary wavefunctions and Quantum Tunneling:

Along with eigenenergies, wavefunctions are also used to probe quantum fingerprints and signatures of classical chaos. Usually wavefunctions provide more information about the dynamics than eigenenergies. In fact, it is the space structure of wavefunction that determines the properties of spectral statistics such as level repulsion (in chaotic systems) and/or clustering (in integrable systems) etc. So far the only proved theorem about the eigenfunctions is Shnirelman's theorem. It agrees with the conjecture of Berry and Voros that the probability density of most eigenstates of a chaotic billiard approaches a uniform distribution. It also agrees with the Porter-Thomas distribution of RMT. Numerical studies of a large number of high-lying eigenstates of billiards have confirmed the Gaussian distribution of local wave functions.

#### Classical trajectories \*Quantum Stationary Wave Functions



Circular billiard

Stadium billiard\*

Circular billiard

Stadium billiard

### Quantum Manifestation of Classical Chaos III: Dynamical evolution of states in Quantum Information:

One of the most important discoveries in quantum chaos is the *dynamical localization*, namely the quantum interference effects suppress the classical diffusive process (in phase space) which may take place in classical systems under external periodic perturbations. It has been shown by many physicists and computer scientists that the dynamical localization can be mapped to the *Anderson localization* for electrons in 1d systems with random impurity. This fact bridges Two different fields Quantum Chaos and Solid State Physics. Dynamical localization has been confirmed in several experiments such as Rydberg atom in a microwave field and an atom moving in a modulated standing wave

- Wave function structure and statistics in quantum billiard,
- Quantitative study of scars (wavefunction localization along the unstable classical periodic orbit) in far semi-classical limit.
- Energy level statistics and wave functions in mixed systems.
- Dynamical localization in quantum billiards.
- Semi-classical propagator for chaotic quantum systems.
- Semi-classical analysis of correlation functions in chaotic eigenstates. (g) Quantum chaos in non-KAM systems.

### Quantum fidelity (quantum Loschmidt's echo) and chaos in Quantum Information:

The definition of classical chaos - sensitive dependence on initial condition - loses its meaning in quantum mechanics, because the unitarity properties of quantum mechanics, namely, the overlap between two evolving wave functions - a natural indicator of distance between them is preserved with time, hence there is no divergence. An alternative definition of chaos the sensitive dependence on perturbation - has been suggested recently. This new definition is meaningful both in classical and quantum mechanics. Classically, even for Small perturbation, one generically expects rapid divergence when the systems are chaotic, as the perturbation, i.e. the difference between equations of motion, soon introduces a small displacement between the trajectories. Quantum mechanically, the overlap between the wave functions begins at unity, and then decays with time, and the rate of this decay is equivalent to a measure of the sensitivity of quantum evolution to perturbations in the equation of motion - which can be used as a signature of quantum chaos.

### Quantum mechanics as a statistical theory (For details see. E. Moyal)

An attempt is made by some computer scientists and theoretical physicists that quantum mechanics is a statistical theory, or more exactly as a form of non-deterministic statistical dynamics. Distribution functions of the complete set of dynamical variables specifying a mechanical system (phase-space distributions), which are fundamental in any form of statistical dynamics, could be expressed in terms of the wave vectors of quantum theory. This is shown to be equivalent to specifying a theory of functions of non-commuting operators, and may hence be considered as an interpretation of *quantum kinematics*. In

the second part, the laws governing the transformation with time of these phase-space distributions are derived from the equations of motion of *quantum dynamics* and found to be of the required form for a dynamical stochastic process. It is also shown that these phase-space transformation equations can be used as an alternative to the Schrödinger equation in the solution of quantum mechanical problems, such as the evolution with time of wave packets, *collision problems and the calculation of transition probabilities in perturbed systems*; an approximation method is derived for this purpose. *Quantum statistics*, deals with the phase-space distribution of members of large assemblies, with a view to applications of quantum mechanics to kinetic theories of matter. Recently, investigation has been carried out in the crossover of the quantum Loschmidt's echo (or fidelity) from the golden rule regime to the perturbation-independent exponential decay regime by using the kicked top model

$$M(t) = \left| \langle \Phi_0 | \exp(iHt) \exp(-iH_0t) | \Phi_0 \rangle \right|^2.$$

Where H is a perturbed Hamiltonian from H<sub>0</sub> which is chaotic. It is shown that the deviation of the perturbation independent decay of the averaged fidelity from the Lyapunov decay results from quantum fluctuations in individual fidelity, which is caused by the coherence in the initial coherent states. With an averaging procedure suppressing the quantum fluctuations effectively, the perturbation-independent decay is found to be close to the Lyapunov decay. This obviously means that the dissipation in quantum information sticks to the axiomatic predications of both Lyapunov decay and the Theory Of Classification.

### Quantum Computer, Quantum Information, Dynamic Thermalization, Quantum Ergodicity, Theory of Classification, Quantum Mechanical process and Quantum Computing

In classical mechanics, chaos severely limits the operation of a reversible computer. Any uncertainty in the initial conditions is magnified exponentially by chaotic dynamics, rendering the outcome of the computation unpredictable. This is why practical computational scheme are irreversible. A quantum computer does not have this option. It relies on the reversible unitary evolution of entangled quantum mechanical states, which does not tolerate dissipation. On the other hand, the exponential gain of quantum computing is due to exponentially large size of Hilbert space which grows exponentially with the number of qubits which are the basis of quantum computers. In order to perform logical operations in quantum computers, these qubits should be coupled. As a consequence, quantum computers represent many body systems with interaction. Similar systems have been recently studied in the field of quantum chaos with applications to different many body systems such as nuclei, complex atoms, quantum dots and quantum spin glasses. It had been found that a sufficiently strong coupling leads to the emergence of quantum Ergodicity and dynamical (internal) thermalization. In this regime the systems eigenstates become very complex and strongly different from the eigenstates of non interacting many body systems. At first glance, one would expect this regime to appear when the coupling is comparable with the spacing between multiparticles levels. This naïve estimate would give an absurdly strong restriction for the coupling strength and therefore a too severe limitation for the realization of quantum computers. This raises the question:

- What limitations quantum chaos might pose on quantum computing?
- What restrictions of quantum chaos might pose on quantum error correction?
- Whether the suppression of quantum chaos (dynamical localization) improve the fidelity for recovery from errors of decoherence.

### Mathematically equivalent formulations of quantum mechanical behaviour and Quantum Information

Oldest and most commonly used formulations is the "transformation theory" proposed by the late Cambridge theoretical physicist Paul Dirac, which unifies and generalizes the two earliest formulations of quantum mechanics - matrix mechanics (invented by Werner Heisenberg) and wave mechanics (invented by Erwin Schrödinger). In Matrix formulation, the instantaneous state of a quantum system encodes the probabilities of its *measurable properties*, or "observables". Examples of observables include energy, position, momentum, and angular momentum. These are the factors that play a vital role in the classification scheme of our paper. Observables can be either continuous (e.g., the position of a particle) or discrete (e.g., the energy of an electron bound to a hydrogen atom) An alternative formulation of quantum mechanics is Feynman's path integral formulation, in which a quantum-mechanical amplitude is considered as equivalent to as a sum over all possible histories between the initial and final states. This is the quantum-mechanical counterpart of the *action principle* in classical mechanics.

### Quantum Mechanical Behaviour, DIRAC Equation, Quantum Information and its Interactions with other scientific theories

The rules of quantum mechanics are fundamental. They assert that the state space of a system is a Hilbert space, and that observables of that system are operators acting on that space - although they do not tell us which Hilbert space or which operators. (In physics, particularly in quantum physics, a system observable is a property of the system state that can be determined by some sequence of physical operations. For example, these operations might involve submitting the system to various electromagnetic fields and eventually reading a value off some gauge. In systems governed by classical mechanics,

any experimentally observable value can be shown to be given by a real-valued function on the set of all possible system states. Physically meaningful observables must also satisfy transformation laws which relate observations performed by different observers in different frames of reference. These transformation laws are automorphism of the state space that is bijective transformations which preserve some mathematical property.)

These can be chosen appropriately in order to obtain a quantitative description of a quantum system. An important guide for making these choices is the correspondence principle, which states that the predictions of quantum mechanics reduce to those of ,become equal to classical mechanics when a system moves to higher energies or - equivalently - larger quantum numbers, i.e. whereas a single particle exhibits a degree of randomness, in systems incorporating millions of particles averaging takes over and, at the high energy limit, the statistical probability of random behaviour approaches zero. In other words, classical mechanics is simply a quantum mechanics of large systems. This "high energy" limit is known as the classical or correspondence limit. One can even start from an established classical model of a particular system, then attempt to guess the underlying quantum model that would give rise to the classical model in the correspondence limit. When quantum mechanics was originally formulated, it was applied to models whose correspondence limit was non-relativistic classical mechanics. For instance, the well-known model of the quantum harmonic oscillator uses an explicitly non-relativistic expression for the kinetic energy of the oscillator, and is thus a quantum version of the classical harmonic oscillator.

Early attempts to merge quantum mechanics with special relativity involved the replacement of the Schrödinger equation with a covariant equation such as the Klein-Gordon equation or the Dirac equation. The Dirac bispinor is represented by four ionic internal states, and position and momentum of the Dirac particle are associated with the respective ionic variables. Simulation of the simplified 1 + 1 case, requiring the manipulation of only two internal levels and one motional degree of freedom. Moreover, relevant quantum-relativistic effects, like the Zitterbewegung and Klein's paradox, the transition from massless to massive fermions, and the relativistic and nonrelativistic limits, via the tuning of controllable experimental parameters. Dirac equation, describes the behaviour of fermions, and predicted the existence of antimatter

While these theories were successful in explaining many experimental results, they had certain unsatisfactory qualities stemming from their neglect of the relativistic creation and annihilation of particles. A fully relativistic quantum theory required the development of quantum field theory, which applies quantization to a field (rather than a fixed set of particles). The first complete quantum field theory, quantum electrodynamics, provides a fully quantum description of the electromagnetic interaction. The full apparatus of quantum field theory is often unnecessary for describing electrodynamic systems. A simpler approach, one that has been employed since the inception of quantum mechanics, is to treat charged particles as quantum mechanical objects being acted on by a classical electromagnetic field. For example, the elementary quantum model of the hydrogen atom describes the electric field of the hydrogen atom using a classical  $-e^2/(4\pi \epsilon_0 r)$  Coulomb potential. This "semi-classical" approach fails if quantum fluctuations in the electromagnetic field play an important role, such as in the emission of photons by charged particles.

Quantum field theories for the strong nuclear force and the weak nuclear force have also been developed. The quantum field theory of the strong nuclear force is called quantum chromodynamics, and describes the interactions of sub nuclear particles such as quarks and gluons. The weak nuclear force and the electromagnetic force were unified, in their quantized forms, into a single quantum field theory (known as electroweak theory), by the physicists Abdus Salam, Sheldon Glashow and Steven Weinberg. It has proven difficult to construct quantum models of gravity, the remaining fundamental force. Semi-classical approximations are workable, and have led to predictions such as Hawking. However, the formulation of a complete theory of quantum gravity is hindered by apparent incompatibilities between general relativity (the most accurate theory of gravity currently known) and some of the fundamental assumptions of quantum theory. The resolution of these incompatibilities is an area of active research, and theories such as String Theory are among the possible candidates for a future theory of quantum gravity. Classical mechanics has also been extended into the complex domain, with complex classical mechanics exhibiting behaviors similar to quantum mechanics.

### Quantum Chaos and Quantum Information:

Predictions of quantum mechanics have been verified experimentally to an extremely high degree of accuracy. According to the correspondence principle between classical and quantum mechanics, all objects obey the laws of quantum mechanics, and classical mechanics is just an approximation for large systems of objects (or a statistical quantum mechanics of a large collection of particles). The laws of classical mechanics thus follow from the laws of quantum mechanics as a statistical average at the limit of large systems or large quantum numbers. However, chaotic systems do not have good quantum numbers, and quantum chaos studies the relationship between classical and quantum descriptions in these systems. Maintaining coherence in Quantum Computers. (W. G. Unruh) is an important phenomenon that has to be addressed with for the efficient functioning of a Quantum Computer. The effect of the inevitable coupling to external degrees of freedom of a quantum computer are examined, by various Physicists (See Unruh). It is found that for quantum calculations (in which the maintenance of coherence over a large number of states is important), not only must the coupling be small but the time taken in the quantum calculation must be less than the thermal time scale, Quantum computers to be more efficient in certain problems involves having the computer be placed in the coherent superposition of a very large number (exponential in L) of "classical states", and having the outputs interfere in such a way that there is a very high probability that on the appropriate reading of the output, one would obtain the required answer. One is replacing exponentiality in time with exponentiality in quantum coherence. This requires that the computer be able to maintain the coherence during the course of the calculation.

The **constraints** placed on the ability to maintain this coherence in the face of coupling to external heat baths. Landauer has long emphasized the necessity of examining **the effect of** both imperfections and of the coupling **to** the external world of any realistic device on the ability of quantum computers to realize their promise. For longer times the condition on the strength of the coupling to the external world becomes much more stringent. Quantum coherence is an essential difference between classical and quantum theories, and is illustrated by the Einstein-Podolsky-Rosen paradox. Quantum interference **involves adding together probability amplitudes**, whereas classical "waves" infer that there is an adding **together of intensities**. For microscopic bodies, the extension of the system is much smaller than the **coherence length**, which gives rise to long-range entanglement and other nonlocal phenomena that are characteristic of quantum systems. Quantum coherence is not typically evident at macroscopic scales - although an exception to this rule **can occur at** extremely low temperatures (i.e. approaching absolute zero), when quantum behavior can manifest itself on more macroscopic scales (see macroscopic quantum phenomena, Bose-Einstein condensate, and Quantum machine).

Many macroscopic properties of a classical system are a direct **consequence** of the quantum behavior of its parts. For example, the stability of bulk matter (which consists of atoms and molecules which **would quickly collapse under** electric forces alone), the rigidity of solids, and the mechanical, thermal, chemical, optical and magnetic properties of matter **are all results** of the interaction of electric charges under the rules of quantum mechanics. While the seemingly "exotic" behavior of matter **posited by** quantum mechanics and relativity theory **become more apparent** when dealing with particles of extremely small size or velocities approaching the speed of light, the laws of classical Newtonian physics remain accurate in predicting the behavior of the vast majority of "large" objects (on the order of the size of large molecules or bigger) at velocities much smaller than the velocity of light.

**Emergence of Quantum Chaos in Quantum Computer Core and How to manage it** has been discussed by B. Georgeot and D. L. Shepelyansky. Generic quantum computer model, **describes a** realistic isolated quantum computer **with fluctuations** in individual qubit energies and residual short-range inter-qubit **couplings**. In the limit where the fluctuations and couplings **are small** compared to one-qubit energy spacing the spectrum **has a** band structure and **a renormalized** Hamiltonian **is obtained** which describes the eigenstate properties inside one band. The studies are concentrated on the central band of the computer ("core") with the highest density of states. **Above a** critical inter-qubit coupling strength, quantum chaos **sets in, leading to** quantum Ergodicity of the computer eigenstates. In this regime the ideal qubit structure **disappears**, the eigenstates **become** complex and the operability of the computer are quickly **destroyed**. Quantum chaos border **decreases** only linearly with the number of qubits  $n$ , although the spacing between multi-qubit states **drops** exponentially with  $n$ . The investigation of time-evolution in the quantum computer **shows that** in the quantum chaos regime, an ideal (noninteracting) state quickly **disappears** and exponentially many states **become** mixed after a short chaotic time scale for which the dependence on system parameters **is determined**. **Below the** quantum chaos border an ideal state can survive for long times and can **be used** for computation. The **results show** that a broad parameter region **does exist** where the efficient operation of a quantum computer is possible. Quantum computers **have problem in** simulation of common quantum systems, since the computation time **grows** exponentially with the number of quantum particles. Therefore for such problems it is natural to **envision a** computer composed from quantum elements (qubits) which **operate** according to the laws of quantum mechanics. In any case, such devices will be in a sense unavoidable since the technological progress **leads to** chips of smaller and smaller size which will eventually reach the quantum scale. At present a quantum computer is viewed as a system of  $n$  qubits (two-level quantum systems), with the possibility of **switching on and off a coupling between them**. The operation of such computers **is based** on reversible unitary transformations in the Hilbert space whose dimension  $N_H = 2^n$  is exponentially large in  $n$ . It has been shown that all unitary operations can be realized with two-qubit transformations. This makes necessary the existence of **a coupling** between qubits. Any quantum algorithm will be a sequence of such fundamental transformations, which form the basis of new quantum logic. Shor constructed a quantum algorithm which is **exponentially faster** than the classical ones. It was also shown by Grover that the searching of an item in a long list is parametrically much faster on a quantum computer. The recent development of error correcting codes showed that a certain amount of noise due to external coupling **could be tolerable** in the operation of a quantum computer. All these exciting developments motivated a great body of experimental proposals to effectively realize such a quantum computer. They include ion traps, nuclear magnetic resonance systems, nuclear spins with **interaction controlled** electronically or by laser pulses, quantum dots, Cooper pair boxes optical lattices and electrons floating on liquid helium. **As a result**, a two-qubit gate has been experimentally realized with cold ions, and the Grover algorithm has been performed for three qubits made from nuclear spins in a molecule. However, to have a quantum computer competitive with a classical one will require a much larger number of qubits. For example, the minimal number of qubits for which Shor's algorithm will become useful is of the order of  $n = 1000$ . Serious **obstacle to** the physical realization of such computers is the quantum decoherence due to the couplings with the external world **which gives** a finite lifetime to the excited state of a given qubit. **The effects** of decoherence and laser pulse shape broadening were numerically simulated in the context of Shor's algorithm, and shown to be quite important for the operability of the computer. However, in a number of physical proposals, for example nuclear spins in two-dimensional semiconductor structures, the relaxation time due to this decoherence process can be many orders of magnitude **larger than** the time required for the gates operation, so that there are hopes to manage this obstacle. Obstacle to the physical realization of quantum computers that was not stressed up to now. This problem arises even if the decoherence time is infinite and the system is **isolated/decoupled** from the external world. Indeed, even **in the absence** of decoherence there are always imperfections in physical systems. **Due to that the** spacing between the two states of each qubit will have fluctuations in some finite detuning interval  $\delta$ . Also, some residual static

**interaction** J between qubits will be unavoidably present (inter-qubit coupling is required to operate the gates). Extensive studies of many-body **interacting systems** such as nuclei, complex atoms, quantum dots and quantum spin glasses have shown that generically in such **systems the interaction leads to** quantum chaos characterized by Ergodicity of the eigenstates and level spacing statistics as in Random Matrix Theory (RMT). In a sense the interaction **leads to** dynamical thermalization **without** coupling to an external thermal bath. If the quantum computer were in such a regime, its operability would be effectively **destroyed** since the noninteracting multi-qubit states representing the quantum register states **will be eliminated** by quantum ergodicity. In this respect, it is important to stress that unavoidably the residual interaction J **will be much larger than** the energy spacing  $\Delta_n$  between adjacent eigenstates of the quantum computer. Indeed the residual interaction J is relatively small so that all NH computer eigenenergies are **distributed in an** energy band of size  $\Delta E \sim n\Delta_0$ , where  $\Delta_0$  is the average energy distance between the two levels of one qubit and n is the total number of qubits in the computer. **By changing the** electrostatic gate potential, the effective electron mass can **be modified up** to a factor of two. In the quantum computer the quantum **chaos sets in only for** couplings J exponentially stronger than  $\Delta_n$ . In fact, it was shown that the critical coupling  $J_c$  for the transition to quantum chaos **decreases** only linearly with the number of qubits n (for short-range inter-qubit coupling): a broad parameter region where a quantum computer can be operated below the quantum chaos border, **exists** when noninteracting multi-qubit states are **very close** to the exact quantum computer eigenstates. Details the **transition to** chaos and how **it effects** the time evolution of the system. **The effects** of **residual interaction** in the presence or absence of fine fluctuations of individual qubit energy spacing are also analyzed in great detail.

#### **Relativity and quantum Information and Quantum machine:**

Quantum mechanics, information theory, and relativity theory are the basic foundations of theoretical physics. The acquisition of information from a quantum system **is the interface of** classical and quantum physics. Essential tools for its description are Kraus matrices and positive operator valued measures (POVMs). Special relativity **imposes severe restrictions** on the transfer of information between distant systems. Quantum entropy is not a Lorentz covariant concept. Lorentz transformations of reduced density matrices for entangled **systems may not be completely** positive maps. Quantum field theory, which is necessary for a consistent description of interactions, implies a fundamental trade-off between detector reliability and localizability. General relativity produces new, counterintuitive effects, in particular when black holes (or more generally, event horizons) are involved. Most of the current concepts in quantum information theory may then require a reassessment

The fundamental properties of quantum information and its applications to computing and cryptography have been greatly illuminated by considering information-theoretic tasks that are provably possible or impossible within non-relativistic quantum mechanics. I describe here a general framework for defining tasks within (special) relativistic quantum theory and illustrate it with examples from relativistic quantum cryptography and relativistic distributed quantum computation. The framework gives a unified description of all tasks previously considered and also defines a large class of new questions about the properties of quantum information in relation to Minkowski causality. It offers a way of exploring interesting new fundamental tasks and applications, and also highlights the scope for a more systematic understanding of the fundamental information-theoretic properties of relativistic quantum theory.

#### **Entanglement generation in relativistic quantum fields and corresponding Quantum Computation. (See Nicolai Friis, Ivette Fuentes)**

A general, analytic recipe is to compute the entanglement **that is generated** between arbitrary, discrete modes of bosonic quantum fields by **Bogoliubov transformations**. One setup allows the complete characterization of the quantum **correlations in all** Gaussian field states. Additionally, it holds for all Bogoliubov transformations. These are commonly **applied in** quantum optics **for the description** of squeezing operations, **relate** the mode decompositions of observers in different regions of curved spacetimes, and **describe** observers moving along non-stationary trajectories. Physicists have focused on a quantum optical example in a cavity quantum electrodynamics setting: **an uncharged scalar field within a cavity provides a** model for an optical resonator, in which **entanglement is created** by **non-uniform acceleration**. Amount of generated entanglement can be **magnified** by initial single-mode squeezing. Applications to quantum fields in curved spacetimes, such as an expanding universe, are also discussed. in many studies.

There exists an intimate **relationship** between quantum mechanics, information theory, and relativity theory. Taken together these are the foundations of present-day theoretical physics, and their interrelationship is an essential part of the theory. The acquisition of information from a quantum system by an observer **occurs at the** interface of classical and quantum physics. Many a author have reviewed the essential tools needed to describe this interface, i.e., Kraus matrices and positive-operator-valued measures. They then discuss how special relativity **impose** severe (e) **restrictions** on the **transfer of information between distant systems and the implications of the fact that quantum entropy is not a Lorentz-covariant concept**. This leads to a discussion of how it comes about that Lorentz transformations of reduced density matrices for entangled systems may not be completely positive maps. Quantum field theory is, of course, necessary for a consistent description of interactions. Its structure implies a fundamental **tradeoff** between detector reliability **and** localizability. Moreover, general relativity **produces new and counterintuitive effects**, particularly when black holes (or, more generally, event horizons) are involved. In this more general context some authors discuss how most of the current concepts in quantum information theory may require a reassessment. How relativistic effects (-) can be **exploited** to improve quantum information tasks, (+) a key topic of immense technological importance already today and more so for the next decades. The vantage point of these investigations is that the world is fundamentally both quantum and relativistic, and that these facts are immensely useful for the design of communication devices that are absolutely safe from eavesdropping, and of quantum computers that can quickly perform difficult computational tasks which overwhelm any classically imaginable computer. Indeed, impressive technological



achievements and promises have already been derived from taking seriously solely the quantum aspects of matter: quantum cryptography and communication have become a technical reality in recent years, but the practical construction of a quantum computer still requires understanding better how to efficiently store, manipulate and read information, without prohibitively large disturbances from the environment. Throwing relativity into the equation fundamentally changes the entire game, as I could show in a series of research papers, one of which was featured in a generally accessible Science article highlighting my work (Cho, Science 2005). I propose to push this exciting line of theoretical research to the point where relativistic effects in quantum information theory can be exploited technologically. Far from yielding only quantitative corrections, relativity **plays a dominant role** in the qualitative behaviour of many physical systems used to implement quantum information tasks in the laboratory. The prototypical example is provided by any system involving light, be it for the transmission or manipulation of quantum information. There is no such thing as a **non-relativistic approximation** to light quanta, so-called photons, since these always travel at the speed of light. While relativistic quantum theory, commonly known as quantum field theory, is a very well studied subject in foundational particle physics, research in **quantum information theory** selectively focused almost exclusively on those aspects one can study without relativity. Thus both unexpected obstacles (such as a relativistic **degradation** of quantum entanglement) and unimagined possibilities for quantum information theory (such as improved **quantum cryptography** and **hypersensitive quantum** measurement devices) have gone unnoticed. , Indeed, the impact of such research extends beyond *pure quantum information theory*, and applications to foundational questions in cosmology and black hole physics have been found. Over the past few years, a new field of high research intensity has emerged, known as **Relativistic Quantum Information (RQI)**. The field of RQI aims to understand the **relationship** between special and general relativity, and quantum information. In particular, so-called **quantum entanglement bits (e-bits)** are **a necessary resource** in all of **quantum communication and quantum computation**. One of the key goals of the field of RQI is to develop a theory of e-bits in realistic spacetimes described by Lorentzian manifolds. This offers exciting new challenges because it combines **information theory with quantum theoretic and general relativistic questions**. Deep questions concerning the **relationship** between information processing and the structure of spacetime are considered. Phenomena of relativistic quantum information, for example, use quantum communication with satellite-based instruments or **Berry's phase tests of the Unruh effect**.

Recently, there has been increased interest in understanding entanglement and quantum communication in black hole spacetimes and in using quantum information techniques to address questions in gravity. Studies on **relativistic entanglement** show the emergence of conceptually important **qualitative differences** to a non-relativistic treatment. For instance, entanglement was found to be an observer-dependent property that changes from the perspective of accelerated observers moving in flat spacetime. Relativistic quantum information theory uses well-known tools coming from quantum information and quantum optics to study quantum effects provoked by gravity to learn information about the spacetime. We can take advantage of our knowledge **about quantum correlations** and **effects produced** by the **gravitational interaction** to set the basis for experimental proposals ultimately aiming at **finding corrections due** to quantum gravity effects, too mild to be directly observed.

**Quantum theory** and **general relativity** are famously at loggerheads. Their mathematical languages are different and conceptual bases are discordant, if not outright conflicting. For more than sixty years this conceptual gap and scant experimental evidence has been preventing **unification of** the two theories. At the close of the last century a seemingly unrelated development of quantum information theory helped to unriddle some of the long-standing conceptual problems in quantum mechanics. While originally discussed in terms of **non-relativistic quantum mechanics**, recent years have seen increasing research interest and activities in placing quantum information in a more rigorous framework of **quantum field theory in** curved spacetimes. As a matter of fact, information theory is often appealed to in black hole physics and gravitational theories, particularly in relation to **the information loss paradox** and the **holographic principle**. **A relationship of "R" and "QI" is twofold**. On the one hand, **quantum field theory** is an **instrumental** tool in proposed designs of **quantum computing and communication**, and to comprehend the meaning and possibilities of **quantum non-locality**, and **entanglement in the quantum vacuum**. On the other hand, what "R" entails in RQI is perhaps best captured by what a relativist does traditionally: encompassing special and general relativity, geometry and topology (of spacetime and gauge fields), quantum field theory in curved spacetime and quantum gravity

### **Black hole information paradox and Quantum Machine:**



The **black hole information paradox results** from the combination of quantum mechanics and general relativity. It suggests that physical INFORMATION could permanently **disappear in a** black hole, **allowing** many physical states **to evolve into** the same state. Loss of information in a black hole **produces** evolution of many physical states in to the same state Or, in other words, evolution of many physical states in to single state must **utilize** the loss of physical information in black hole..This is controversial because it violates a commonly assumed tenet of science—that *in principle* complete information about a physical system at one point in time should **determine** its state at any other time A fundamental postulate of quantum

mechanics is that complete information about a **system is encoded** in its wave function. The evolution of the wave function is **determined(e) by** a unitary operator. And unitarity **implies** that information is conserved in the quantum sense.

There are two main principles in play: **quantum determinism, and reversibility**. Quantum determinism means that given a present wave function, its future changes are uniquely **determined by** the evolution operator. Reversibility refers to the fact that the evolution operator **has an** inverse, meaning that the past wave functions are similarly unique. The combination of the two means that information must always be preserved.

Starting in the mid 1970's, Stephen Hawking and Jacob Bekenstein put forward theoretical arguments based on general relativity and quantum that appeared to be inconsistent with information conservation. Specifically, Hawking's calculations indicated that black hole evaporation via Hawking radiation **does not** preserve information. Today, many physicists believe that the holographic principle (specifically the AdS/CFT duality) **demonstrates that** Hawking's conclusion was incorrect, and that information is in fact preserved. In 2004 Hawking himself conceded a bet he had made, agreeing that black hole evaporation does in fact preserve information. Black hole evaporation **produces** the conservation and preservation of information. Conservation and preservation of information is attributable and ascribable to the entropic entecote of (e) Black hole evaporation.

Einstein himself is well known for rejecting some of the claims of quantum mechanics. While clearly contributing to the field, he did not accept many of the more "philosophical consequences and interpretations" of quantum mechanics, such as the **lack of deterministic** causality. He is famously quoted as saying, in response to this aspect, "My God does not play with dice". He also had difficulty with the assertion that a single subatomic particle **can occupy** numerous areas of space at one time. However, he was also the first to notice some of the apparently exotic **consequences of** entanglement, and **used** them to formulate the Einstein-Podolsky-Rosen paradox in the hope of showing that quantum mechanics had unacceptable implications. This was 1935, but in 1964 it was shown by John Bell (see Bell inequality) that - although Einstein was correct in identifying seemingly **paradoxical implications** of quantum mechanical non locality - these implications could be experimentally tested. Alain Aspect's initial experiments in 1982, and many subsequent experiments since, have definitively verified quantum entanglement.

According to the paper of J. Bell and the Copenhagen interpretation - the common interpretation of quantum mechanics by physicists since 1927 - and contrary to Einstein's ideas, quantum mechanics was **not**, at the same time:

#### **EPR EXPERIMENT, QUANTUM STATES, QUANTUM COMPUTATION, QUANTUM ENTANGLEMENT:**

The Einstein-Podolsky-Rosen paradox shows in any case that there exist experiments by which one can measure the state of one particle and instantaneously **change the** state of its entangled partner - although the two particles can be an arbitrary distance apart. However, **this effect** does **not violate** causality, since **no transfer of** information happens. Quantum entanglement forms the **basis** of quantum cryptography, which is used in high-security commercial applications in banking and government.

Gravity **is negligible** in many areas of particle physics, so that **unification between** general relativity and quantum mechanics is not an urgent issue in those particular applications. However, the lack of a correct theory of **quantum gravity** is an important issue in cosmology and the search by physicists for an elegant "Theory of Everything" (TOE). Consequently, resolving the inconsistencies between both theories has been a major goal of 20th and 21st century physics. Many prominent physicists, including Stephen Hawking, have labored for many years in the attempt to discover a theory underlying **everything**. This TOE would combine not only the different models of subatomic physics, but also derive the four fundamental forces of nature - the strong force, electromagnetism, the weak force, and gravity - from a single force or phenomenon. While Stephen Hawking was initially a believer in the Theory of Everything, after considering Gödel's Incompleteness Theorem, he has concluded that one is not obtainable, and has stated so publicly in his lecture "Gödel and the End of Physics" (2002). One of the leading authorities continuing the search for a coherent TOE is Edward Witten, a theoretical physicist who formulated the groundbreaking M-theory, which is an attempt at describing the super symmetrical based string theory. **M-theory posits** that our apparent 4-dimensional spacetime is, in reality, actually an 11-dimensional spacetime containing 10 spatial dimensions and 1 time dimension, although 7 of the spatial dimensions are - at lower energies - **completely "compactified" (or** infinitely curved) and not readily **amenable to** measurement or probing.

#### **ATTEMPTS AT UNIFIED THEORY AND ITS IMPLICATIONS ON QUANTUM COMPUTING:**

The quest **to unify the** fundamental forces through quantum mechanics is still ongoing. Quantum electrodynamics (or "quantum electromagnetism"), which is currently (in the perturbative regime at least) the most accurately tested physical theory<sup>3</sup> has been successfully merged with the weak nuclear force into the electroweak force and work is currently being done to merge the electroweak and strong force into the electro strong force. Current predictions state that at around  $10^{14}$  GeV the three aforementioned forces are fused into a single unified field,<sup>[37]</sup> Beyond this "grand unification," it is speculated that it may be possible to merge gravity with the other three gauge symmetries, expected to occur at roughly  $10^{19}$  GeV. However — and while special relativity is parsimoniously incorporated into quantum electrodynamics — the expanded general relativity, currently the best theory describing the gravitation force, has not been fully incorporated into quantum theory.

Albert Einstein, himself one of the founders of quantum theory, disliked this **loss of determinism in measurement**. Einstein held that there should be a theory underlying quantum mechanics and, consequently, that the present theory was incomplete. He produced a series of objections to the theory, the most famous of which has become known as the Einstein-Podolsky-

Rosen paradox. John Bell showed that this "EPR" paradox led to experimentally testable differences between quantum mechanics and local realistic theories. Experiments have been performed confirming the accuracy of quantum mechanics, **thereby demonstrating that** the physical world cannot be described by any local realistic theory. The *Bohr-Einstein debates* provide a vibrant critique of the Copenhagen Interpretation from an epistemological point of view.

The *Everett many-worlds interpretation*, formulated in 1956, holds that *all* the possibilities described by quantum theory *simultaneously* occur in a multiverse composed of mostly **independent parallel universes**. This is not accomplished by introducing some "new axiom" to quantum mechanics, but on the contrary, **by removing** the axiom of the **collapse of the wave packet**. All of the possible consistent states of the measured system and the measuring apparatus (including the observer) are present in a *real* physical - not just formally mathematical, as in other interpretations - **quantum superposition**. Such a superposition of consistent state **combinations of different** systems is called an entangled state. While the multiverse is deterministic, we perceive non-deterministic behavior **governed** by probabilities, **because** we can observe only the universe (i.e., the consistent state contribution to the aforementioned superposition) that we, as observers, inhabit. Everett's interpretation is perfectly **consistent with John** Bell's experiments and makes them intuitively understandable. However, according to the theory of quantum decoherence, these "parallel universes" **will never be accessible to us**. The inaccessibility can be understood as follows: once a measurement is done, the measured system **becomes entangled** with *both* the physicist who measured it *and* a huge number of other particles, some of which are photons flying away at the speed of light towards the other end of the universe. In order to prove that the wave function **did not collapse**, one would have to bring *all* these particles back and measure them again, together with the system that was originally measured. Not only is this completely impractical, but even if one *could* theoretically do this, it would **destroy any evidence that the original measurement took place** (to include the physicist's memory).

Quantum mechanics had enormous success in explaining many of the features of our world. The **individual behaviors** of the subatomic particles that make up all forms of matter (electrons, protons, neutrons, photons, and others) can often **only be satisfactorily described** using quantum mechanics. Quantum mechanics has **strongly influenced** string theories, candidates for a *Theory of Everything* (see *reductionism*), and the **multiverse hypotheses**.

### **QUANTUM TUNELLING, QUANTUM COHERENCE, AND QUANTUM MACHINE**

**Quantum Tunneling of Magnetization and Related Phenomena in Molecular Materials (SEE Dante Gatteschi dRoberta Sessoli)**

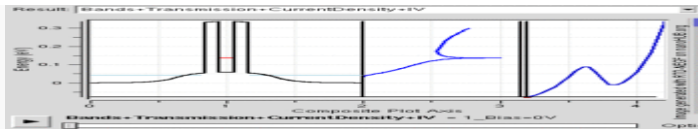
The fields in which chemistry may have a fundamental role are increasing in number, and it is becoming more and more difficult (though stimulating) for synthetic chemists to cope with requests of new compounds with highly sophisticated properties. The movement from simplicity to complexity in molecular chemistry is one which cannot be stopped, and supramolecular chemistry is just one clear example of that. The last few years have seen the opening of many new research fields in which molecular materials have been forced, through chemical ingenuity, to acquire properties which are associated with classic inorganic materials. Perhaps the most convincing evidence for this has been the development of purely organic materials, which behave as conductors and superconductors. In addition, organic materials, because of their photorefractive and nonlinear-optical properties, are now even used to make lasers. Magnetism is certainly one of the fundamental properties of matter, inextricably associated with electrical properties. One of the main difficulties in developing molecule-based magnets is that bulk magnetism is intrinsically a three-dimensional (3D) property in that only if a 3D lattice of interacting magnetic centers is assembled may the material show spontaneous magnetization below a critical temperature. The problem with molecular materials is that the design of genuine 3D connected lattices is not obvious, because the intrinsic low symmetry of the individual molecules tends to favor 1D or 2D arrangements. Here the difference between molecular magnets and conductors is striking, because for conductors exciting properties can be found even if 1D arrangements of individual molecules are obtained in the solid state. However, this has not scared the synthetic chemists and genuine room-temperature molecule based magnets are now available. The difficulties in forming 3D magnets with molecular materials have been turned into advantages when it was realized that oligonuclear compounds, comprising a large, but finite, number of magnetic centers may have unique magnetic properties which have made them almost ideal systems for observing quantum size effects in magnets.

The magnetic centers can be transition-metal or rare-earth ions, or even organic radicals. The observation of quantum phenomena in mesoscopic matter provides, in principle, confirmation to the so-called Copenhagen interpretation, which assumes that there is a continuous transition from the field of small objects, where quantum mechanics is required, to macroscopic objects, where classical physics operates well. Materials the properties of which are intermediate between classical and quantum nature, or where there is coexistence of the two, may be used for completely new types of devices. A particularly interesting field is that of **quantum computing**, where information can be handled taking advantage of quantum Tunneling. Molecules comprising a large number of coupled paramagnetic centers are attracting much interest because they may show properties which are intermediate between those of simple paramagnets and classical bulk magnets and provide unambiguous evidence of quantum size effects in magnets. To date, two cluster families usually referred to as Mn<sub>12</sub> and Fe<sub>8</sub>, have been used to test theories. However, it is reasonable to predict that other classes of molecules will be discovered which have similar or superior properties. To do this it is necessary that synthetic chemists have a good understanding of the correlation between the structure and properties of the molecules, for this it is necessary that concepts such as **quantum tunneling, quantum coherence, quantum oscillations are understood**. In fact it gives rise to magnetic hysteresis, which is one condition for storing information in a particle. Under this respect therefore Mn<sub>12</sub>ac behaves like a classical magnet. However, it is still small enough to also show large quantum effects. In fact Mn<sub>12</sub>ac, and other molecules which have since

been investigated, provide the best examples to date of the observation of quantum effects, such as the tunneling of the magnetization, in magnets. These molecules are now often called <sup>TM</sup>single-molecule magnets]

The interactions with the environment will tend to localize the particles, because the interactions will make one well more <sup>TM</sup>attractive] than the other (that is, reduce the energy of one of the wells). In the case of strong coupling with the environment, which means that this interaction is much larger than the tunnel splitting, the particle will stay localized in one of the two wells, and will not tunnel. For intermediate coupling the particle can tunnel, but jumping incoherently from one well to the other. This means that one particle will tunnel, and localize for some time in the other well, and then tunnel again, but in an irregular way. The third case is that of weak coupling when the tunnel splitting is large compared to the interaction with the environment, and the particle oscillates coherently between the two minima. The conditions for observing coherent tunneling are severe. Evidence for coherent tunneling is the observation of energy absorption at a frequency corresponding to the tunnel splitting. In the above experiment of calyx a peak was observed in the proton-spin lattice-relaxation rate at a field corresponding to the tunneling frequency of 35 MHz

Quantum mechanics is also critically important for understanding how individual atoms combine covalently to form molecules. Relativistic quantum mechanics can, in principle, mathematically describe most of chemistry. Quantum mechanics can also provide quantitative insight into ionic and covalent bonding processes by explicitly showing which molecules are energetically favorable to which others, and the magnitudes of the energies involved. Furthermore, most of the calculations performed in modern computational chemistry rely on quantum mechanics.



A working mechanism of a resonant tunneling diode device, based on the (e)phenomenon of quantum tunneling through potential barriers

A great deal of modern technological inventions operates at a scale where quantum effects are significant. Examples include the laser, the transistor (and thus the microchip), the electron microscope, and magnetic resonance imaging (MRI). The study of semiconductors led to the invention of the diode and the transistor, which are indispensable parts of modern electronics systems and devices.\

## SIMULATION OF THE REALITY-IS THIS WORLD SIMULATED BY A GIANT QUANTUM COMPUTER?

*A Quantum-Digital Universe (See Giacomo Mauro D'Ariano)*

Can Reality be simulated by a huge Quantum Computer? Do we believe that Reality is made of something more than interacting quantum systems? The idea that the whole Physics is ultimately quantum computation; strong quantum version of the Church-Turing hypothesis well synthesized by the Wheeler's coinage it from bit is very appealing. It is theoretically very parsimonious and curmudgeonly- an Occam razor's quality-guaranteed description of the world. But, if this is the case, then we need to understand the entire Physics as emergent(e) from the quantum computation. Universe (=) is a Quantum Field. Particles are(=) just states of the Field: they can be created and (e)&(eb)annihilated. We have indeed a beautiful Grand Unified Field Theory, and we are looking forward to see the Higg's boson at the LHC.

But what is the Quantum Field made of? Ultimately, Quantum Field is (e)made of quantum systems that are interacting,(each Quantum system located at a different position in space). Quantum field is((e)) a continuum. But is Reality actually continuous? We don't know: but it looks easier to think to Reality or Objective Reality and in fact the Subjective experience as a continuum. Now, suppose that this is not the case, namely Reality is (e&eb)ultimately discrete, and the continuum is only a mathematical fiction. Then, what else are out there more than interacting quantum systems? Is it space? No, space is "nothingness". Is it Relativity? No, that's not a "thing": Or is it? Can it not be classified based on the tensorial entities it is made of for a particle or set of particles it is meant of? That despite the fact that the 'theory' is a 'generalization' one; it is a way of looking at things. We thus come to the conclusion that Reality is made(e) only of "interacting quantum systems", and this is (=)precisely what we call a quantum computer; the interacting quantum states. David Deutsch in his seminal paper Quantum theory, the Church-Turing principle and the universal quantum computer rephrased the Church-Turing hypothesis as a truly physical principle. In short: every piece of physical reality can be perfectly 'simulated' by a quantum computer. But now: what is the difference between Reality and its simulation? Reality is indistinguishable from its simulation implies then it is its simulation. But how would you know that a situation is simulation or the Objective Reality. If you are followed by Supari Mafia, Crime Syndicates, hoodlum mugger aggregates, raucous ribaldry congregates, loudish, jeerish elements, the veritable string ray sachrochusus imperators, herrenvolk sandhurst lucubration cormorants, that cause traffic jams, create havoc in the neighborhood with the connivance and contrivance of the neighbours, or some hundred vehicles are parked at your door; somebody is keeping vigil while' casually' sitting in the park. Then? It is here consciousness comes in. Consciousness of what exactly the truth is comes from 'knowing'. How would you 'know'? From the subjective experiences of the past. That you are the victim. So it is 'consciousness' that produces 'objective reality' Next paper exactly deals with this: 'Consciousness' and 'Objective reality'; and 'consciousness' and 'Subjective experience' 'The Universe is really a huge quantum computer: the computational universe of Seth Lloyd .But we have more than that. Quantum Theory is (=)ultimately a "theory of information", an idea that has been hanging around for many years since the Wheeler's it from bit, and which has been also recently proved mathematically. Therefore, if we adopt the Deutsch's Church-Turing principle, the notion of Information becomes(e) the new big paradigm for Physics. And Physics can be classified based on the theoretical

parameters that are involved in a Theory; notwithstanding ‘generalizationalness’ of the theory. Take for example Heisenberg’s Principle of Uncertainty. Can we not have positions and momentums that are different to different particles? That the product is always less than  $h/2$  does not mean there cannot be total quantum of ‘positions and momentums’ of the particles or for that matter there cannot be ‘total gravitational field’. The scientific approach behind the computational paradigm. The Informationalism can be ultimately regarded as a new scientific approach, very close to the spirit of Niels Bohr and the Copenhagen school. Far from being speculative, the approach is truly operational, namely everything must be defined by a precise procedure—ultimately in terms of accurate quantum measurements. The real entities are the events, facts of the world describable by the basic language obeying the rules of predicate logic (the “facts” of Wittgenstein’s Tractatus). Formulating a Theory of the observed (or potentially observable) events means building up a network of input-output connections between them. In a causal theory these connections are causal links: Quantum Theory is exactly a theory of this kind. Translating these terms into computer-programming language, the events are the subroutines, and the causal links are the registers where information is written and read. Like “Transfer Scrolls” in the Bank. Total conservation and preservation of assets and liabilities does not mean that there cannot be individual debits and credits. In fact individual debits and credits are conserved; then the summary sheets of each department is conserved; then the General Ledger which gives the double entries of ‘inflow’ and ‘outflow’ are recorded gives the total quantum of energy transformed from one to another is recorded. This ‘General Ledger’ is the “General Theory Of Every Thing” the Quantum Mechanical Operations, Relativistic operations, or nonoperationability of accounts themselves which are taken in to consideration also but in such a case there are no inflows and out flows. Translating into physical terms: the links are the systems and the events are the transformations. There absolutely is no difference except in that there might not be (?) ‘Interest’ in the energy transactions or for that matter Nature might not be making any “Profit “out of each transaction. But still all these “inflow” terms are given ‘Account Heads’ and are ‘invoked’ and ‘revoked’ based on the concomitant and corresponding “Debits and Credits”; Whenever it is eschewed then there shall be no “transactions” under this head. It is in the light of the foregoing consideration the word or notion of “event” must be regarded as truly primordial: events do not (e(eb)) happen in spacetime, they(eb) build-up space-time. Stated in other words: space-time is our(e&eb) way of organizing events. The idea of deriving the geometry of space-time from apurely “causal structure”(this is the source) has been also hanging around for more than two decades after Raphael Sorkin opened the causal sets program.

A portion of a quantum circuit (left) and its causal network representation (right). The hexagon (and the corresponding circle on the right) represents a tile which allows to recover the whole circuit upon translation. This is the equivalent of a Physical Law. Physicists often identify Theory with Reality, but Theory is only our way to connect phenomena, to relate input with an output. Input and output are linked through cascades of local interactions, namely events that involve only a finite number of systems. In the quantum computer the subroutines (the potential events) are the unitary transformations of the gates, and the causal links are the quantum systems—the so-called qubits. In Fig. 1 a piece of quantum circuit is represented. The gate(box) performing the unitary transformation A reads information from two input registers (wires) which in turn are the output of two gates performing the unitary B, and so forth. What is a Physical Law in this causal-network framework? It is a piece of network—a set of events (gates) along with causal links (wires) connecting them—by translating which we can build up the whole unbounded periodic network, corresponding to our supposition that the law is true everywhere and ever. Such representation of the physical law contains only its logical essence, stripped of the “conventional” part (e. g. the conventionality of simultaneity the informational paradigm is a huge change of ontology: there is no stuff that supports the qubits, but stuff itself is made(e) of qubits! This is a change of perspective that is hard to swallow. Those who strongly believe in the reality of space-time with “objects” inside it in. Another objection is that, once we have the computer, we still need to provide it with the software. True: but this is the same challenge of grand unification of quantum field theory, and here at least we have a simple common “programmable” background, and we may hope to find unification in new kinds of principles, related e. g. to the topological nature of the network. The principles must be simple: the software must be simple. But the computational grand-unification, being naturally a lattice theory, would also have the great bonus of avoiding all problems due to the continuum which plague quantum field theory (ultraviolet divergences, the Feynman path integral, nonlocalizability of measurements, and many more). On the other hand, the digital theory will likely miss some of the simplicity of the continuum, whence finding easy ways to interpolate digital with analog must have top priority. Recovering the whole Physics as emergent from the quantum information processing is a large program: we need to build up a complete dictionary that translates all physical notions into information-theoretic words. And we want more than that: we want to know if the digital character of Reality is experimentally detectable at some scale. Spacetime, a physical notion, can(e) emerge from the quantum computation, and how the quantum-digital nature of Reality leads to physical (eb)consequences that are in-principle detectable. Current quantum field theory is indeed a kind of “thermodynamic” limit, valid at the Fermi scale, of a deeper theory that holds at the Planck scale, where the quantum field is replaced by a giant quantum computer. We’ll see that the free-flow of quantum information is described by digital version of the Dirac equation and this also provides informational interpretations for inertial mass and Planck constant. At the same time, the notion of Hamiltonian is emerging, and, the quantum field can be eliminated in favor of pure qubits. Some of these ideas for the moment plainly work in one space dimension, and are only a starting point: later in the paper we will see routes to be explored for larger space dimensions. The free flow of information is the Dirac equation. One striking feature of the computational paradigm is that Lorentz covariance is a free bonus. As a matter of fact, Lorentz covariance must emerge(e) from the computation if this is able to simulate” Reality”. And, the Dirac equation turns out to be just the freedom of quantum informations mentioned, we will restrict to one space dimension, and incorporation of larger dimensions later. In the quantum computer information can flow in a fixed direction only at the maximum speed of one-gate-per-step. In the digital world there is no physical unit: time and space are measured by counting, and the digital-analog conversion factors

*will be given by a time  $\tau$  expressed in seconds and length*  $a$  expressed in meters, which can be interpreted as the minimal space distance and time-distance between events, respectively. We may think to as providing the Planck scale, namely 0.1 mm compared to an electron as huge as an entire galaxy! In analog units, the maximal speed is then given by  $c = a/\tau$ . Mathematically information flow in the two directions by the two field operators.

Various present and future specialized applications of magnets require monodisperse, small magnetic particles, and the discovery of molecules that can function as Nanoscale magnets was an important development in this regard. These molecules act as single-domain magnetic particles that, below their blocking temperature, **exhibit** magnetization hysteresis, a classical property of macroscopic magnets. Such 'single-molecule magnets' (SMMs)<sup>4</sup> straddle the interface between classical and quantum mechanical behaviour because they also display quantum tunnelling of magnetization<sup>5,6</sup> and quantum phase interference<sup>7</sup>. Quantum tunnelling of magnetization can be advantageous for some potential applications of SMMs, for example, in providing the quantum superposition of states required for quantum computing<sup>8</sup>. However, it is a disadvantage in other applications, such as information storage, where it would lead to information loss. Thus it is important to both understand **and control** the quantum properties of SMMs. Here we report a supramolecular SMM dimer in which antiferromagnetic coupling between the two components results in quantum behaviour different from that of the individual SMMs. Our experimental observations and theoretical analysis suggest a means of tuning the quantum tunnelling of magnetization in SMMs. This system may also prove useful for studying quantum tunnelling of relevance to mesoscopic antiferromagnets. Researchers are currently seeking robust methods of **directly manipulating** quantum states. Efforts are being made to more fully **develop** quantum cryptography, which will **theoretically allow** guaranteed secure transmission of information. A more distant goal is the development of quantum computers, which are expected to perform certain computational tasks exponentially faster than classical computers. Another active research topic is quantum teleportation, which deals with techniques to transmit quantum information over arbitrary distances

### **QUANTUM COMPUTATION AND PAULI'S EXCLUSION PRINCIPLE:**

What is quantum computing? Alan Turing thought about this in 1936 with regard (implicitly) to classical mechanics, and gave the world the paradigm classical computer: the Turing machine. In a trivial sense, everything is a quantum computer. (A pebble **is a** quantum computer **for calculating** the constant-position function - you get the idea.) And of course, today's computers exploit quantum *effects* (like electrons tunneling through barriers) to help do the right thing and do it fast. For that matter, both the computer and the pebble **exploit a** quantum effect - **the "Pauli exclusion principle"**, which **holds(e) up** ordinary matter against collapse by bringing about the kind of degeneracy we call chemistry - just to remain stable solid objects. But quantum computing is much more than that.

The most exciting really new feature of quantum computing is **quantum parallelism**. A quantum system is in general not in *one* "classical state", but in a "quantum state" consisting (crudely speaking) of a **superposition of many(e&eb) classical or classical-like** states. This superposition is not just a figure of speech, covering up our ignorance of which classical-like state it's "really" in. But actually you need the whole superposition to get the time evolution right. The system *really is* in some sense in all the classical-like states at once! If the superposition can be protected from unwanted entanglement **with its** environment (known as *decoherence*), a quantum computer can **output results dependent on details** of *all* its classical-like states. This is quantum parallelism - parallelism on a serial machine. And if that wasn't enough, machines that would already, in architectural terms, qualify as parallel can benefit from quantum parallelism too -

### **QUANTUM TUNELLING AND QUANTUM MACHINE:.**

Physicists Andrew Cleland and John Martinis from the University of California at Santa Barbara and their colleagues designed the machine -- a tiny metal paddle of semiconductor, visible to the naked eye -- and coaxed it into dancing with a quantum groove. First, they cooled the paddle until it reached its "ground state," or the lowest energy state permitted by the laws of quantum mechanics (a goal long-sought by physicists). Then they raised the widget's energy by a single quantum to produce a purely quantum-mechanical state of motion. They even managed to put the gadget in both states at once, so that it literally vibrated a little and a lot at the same time -- a bizarre phenomenon allowed by the weird rules of quantum mecha space with a complex metric permitted by quantum cosmology is given; two regions, one containing no closed causal curve and one containing closed time like curves, are separated by a complex region. Through quantum tunneling one can travel from one region to the other. The vacuum polarization stress-energy tensor converges everywhere so the space is stable. This challenges Hawking's chronology protection conjecture and highlights on building a time machine.

### **Superpositional quantum computation**

Superpositional quantum computations **exploit the(e) fact** that a coherent quantum state is a **superposition(e&eb)** of  $n$  distinct states,  $\alpha_i$ , each weighted by some complex scalar  $\alpha_i$ . Under certain conditions, this quantum state decoheres, and the particle adopts one of the  $\alpha_i$  as its determinate state, with a probability that is determined by the ratios of the  $\alpha_i$ . The idea proposed in [Feynman, 1982] and developed in, e.g., [Deutsch, 1985], is that if such superpositional states were used to implement the states of a computer, then **various registers or memory locations** in the computer would not be conventional bits with a determinate value of 1 or 0, but would instead be quantum bits - *qubits* - which are superpositions of both the 0 and 1 values.

A key advantage to this would be that  $n$  qubits **could (e)be used** to perform  $2^n$  computations in parallel, one computation for **each (e&eb)combination** of values of the superposed states. However, there are two principal difficulties **in (e)exploiting** this proposal. First, there is the problem of maintaining the coherence (=superpositionality) of a qubit while performing computations on it: the danger is that the kind of physical processes necessary to implement the relevant bit operations are

such that they would cause the quantum state to collapse or decohere. But even supposing one can perform such operations while maintaining the coherence of the qubit, there is still the difficulty of exploiting the superpositionality of the qubit in a way that can perform effective computational work.

The only specific idea of how this can be done was proposed by Shor [Shor, 1994]. Shor describes how to initialize a superpositional quantum state with a random number  $x$  and a number  $n$  to be factored into primes. He then describes how to transform(e&eb) that state into another which is such that the probability distribution of the measurements of the state is a simple function of a number  $r$  which is a reliable guide to the prime factors of  $n$ . A few collapses, then, of this system allows one to calculate  $r$  and thus factorize  $n$ . If this algorithm could be implemented in a real quantum computational system, one could then produce the prime factors of large (e.g., 159-digit) numbers in seconds. Since current cryptography technology relies on the fact that such numbers would take a single computer many years to factor, Shor's algorithm has generated much interest in quantum computation. However, it has proven difficult to generalize this exploitation of the qubit to other applications. The general problem of how to use a superpositional state to do computational work remains. Quantum tunneling is vital to the operation of many devices - even in the simple light switch, as otherwise the electrons in the electric current could not penetrate the potential barrier made up of a layer of oxide. Memory chips found in USB drives(e) use quantum tunneling to(e) erase their memory cells.

## BLACK BODY RADIATION QUANTUM COMPUTATION

**Hawking radiation** is black body radiation that is predicted to be emitted by black holes, due to quantum effects near the event horizon. It is named after the physicist Stephen Hawking, who provided a theoretical argument for its existence in 1974, and sometimes also after the physicist Jacob Bekenstein who predicted that black holes should have a finite, non-zero temperature and entropy. Hawking's work followed his visit to Moscow in 1973 where Soviet scientists Yakov Zeldovich and Alexei Starobinsky showed him that according to the quantum mechanical uncertainty principle, rotating black holes should create and emit particles. Hawking radiation reduces the mass and the energy of the black hole and is therefore also known as **black hole evaporation**. Because of this, black holes that lose more mass than they gain through other means are expected to shrink and ultimately vanish. Micro black holes (MBHs) are predicted to be larger net emitters of radiation than larger black holes and should shrink and dissipate faster.

Quantum coherence and nonlocality were long regarded as primary(eb) manifestations of the counterintuitive nature of quantum theory. They are now also coming to be recognized as a potentially(e) valuable resource for information processing and communication. In contact with its environment, a quantum system can(e) lose its ability to (eb)exhibit coherence and nonlocality. The process responsible for *this transition* to effectively classical behavior is known as decoherence. While shedding new light on the origins of "the classical" decoherence makes difficult to take advantage of the full potential offered by the quantum in communication and, especially, in computation. A challenge for physics is therefore to understand more thoroughly the reasons for decoherence and to devise means to preserve(e&eb) it. The discovery by Peter Shor that quantum computers can factor large numbers much more efficiently than their classical counterparts has brought the whole field to the limelight. In quantum computers coherence must e(e&eb)be preserved throughout the calculation. The notion of quantum operations, reversible measurement and the information theoretic notions are described extensively in literature demonstrates how quantum communication can be done without (e) requiring energy extending the analogous classical result. Generalisation of the quantum factoring algorithm. Zalka demonstrates how a quantum computer can be utilized(e) for efficiently simulating quantum mechanical systems. Cleve et al., identify common pattern of quantum algorithms. Quantum information is extremely fragile. Not only there is little energy between the states  $|0\rangle$  and  $|1\rangle$  but any superpositions are also allowed. Superpositions with the different phases have the same energy and therefore become exceedingly fragile. This fragility has been thought to imply the demise of quantum computers. Fortunately quantum error correction codes have been discovered thus giving hope that it may be possible to build quantum computers robust against imperfections. The third part of these proceedings deals with errors and quantum error correction. Paz and Zurek analyze the effect of errors on the factoring algorithm. Knill et al. introduce error correction and demonstrate an accuracy threshold theorem. A similar theorem is also proved and analyzed by Preskill. Finally, all these theoretical constructions would be like sand castles if it would not be possible to build quantum computers. Wineland gives a review of the ion trap quantum computer. Walther surveys single atom experiments in cavities and traps. And finally Gershenfeld et al. analyze a new system to realize a quantum computer: Nuclear Magnetic Resonance. While quantum mechanics primarily applies to the atomic regimes of matter and energy, some systems exhibit quantum on a large scale - super fluidity, the frictionless flow of a liquid at temperatures near absolute, is one well-known example. Quantum theory also provides accurate descriptions for many previously unexplained phenomena, such as black body radiation and the stability of the orbitals of electrons in atoms. It has also given insight into the workings of many different biological systems, including receptors and protein structures. Recent work on photosynthesis has provided evidence that quantum correlations play an essential role in this basic fundamental process of the plant kingdom. Even so, classical physics can often provide good approximations to results otherwise obtained by quantum physics, typically in circumstances with large numbers of particles or large quantum numbers.

## Examples

Topological quantum computer

A **topological quantum computer** is a theoretical quantum computer that(e) employs two-dimensional quasiparticles called anyons, whose world lines cross(e&eb) over one another to form braids in a three-dimensional spacetime (i.e., one temporal plus two spatial dimensions). These braids form(eb) the logic gates that make(eb)

up the computer. The advantage of a quantum computer based (e)on quantum braids over using (e)trapped quantum particles is that the former is much more stable. The smallest perturbations can (e)cause a quantum particle to decohere and introduce errors in the computation, but such small perturbations do (e&e)not change the topological properties of the braids. This is like the effort required to cut a string and reattach the ends to form a different braid, as opposed to a ball (representing an ordinary quantum particle in four-dimensional spacetime) simply bumping into a wall. While the elements of a topological quantum computer originate in a purely mathematical realm, experiments in 2002 by Michael H. Freedman along with Zhenghan Wang, both with Microsoft, and Michael Larsen of Indiana University indicate these elements can be (e) created in the real world using semiconductors made of gallium arsenide near absolute zero and subjected to strong magnetic fields.

Anyons are quasiparticles in a two-dimensional space. Anyons are not strictly fermions or bosons, but do share the characteristic of fermions in that they cannot occupy the same state. Thus, the world lines of two anyons cannot (e&e)cross or merge. This allows braids to be made that make up a particular circuit. In the real world, anyons form (e) from the excitations in an electron gas in a very strong magnetic field, and carry fractional units of magnetic flux in a particle-like manner. This phenomenon is called the fractional quantum Hall effect. The electron "gas" is sandwiched between two flat plates of gallium arsenide, which (e)create the two-dimensional space required for anyons, and is cooled (e) and subjected to intense transverse magnetic fields.

When anyons are (e&e) braided, the transformation of the quantum state of the system depends only on the topological class of the anyons' trajectories (which are classified according to the braid group). Therefore, the quantum information which is stored in the state of the system is (e&e) impervious to small errors in the trajectories. In 2005, Sankar Das Sarma, Michael Freedman, and Chetan Nayak proposed a quantum Hall device which would realize a topological qubit. In a key development for topological quantum computers, in 2005 Vladimir J. Goldman, Fernando E. Camino, and Wei Zhou were said to have created the first experimental evidence for using fractional quantum Hall effect to (e)create actual anyons, although others have suggested their results could be the product of phenomena not involving anyons. It should also be noted that nonabelian anyons, a species required for topological quantum computers, have yet to be experimentally confirmed.

Topological quantum computers are equivalent in computational power to other standard models of quantum computation, in particular to the quantum circuit model and to the quantum Turing machine model. That is, any of these models can efficiently simulate any of the others. Nonetheless, certain algorithms may be a more natural fit to the topological quantum computer model. For example, algorithms for evaluating the Jones polynomial were first developed in the topological model, and only later converted and extended in the standard quantum circuit model.

### Topological Quantum Computations

To live up to its name, a topological quantum computer must (e) provide the unique computation properties promised by a conventional quantum computer design, which uses trapped quantum particles. Fortunately in 2002, Michael H. Freedman along with Zhenghan Wang, both with Microsoft, and Michael Larsen of Indiana University proved that a topological quantum computer can, in principle, perform any computation that a conventional quantum computer can do.

They found that conventional quantum computer device, given a flawless (error-free) operation of its logic circuits, will give a solution with an absolute level of accuracy, whereas a topological quantum computing device with flawless operation will (e) give the solution with only a finite level of accuracy. However, any level of precision for the answer can be obtained by adding more braid twists (e&e) of logic circuits) to the topological quantum computer, in a simple linear relationship. In other words, a reasonable (+x) increase in elements (braid twists) can (e) achieve a high degree of accuracy in the answer. Actual computation [gates] is done (e) by edge states of fractional quantum Hall effect. This makes models one dimensional anyons important. In one space dimension anyons are defined algebraically.

### Error correction and control

Even though quantum braids are inherently more (e) stable than trapped quantum particles, there is still a need to control (e&e) for error inducing (e) thermal fluctuations, which (e) produce random stray pairs of anyons which (e&e) interfere with adjoining braids. Controlling these errors is simply a matter of (e&e) separating the anyons to a distance where the rate of interfering strays drops (e) to near zero. Simulating the dynamics of a topological quantum computer may be a promising method of implementing fault-tolerant quantum computation even with a standard quantum information processing scheme. Raussendorf, Harrington, and Goyal have studied one model, with promising simulation results.

### Free particle

For example, consider a free particle. In quantum mechanics, there is wave-particle duality, so the properties of the particle can be described as the properties of a wave. Therefore, its quantum state can be represented as a wave of arbitrary shape and extending over space as a wave function. The position and momentum of the particle are observables. The Uncertainty Principle states that both the position and the momentum cannot simultaneously be measured with complete precision simultaneously. However, one can measure the position (alone) of a moving free particle, creating an eigenstate of position with a wavefunction that is very large (a Dirac delta) at a particular position  $x$ , and zero everywhere else. If one performs a position measurement on such a wavefunction, the resultant  $x$  will be obtained with 100% probability (i.e., with full certainty, or complete precision). This is called an

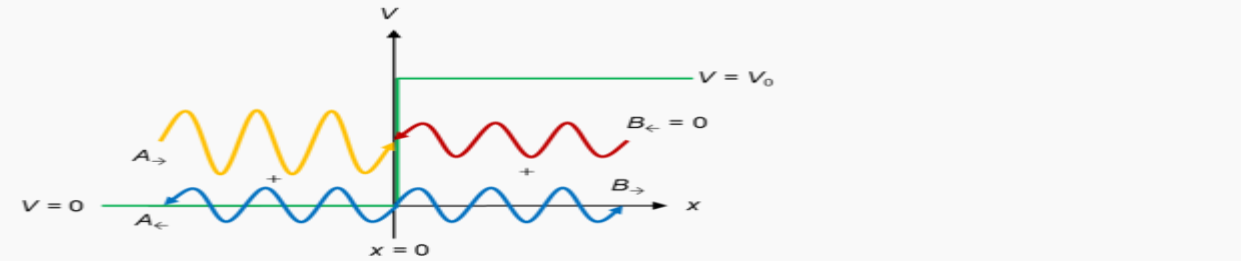


eigenstate of position - or, stated in mathematical terms, a *generalized position eigenstate (eigendistribution)*. If the particle is in an eigenstate of position, then its momentum is completely unknown. On the other hand, if the particle is in an eigenstate of momentum, then its position is completely unknown. In an eigenstate of momentum having a plane wave form, it can be shown that the wavelength is equal to  $h/p$ , where  $h$  is Planck's constant and  $p$  is the momentum of the eigenstate



3D confined electron wave functions for each eigenstate in a Quantum Dot. Here, rectangular and triangular-shaped quantum dots are shown. Energy states in rectangular dots are more ‘s-type’ and ‘p-type’. However, in a triangular dot, the wave functions are mixed due to confinement symmetry.

**Step potential and Schrodinger’s Equation:**



Scattering at a finite potential step of height  $V_0$ , shown in green. The amplitudes and direction of left- and right-moving waves are indicated. Yellow is the incident wave, blue are reflected and transmitted waves, red does not occur  $> V_0$  for this figure.

The potential in this case is given by:

$$V(x) = \begin{cases} 0, & x < 0, \\ V_0, & x \geq 0. \end{cases}$$

The solutions are superpositions of left- and right-moving waves:

$$\psi_1(x) = \frac{1}{\sqrt{k_1}} \left( A_{\rightarrow} e^{ik_1x} + A_{\leftarrow} e^{-ik_1x} \right) \quad x < 0$$

$$\psi_2(x) = \frac{1}{\sqrt{k_2}} \left( B_{\rightarrow} e^{ik_2x} + B_{\leftarrow} e^{-ik_2x} \right) \quad x > 0$$

where the wave vectors are related to the energy via

$$k_1 = \sqrt{2mE/\hbar^2}, \text{ and}$$

$$k_2 = \sqrt{2m(E - V_0)/\hbar^2}$$

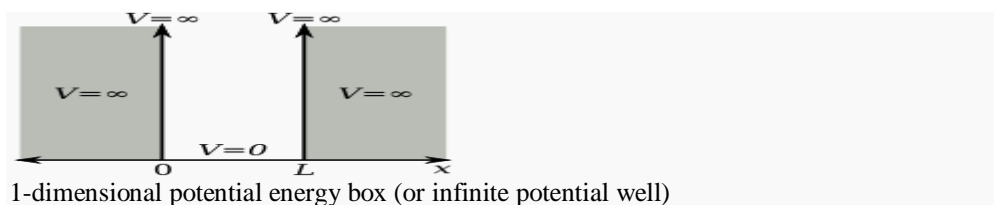
and the coefficients A and B are determined from the boundary conditions and by imposing a continuous derivative on the solution.

Each term of the solution can be interpreted as an incident, reflected, or transmitted component of the wave, allowing the calculation of transmission and reflection coefficients. In contrast to classical mechanics, incident particles with energies higher than the size of the potential step are still partially reflected.

**Rectangular potential barrier**

This is a model for the quantum tunneling effect, which has important applications to modern devices such as flash memory and the scanning tunneling microscope.

**Particle in a box**



The particle in a one-dimensional potential energy box is the most simple example where restraints lead to the quantization of energy levels. The box is defined as having zero potential energy everywhere *inside* a certain region, and infinite potential energy everywhere *outside* that region. For the one-dimensional case in the  $x$  direction, the time-independent Schrödinger equation can be written as

$$-\frac{\hbar^2}{2m} \frac{d^2\psi}{dx^2} = E\psi.$$

Writing the differential operator

$$\hat{p}_x = -i\hbar \frac{d}{dx}$$

the previous equation can be seen to be evocative of the classic kinetic energy analogue

$$\frac{1}{2m} \hat{p}_x^2 = E$$

with  $E$  as the energy for the state  $\psi$ , which in this case coincides with the kinetic energy of the particle.

The general solutions of the Schrödinger equation for the particle in a box are:

$$\psi(x) = Ae^{ikx} + Be^{-ikx} \quad E = \frac{\hbar^2 k^2}{2m}$$

or, from Euler's formula,

$$\psi(x) = C \sin kx + D \cos kx.$$

The presence of the walls of the box determines the values of  $C$ ,  $D$ , and  $k$ . At each wall ( $x = 0$  and  $x = L$ ),  $\psi = 0$ . Thus when  $x = 0$ ,

$$\psi(0) = 0 = C \sin 0 + D \cos 0 = D$$

and so  $D = 0$ . When  $x = L$ ,

$$\psi(L) = 0 = C \sin kL.$$

$C$  cannot be zero, since this would conflict with the Born interpretation. Therefore,  $\sin kL = 0$ , and so it must be that  $kL$  is an integer multiple of  $\pi$ . And additionally,

$$k = \frac{n\pi}{L} \quad n = 1, 2, 3, \dots$$

The quantization of energy levels follows from this constraint on  $k$ , since

$$E = \frac{\hbar^2 \pi^2 n^2}{2mL^2} = \frac{n^2 h^2}{8mL^2}.$$

### Finite potential well

This is the generalization of the infinite potential well problem to potential wells of finite depth.

### Harmonic oscillator

Some trajectories of a harmonic oscillator (i.e. a ball attached to a spring) in classical mechanics (A-B) and quantum mechanics (C-H). In quantum mechanics, the position of the ball is represented by a wave (called the wavefunction), with the real part shown in blue and the imaginary shown in red. Some of the trajectories (such as C, D, E, and F) are standing waves (or "stationary states"). Each standing-wave frequency is proportional to a possible energy of the oscillator. This "energy quantization" does not occur in classical physics, where the oscillator can have *any* energy.

As in the classical case, the potential for the quantum harmonic oscillator is given by:

$$V(x) = \frac{1}{2} m\omega^2 x^2$$

This problem can be solved either by solving the Schrödinger equation directly, which is not trivial, or by using the more elegant "ladder method", first proposed by Paul Dirac. The eigenstates are given by:

$$\psi_n(x) = \sqrt{\frac{1}{2^n n!}} \cdot \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} \cdot e^{-\frac{m\omega x^2}{2\hbar}} \cdot H_n\left(\sqrt{\frac{m\omega}{\hbar}} x\right), \quad n = 0, 1, 2, \dots$$

where  $H_n$  are the Hermite polynomials:

$$H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n} (e^{-x^2})$$

and the corresponding energy levels are

$$E_n = \hbar\omega \left( n + \frac{1}{2} \right)$$

This is another example which illustrates the quantization of energy for bound states.

### **Quantum coherence and Quantum Computation:**

In quantum mechanics, all objects have wave-like properties (see de Broglie waves). For instance, in Young's Double-slit experiment electrons can be used in the place of light waves. Each electron's wave-function goes through both slits, and hence has two separate split-beams that contribute to the intensity pattern on a screen. According to standard wave theory [Fresnel, Huygens] these two contributions give rise to an intensity pattern of bright bands due to constructive interference, interlaced with dark bands due to destructive interference, on a downstream screen. (Each split-beam, by itself, generates a diffraction pattern with less noticeable, more widely spaced dark and light bands.) This ability to interfere and diffract is related to coherence (classical or quantum) of the wave. The association of an electron with a wave is unique to quantum theory.

The effect of the inevitable coupling to external degrees of freedom of a quantum computer are extensively examined. It is found that for quantum calculations (in which the maintenance of coherence over a large number of states is important), not only must the coupling ***be small but the*** time taken in the quantum calculation ***must be less than*** the thermal time scale,  $\hbar/kBT$ . For longer times the condition on the strength of the coupling to the external world becomes much more stringent.

When the incident beam is represented by a quantum pure state, the split beams downstream of the two slits are represented as a superposition of the pure states representing each split beam. (This has nothing to do with two particles or Bell's inequalities) relevant to an entangled state: a 2-body state, a kind of coherence between two 1-body states.) The quantum description of imperfectly coherent paths is called a mixed state. A perfectly coherent state has a density matrix (also called the "statistical operator") that is a projection onto the pure coherent state, while a mixed state is described by a classical probability distribution for the pure states that make up the mixture.

Large-scale (macroscopic) quantum coherence leads to novel phenomena, the so-called macroscopic quantum phenomena. For instance, the laser, superconductivity, and superfluidity are examples of highly coherent quantum systems, whose effects are evident at the macroscopic scale. The macroscopic quantum coherence (Off-Diagonal Long-Range Order, ODLRO) [Penrose & Onsager (1957), C. N. Yang (1962)] for laser light, and super fluidity, is related to first-order (1-body) coherence/ODLRO, while superconductivity is related to second-order coherence/ODLRO. (For fermions, such as electrons, only even orders of coherence/ODLRO are possible.) Super fluidity in liquid He4 is related to a partial Bose-Einstein condensate. Here, the condensate portion is described by a multiply occupied single-particle state. [e.g., Cummings & Johnston (1966)]

On the other hand, the Schrödinger's cat thought experiment highlights the fact that quantum coherence cannot be arbitrarily applied to macroscopic situations. In order to have a quantum superposition of dead and alive cat, one needs to have pure states associated with aliveness and pure states associated with death, which are then superposed. Given the problem of defining death (absence of EEG, heartbeat,) it is hard to imagine a set of quantum parameters that could be used in constructing such superposition. In any case, this is not a good topic for a description of quantum coherence. [Ref.: Fresnel, Huygens, R. Glauber (1963)]

Regarding the occurrence of quantum coherence at a macroscopic level, it is interesting to note that the classical electromagnetic field exhibits macroscopic quantum coherence. The most obvious example is carrier signals for radio and TV. They satisfy Glauber's quantum description of coherence.

### **CONSERVATION LAWS AND QUANTUM COMPUTATION(INFORMATION)**

The Wigner-Araki-Yanase theorem shows that conservation laws ***limit (e)*** the accuracy of measurement. Ozawa generalized the argument to show that conservation laws ***limit (e) the*** accuracy of quantum logic operations. A rigorous lower bound was obtained of the error probability of any physical realization of the controlled-NOT gate ***under the(e) constraint*** that the computational basis is represented by a component of spin and that physical implementations ***obey e(e&eb) the*** angular momentum conservation law. The lower bound ***is shown to be*** inversely proportional to the number of ancilla qubits or the strength of the external control field. Since the discovery of Shor's algorithm, physical realization of quantum computers is one of the major topics in physics. One of the formidable ***obstacles to(e) the*** realization of quantum computers is the decoherence induced by the environment. The theory of quantum error correction and the theory of fault-tolerant quantum computing have been developed to ***overcome*** this difficulty. One of the main achievements of this field is the threshold theorem: Provided the noise in individual quantum gates ***is below a certain(e) threshold*** it is possible to efficiently perform an arbitrarily large quantum computing. However, the threshold is rather demanding and the problem turns to whether there is any fundamental limit for implementing quantum gates. Recently, Lloyd and Ng have discussed how fundamental constants ***provide limits (e)on*** speed and memory of quantum computers. MASANAO OZAWA proposed another approach based on conservation laws. If we consider the ultimate performance of computing allowed by the laws of physics, elementary quantum gates should be isolated and small, so that the corresponding unitary operators should satisfy fundamental symmetries, or conservation laws. From this point of view, it is likely that the degree of conflict with a conservation law ***depends on the*** nature of its logic to be performed and that the imperfection can be ***reduced by*** increasing the size of implementation. However, no serious investigation has ever taken place. Ozawa model takes qubits as spin 1/2 objects and investigate the quantum limit ***induced by the*** angular momentum conservation law. He shows that although the

SWAP gate has no conflict with the conservation law, the controlled-NOT gate, which is one of the universal quantum logic gates, cannot be implemented by any 2-qubit rotationally invariant unitary operation within error probability  $1/16$ . Thus, to obtain more accuracy, we need to blow up the unitary operation to an ancilla system. Then, the size of an implementation of the quantum gate is defined as the total number of qubits in the computational basis and the ancilla.

### **Conservation-law-induced quantum limits for physical (e&eb)realizations of the quantum NOT gate**

In recent investigations, it has been found that conservation laws generally lead(eb) to precision limits on quantum computing. Lower bounds of the error probability have been obtained for various logic operations from the commutation relation between the noise operator and the conserved quantity or from the recently developed universal uncertainty principle for the noise-disturbance trade-off in general measurements. However, the problem of obtaining the precision limit to realizing the quantum NOT gate has eluded a solution from these approaches. Tokishiro Karasawa\* and Masanao Ozawa† developed a method for this problem based on analyzing the trace distance between the output state from the realization under consideration and the one from the ideal gate. Using the mathematical apparatus of orthogonal polynomials, they(eb) obtained a general lower bound on the error probability for the(eb&e) realization of the quantum NOT gate in terms of the number of qubits in the control system under conservation of the total angular momentum of the computational qubit plus the control system along the direction(e) used to encode the computational basis. The lower bound turns out to be more stringent than one might expect from previous results. Their method is expected to lead to more accurate estimates for physical realizations of various types of quantum computations under conservation laws and to contribute to related problems such as the accuracy of programmable quantum processors.

### **Conservation Laws, Uncertainty Relations, and Quantum Limits of Measurements**

The uncertainty relation between the noise operator and the conserved quantity leads(eb) to a bound on the accuracy of general measurements. The bound extends the assertion by Wigner, Araki, and Yanase that conservation laws limit the (e)accuracy of “repeatable,” or “nondisturbing,” measurements to general measurements, and improves the one previously obtained by Yanase for spin measurements. The bound represents an obstacle to making a small quantum computer. It was found that conservation laws put a(e) limit on measurements of quantum mechanical observables. In 1960, Araki and Yanase proved the following assertion known as the Wigner-Araki-Yanase (WAY) theorem: Observables which do not commute with bounded additive conserved quantities have(e) no “exact” measurements. Subsequently, Yanase found a bound for the accuracy of measurements of the x component of spin in terms of the “size” of the apparatus, where the size is characterized by the mean square of the z component of the angular momentum. Yanase and Wigner concluded from this result that in order to increase the accuracy of spin measurement one ha(e)s to use a very large measuring apparatus. In the WAY theorem, for a measurement to be “exact” the following two conditions are(eb) required to be satisfies:

- (i) the Born statistical formula (BSF) and
- (ii) The repeatability hypothesis (RH), asserting that, if an observable is measured twice in succession in a system, then we obtain the same value each time.

Yanase’s bound does not assume the RH. Instead, a condition, to be referred to as Yanase’s condition, is assumed that the probe observable, the observable in the apparatus to be measured after the measuring interaction, commutes with the conserved quantity, to ensure the measurability of the probe observable. Elaborating the suggestions given by Stein and Shimony, Ohira and Pearle constructed a simple measuring interaction that satisfies the conservation law and the BSF, assuming the precise probe measurement, but does not satisfy the RH. Based on their model, Ohira and Pearle claimed that it is possible to have an accurate measurement of the spin component regardless of the size of the apparatus, if the RH is abandoned. However, their model does not satisfy Yanase’s condition, so that the problem remains as to the measurability of the probe observable. Yanase’s argument, however, assumes a large (but of variable size) measuring apparatus having the continuous angular momentum from the beginning for technical reasons and concludes that accurate measurement requires(e) a very large apparatus. To avoid a circular argument, a rigorous derivation without such an assumption is still demanded. Moreover, Wigner pointed out the necessity for generalizing the bound to general quantum systems other than +spin systems, as well as including all additive conservation laws. In order to accomplish the suggested generalization, a new approach to the problem is proposed by Ozawa based on uncertainty relation between the conserved quantity and the noise operator, defined as the difference between the post-measurement probe and the measured quantity. We obtain a bound for the mean-square error of general measuring interactions imposed by any additive conservation laws without assuming the RH. This bound also clarifies the trade-off between the size and the commutativity of the noise operator with the conserved quantity, unifying the suggestion by WAY and others and the one suggested by Ohira and Pearle. For spin measurements, this bound with Yanase’s condition leads to a tight bound for the error probability of spin measurement, which improves Yanase’s bound.

### **Quantum decoherence**

**Protection of Quantum Information Encoded in Decoherence Free States against Exchange Errors** (See Daniel A. Lidar, David Bacon, Julia Kempe and K. Birgitta Whaley) calls for Decoherence Sub spaces The exchange interaction between identical qubits in a quantum information processor gives (eb)rise to unitary two-qubit errors. Decoherence free subspaces (DFSs) for collective decoherence undergo Pauli errors under exchange, which however do not take the decoherence Free states outside of the DFS. In order to protect DFSs against these errors it is sufficient to employ a recently proposed concatenated DFS-quantum error correcting code scheme [D.A. Lidar, D. Bacon and K.B. Whaley, Phys. Rev. Lett. 82, 4556 (1999)].

**Preserving  $e$  &  $eb$  the** coherence of quantum states and **controlling  $e$  &  $eb$  their** unitary evolution is one of the fundamental goals of Quantum Information Processing. When the system Hamiltonian is invariant under particle permutations, the exchange operator  $E_{ij}$  interchanging particles  $i$  and  $j$  is a constant of the motion, and definite symmetry of a state will be conserved. Models of quantum computers based on identical bosons or fermions must of course respect this elementary requirement. It was pointed out in a recent paper that active quantum error correcting codes (QECCs) designed to **correct  $e$  &  $eb$  independent** single-qubit errors, will **fail  $e$  for** identical particles in the presence of exchange errors. The reason is that exchange acts as a two-qubit error which has the same effect as a simultaneous bit flip on two different qubits. Of course, QECCs dealing explicitly with multiple-qubit errors are also available, so that exchange errors can readily be dealt with provided one accepts longer code words than are needed to deal with single-qubit errors. For example, in Ref. [2] a nine-qubit code is presented **which can  $e$  &  $eb$  correct** all single-qubit errors and all Pauli exchange errors. This is to be compared with the five-qubit "perfect" code which protects (only) against all single-qubit errors. While the nine-qubit code is longer than the "perfect" code, it is shorter than a code **required to  $e$  protect** against all two-qubit errors. A different error model which has been considered by several authors is that in which qubits undergo collective, rather than independent errors. The underlying physics of this model has a rich history: it dates back at least to Dicke's quantum optics work on super radiance of atoms **coupled  $e$  &  $eb$  to a** radiation field, **where it  $eb$  arose** in the consideration of systems confined to a region whose linear dimensions are small compared to the shortest wavelength of the field. The model was later treated extensively by Agarwal in the context of spontaneous emission. It was only recently realized, however, that in the collective decoherence model **there  $eb$  exist** large decoherence-free subspaces (DFSs), which are "quiet" Hilbert subspaces in which no environmentally-induced errors **occur** at  $eb$  all. Such subspaces offer **a passive  $e$  protection** against decoherence. Collective decoherence is an assumption **about the  $e$  manner** in which the environment couples to the system: instead of independent errors, as assumed in the active QECC approach, one assumes that errors are strongly correlated, in the sense that all qubits can be permuted without affecting the coupling between system and bath. This is clearly a very strong assumption, and it may not hold exactly in a realistic system-bath coupling scenario. To deal with this limitation, it has been shown recently how DFSs can be stabilized in the presence of errors that perturb the exact permutation symmetry, by concatenating DFSs with QECCs. Concatenation is a general technique that is useful for achieving fault tolerant quantum computation and trades stability of quantum information for the price of longer code words. Effect of exchange errors on DFSs for collective decoherence. These errors are fundamentally different from those induced by the system-bath coupling, since they originate entirely from the internal system Hamiltonian. It is shown that by use of the very same concatenation scheme (which was designed originally to deal with system-bath induced errors), a DFS can be stabilized in the presence of exchange errors as well. DFS is invariant under such errors, and conclude that concatenation with a QECC can generally stabilize DFSs against exchange.

In quantum mechanics, **quantum decoherence** is **the  $e$  loss of** coherence or ordering of the phase angles between the components of a system in a quantum superposition. **A  $eb$  consequence** of this dephasing **leads  $eb$  to** classical or probabilistically additive behavior. Quantum decoherence gives **the appearance** of wave function collapse (the **reduction  $e$  of** the physical possibilities into a single possibility as seen by an observer) and justifies the framework and intuition of classical physics as an acceptable approximation: decoherence is the mechanism by which the classical limit emerges out of a quantum starting point and **it determines** the location of the quantum-classical boundary. Decoherence **occurs  $eb$  when** a system **interacts  $e$  &  $eb$  with its environment** in a thermodynamically irreversible way. **This  $e$  prevents** different elements in the quantum superposition of the system+environment's wavefunction from interfering with each other. Decoherence has been a subject of active research since the 1980s

Decoherence can be viewed as **the  $e$  loss of** information from a system into the environment (often modeled as a heat bath) since every system is loosely coupled with the energetic state of its surroundings. Viewed in isolation, the system's dynamics are non-unitary (although the combined system plus environment evolves in a unitary fashion). Thus the dynamics of the system alone are irreversible. As with any coupling, entanglements are **generated  $eb$  between** the system and environment, which **have  $e$  &  $eb$  the effect of** sharing quantum information with—or **transferring it** to—the surroundings.

Decoherence does **not generate** actual wave function collapse. It only provides an explanation for the **appearance** of the wavefunction collapse, as the quantum nature of the system **"leaks"  $-$  &  $+$**  into the environment. That is, components of the wavefunction **are  $e$  &  $eb$  decoupled** from a coherent system, and acquire phases from their immediate surroundings. In other words, wave function acquires phase (+) from the environment (-) A total superposition of the global or universal wavefunction still exists (and remains coherent at the global level), but its ultimate fate remains an interpretational issue. Specifically, decoherence does not attempt to explain the measurement problem. Rather, decoherence provides an explanation for the transition of the system to a mixture of states that seem to correspond to those states observers perceive. Moreover, our observation tells us that this mixture looks like a proper quantum ensemble in a measurement situation, as we observe that measurements lead to the "realization" of precisely one state in the "ensemble".

Decoherence represents a challenge for the practical realization of quantum computers, since they are expected to rely heavily on the undisturbed evolution of quantum coherences. Simply put; they require that coherent states be preserved and that decoherence is managed, in order to actually perform quantum computation.

Mechanisms

To examine how decoherence operates, an "intuitive" model is presented. The model requires some familiarity with quantum theory basics. Analogies are made between visualisable classical phase spaces and Hilbert spaces. A more rigorous

derivation in Dirac notation shows how decoherence destroys interference effects and the "quantum nature" of systems. Next, the density matrix approach is presented for perspective.

### Phase space picture

An  $N$ -particle system can be represented in non-relativistic quantum mechanics by a wavefunction,  $\psi(x_1, x_2, \dots, x_N)$ . This has analogies with the classical phase space. A classical phase space contains a real-valued function in  $6N$  dimensions (each particle contributes 3 spatial coordinates and 3 momenta). Our "quantum" phase space conversely contains a complex-valued function in a  $3N$  dimensional space. The position and momenta do not commute but can still inherit much of the mathematical structure of a Hilbert space. Aside from these differences, however, the analogy holds.

Different previously-isolated, non-interacting systems occupy different phase spaces. Alternatively we can say they occupy different, lower-dimensional subspaces in the phase space of the joint system. The *effective* dimensionality of a system's phase space is the number of *degrees of freedom* present which—in non-relativistic models—is 6 times the number of a system's *free* particles. For a macroscopic system this will be a very large dimensionality. When two systems (and the environment would be a system) start to interact, though, their associated state vectors are no longer constrained to the subspaces. Instead the combined state vector time-evolves a path through the "larger volume", whose dimensionality is the sum of the dimensions of the two subspaces. A square (2-d surface) extended by just one dimension (a line) forms a cube. The cube has a greater volume, in some sense, than its component square and line axes. The extent two vectors interfere with each other is a measure of how "close" they are to each other (formally, their overlap or Hilbert space scalar product together) in the phase space. When a system couples to an external environment, the dimensionality of, and hence "volume" available to, the joint state vector increases enormously. Each environmental degree of freedom contributes an extra dimension.

Marcos Saraceno, in his paper represents both the states and the evolution of a quantum computer in phase space using the discrete Wigner function. Properties of the phase space representation of quantum algorithms are studied by many authors: apart from analyzing important examples, such as the Fourier Transform and Grover's search, the conditions for the existence of a direct correspondence between quantum and classical evolutions in phase space. Wigner function in a given phase space point could be measured by means of a tomographic method that, itself, can be interpreted as a simple quantum algorithm. Quantum mechanics can be formulated in phase space, the natural arena of classical physics. For this we can use the Wigner function, which is a distribution enabling to represent quantum states and temporal evolution in the classical phase space scenario. Generalization of the familiar Wigner representation of quantum mechanics to the case of a system with a finite,  $N$ -dimensional, Hilbert space. Phase space representation of both the states and the evolution of a quantum computer, has been investigated by various researchers..

One can ask if there are potential advantages in(e) using a phase space representation for a quantum computer. The use of this(e) approach is quite widespread in various areas of physics (such as quantum optics, and has been fruitful, for example, in analyzing issues concerning the classical limit of quantum mechanics. In answering the above question one should have in mind that a quantum algorithm can be simply thought of as a quantum map acting in a Hilbert space of finite dimensionality (a quantum map should be simply thought of as a unitary operator that is applied successively to a system. Therefore, any algorithm is clearly amenable to a phase space representation. Whether this representation will be useful or not will depend on properties of the algorithm. Specifically, algorithms become interesting in the large  $N$  limit (i.e. when operating on many qubits). For a quantum map this is the semi classical limit where regularities may arise in connection with its classical behavior. Unraveling these regularities, when they exist, becomes an important issue which can be naturally accomplished in a phase space representation. This representation may be useful to analyze some classes of algorithms. Moreover, the phase space approach may allow one to establish contact between the vast literature on quantum maps (dealing with their construction, the study of their semi classical properties, etc) and that of quantum algorithms. This, in turn, may provide hints to develop new algorithms and ideas for novel physics simulations. As a first application of these ideas several properties of quantum algorithms in phase space were studied by Marcos; Scientists analyzed under what circumstance it is possible to establish a direct classical analogue for a quantum algorithm (exhibiting interesting examples of this kind, such as the Fourier transform and other examples which naturally arise in studies of quantum maps). Marcos et al. have shown that, quite surprisingly, Grover's search algorithm can be represented in phase space and interpreted as a simple quantum map. To define Wigner functions for discrete systems various attempts can be found in the literature. Most notably, Wootters proposes a definition that has all the desired properties only when  $N$  is a prime number. His phase space is an  $N \times N$  grid (if  $N$  is prime) and a Cartesian product of spaces corresponding to prime factors of  $N$  in the general case

The original system's wavefunction can be expanded arbitrarily as a sum of elements in a quantum superposition. Each expansion corresponds to a projection of the wave vector onto a basis. The bases can be chosen at will. Let us choose any expansion where the resulting elements interact with the environment in an element-specific way. Such elements will—with overwhelming probability—be rapidly separated from each other by their natural unitary time evolution along their own independent paths. After a very short interaction, there is almost no chance of any further interference. The process is effectively irreversible. The different elements effectively become "lost" from each other in the expanded phase space created by coupling with the environment; in phase space, this decoupling is monitored through the Wigner quasi-probability distribution. The original elements are said to have decohered. The environment has effectively selected out those expansions or decompositions of the original state vector that decohere (or lose phase coherence) with each other. This is called "environmentally-induced-super selection", or einselection. The decohered elements of the system no longer

exhibit quantum interference between each other, as in a double-slit experiment. Any elements that decohere from each other via environmental interactions are said to be quantum entangled with the environment. The converse is not true: not all entangled states are decohered from each other.

Any measuring device or apparatus acts as an environment since, at some stage along the measuring chain, it has to be large enough to be read by humans. It must possess a very large number of hidden degrees of freedom. In effect, the interactions may be considered to be quantum measurements. As a result of an interaction, the wave functions of the system and the measuring device become entangled with each other. Decoherence happens when different portions of the system's wavefunction become entangled in different ways with the measuring device. For two einselected elements of the entangled system's state to interfere, both the original system and the measuring in both elements device must significantly overlap, in the scalar product sense. If the measuring device has many degrees of freedom, it is *very* unlikely for this to happen.

As a consequence, the system behaves as a classical statistical ensemble of the different elements rather than as a single coherent quantum superposition of them. From the perspective of each ensemble member's measuring device, the system appears to have irreversibly collapsed onto a state with a precise value for the measured attributes, relative to that element.

### Dirac notation

Using the Dirac notation, let the system initially be in the state  $|\psi\rangle$  where

$$|\psi\rangle = \sum_i |i\rangle \langle i|\psi\rangle$$

where the  $|i\rangle$ s form an einselected basis (environmentally induced selected eigen basis<sup>[4]</sup>); and let the environment initially be in the state  $|\epsilon\rangle$ . The vector basis of the total combined system and environment can be formed by tensor multiplying the basis vectors of the subsystems together. Thus, before any interaction between the two subsystems, the joint state can be written as:

$$|before\rangle = \sum_i |i\rangle |\epsilon\rangle \langle i|\psi\rangle.$$

where  $|i\rangle |\epsilon\rangle$  is shorthand for the tensor product:  $|i\rangle \otimes |\epsilon\rangle$ . There are two extremes in the way the system can interact with its environment: either (1) the system loses its distinct identity and merges with the environment (e.g. photons in a cold, dark cavity get converted into molecular excitations within the cavity walls), or (2) the system is not disturbed at all, even though the environment is disturbed (e.g. the idealized non-disturbing measurement). In general an interaction is a mixture of these two extremes, which we shall examine:

### System absorbed by environment

If the environment absorbs the system, each element of the total system's basis interacts with the environment such that:

$$|i\rangle |\epsilon\rangle \text{ evolves into } |\epsilon_i\rangle$$

and so

$$|before\rangle \text{ evolves into } |after\rangle = \sum_i |\epsilon_i\rangle \langle i|\psi\rangle$$

where the unitarity of time-evolution demands that the total state basis remains orthonormal and in particular their scalar or inner products with each other vanish, since  $\langle i|j\rangle = \delta_{ij}$ :

$$\langle \epsilon_i | \epsilon_j \rangle = \delta_{ij}$$

This orthonormality of the environment states is the defining characteristic required for einselection.

### System not disturbed by environment

This is the idealized measurement or undisturbed system case in which each element of the basis interacts with the environment such that:

$$|i\rangle |\epsilon\rangle \text{ evolves into the product } |i, \epsilon_i\rangle = |i\rangle |\epsilon_i\rangle$$

i.e. the system disturbs the environment, but is itself *undisturbed* by the environment.

and so:

$$|before\rangle \text{ evolves into } |after\rangle = \sum_i |i, \epsilon_i\rangle \langle i|\psi\rangle$$

where, again, unitarity demands that:

$$\langle i, \epsilon_i | j, \epsilon_j \rangle = \langle i|j\rangle \langle \epsilon_i | \epsilon_j \rangle = \delta_{ij} \langle \epsilon_i | \epsilon_j \rangle = \delta_{ij}$$

and *additionally* decoherence requires, by virtue of the large number of hidden degrees of freedom in the environment, that

$$\langle \epsilon_i | \epsilon_j \rangle \approx \delta_{ij}$$

As before, this is the defining characteristic for decoherence to become einselection. The approximation becomes more exact as the number of environmental degrees of freedom affected increases.

Note that if the system basis  $|i\rangle$  were not an einselected basis then the last condition is trivial since the disturbed environment is not a function of  $i$  and we have the trivial disturbed environment basis  $|\epsilon_j\rangle = |\epsilon'\rangle$ . This would correspond to the system basis being degenerate with respect to the environmentally-defined-measurement-observable. For a complex environmental interaction (which would be expected for a typical macro scale interaction) a non-einselected basis would be hard to define.

### **DIRAC FORMALISM AND QUANTUM COMPUTER:**

G. Benenti et al, modeled an isolated quantum computer as a two-dimensional lattice of qubits (spin halves) with fluctuations in individual qubit energies and residual short-range inter-qubit couplings. In the limit when fluctuations and couplings are small compared to the one-qubit energy spacing, the spectrum has a band structure and the quantum computer core (central band) with the highest density of states. Above a critical inter-qubit coupling strength, quantum chaos sets in, leading to quantum Ergodicity of eigenstates in an isolated quantum computer. The onset of chaos results in the interaction induced dynamical thermalization and the occupation numbers well described by the Fermi-Dirac distribution. This thermalization **destroys the** noninteracting qubit structure and sets serious requirements for the quantum computer operability.

The key ingredient of a quantum computer is that it can simultaneously follow all of the computation paths corresponding to the distinct classical inputs and **produce** a state which **depends on the** interference of these paths. **As a result**, some computational tasks can be performed much more efficiently than on a classical computer. Shor constructed a quantum algorithm **which performs large** number factorization into prime factors exponentially faster than any known classical algorithm. It was also shown by Grover that the search of an item in an unstructured list **can be done with a** square root speedup over any classical algorithm. These results motivated a great body of experimental proposals for a construction of a realistic quantum computer. At present, quantum gates were **realized with** cold ions and the Grover algorithm **was performed** for three qubits made from nuclear spins in a molecule. For proper operation ability, it is essential for the quantum computer to remain **coherent** during the computation process. Hence, a serious obstacle to its physical realization is the quantum decoherence due to the coupling with the external world. In spite of that, in certain physical proposals, for example nuclear spins in two-dimensional semiconductor structures, the decoherence time can be many orders of magnitude **larger than the time** required for the gate operations. **As a result**, one can analyze the operation of an isolated quantum computer **decoupled** from the external world. However, even if the quantum computer is isolated from the external world and the decoherence time is infinite, a proper operation ability of the computer is not guaranteed. As a matter of fact, one has to face a many-body **problem for a system** of  $n$  interacting qubits (two level systems): any computer operation { a unitary transformation in the Hilbert space of size  $NH = 2^n$  } can **be decomposed** into two-qubit gates such as controlled-NOT and single qubit rotations. Due to the unavoidable **presence of imperfections**, the spacing between the two states of each qubit **fluctuates** in some detuning interval. Also, some residual **interaction** between qubits necessarily remains when the **two-qubit coupling is used** to operate the gates. In an isolated quantum computer was modeled as a qubit lattice with fluctuations in individual qubit energies and residual short-range inter-qubit couplings. Similarly to previous studies of interacting many-body systems **such as nuclei, clusters, complex atoms, quantum dots, and quantum spin glasses**, the **interaction leads to** quantum chaos characterized by Ergodicity of the eigenstates and level spacing statistics as in Random Matrix Theory. The transition to chaos takes place when strength is of the order of the energy spacing between directly coupled states. This border is exponentially larger than the energy level spacing in a quantum computer.

This means that a strong enough **interaction plays the role** of a heat bath, thus **leading to** dynamical Thermalization for an isolated system. In such a regime, a quantum computer eigenstate is composed by an exponentially large (with  $n$ ) number of noninteracting multiqubit states representing the quantum register states. **As a result**, exponentially many states of the computation basis **are mixed** after a chaotic time scale and the computer operability **is destroyed**. We note that the dynamical thermalization has been discussed in other many-body interacting systems. **The dependence of the** critical coupling for the onset of dynamical thermalization on the number of qubits. Illustration of the equivalence between the chaos border and the thermalization border is also shown by Benenti.

### **CONDENSED MATTER AND QUANTUM COMPUTATION A BRIEF OVERLOOK OF THE WORK DONE:**

#### **•Christoph Kloeffer and Daniel Loss: Prospects for Spin-Based Quantum Computing**

•Experimental and theoretical progress toward quantum computation with spins in quantum dots (QDs) is reviewed, with particular focus on QDs formed in GaAs heterostructures, on nanowire-based QDs, and on self-assembled QDs. We report on a remarkable evolution of the field where decoherence, one of the main(e) challenges for realizing quantum computers, no longer seems to be the stumbling block it had originally been considered. General concepts, relevant quantities, and basic requirements for spin-based quantum computing are explained; opportunities and challenges of spin-orbit interaction and nuclear spins are reviewed.

#### **•Cotunneling in the 5/2 fractional quantum Hall regime: Robert Zielke, Bernd Braunecker, and Daniel Loss.**

•Cotunneling in the 5/2 fractional quantum Hall regime allows to test the Moore-Read wave function, proposed for this regime, and to probe the nature of the fractional charge carriers. Cotunneling current for electrons that tunnel between two quantum Hall edge states via(e&eb) a quantum dot and for quasiparticles with fractional charges  $e/4$  and  $e/2$  that tunnel via



an antidot. While electron cotunneling is strongly suppressed(e), the quasiparticle tunneling shows(eb) signatures characteristic for the Moore-Read state. Cotunneling between Laughlin states, lead to the fact that electron-transport between Moore-Read states and the one between Laughlin states at filling factor 1/3 have identical voltage dependences.

**•Non-abelian Majoranas and braiding in inhomogeneous spin ladders Fabio L. Pedrocchi, Suhas Gangadharaiah (Bhopal), Stefano Chesi (Montreal), and Daniel Loss.**

An inhomogeneous open spin ladder, related to the Kitaev honeycomb model, which can be tuned between topological and non-topological phases. In extension of Lieb's theorem, it has been shown numerically that the ground state of the spin ladder is either vortex-free or vortex-full. At the phase-boundaries single Majorana states emerge which are proven to be robust against local perturbations and to obey non-abelian braiding statistics. It is also shown that a network of spin ladders provides a promising platform for topological quantum computing.

**•Majorana qubit decoherence by quasiparticle poisoning Diego Rainis and Daniel Loss.Phys. Rev. B 85, 174533 (2012);**

•Consideration is given to the problem of quasiparticle poisoning in a nanowire-based realization of a Majorana qubit, where a spin-orbit-coupled semiconducting wire is placed on top of a (bulk) superconductor. By making use of recent experimental data exhibiting evidence of a low-temperature residual nonequilibrium quasiparticle population in superconductors, it is shown by means of analytical and numerical calculations that the dephasing time due to the tunneling of quasiparticles into the nanowire may be problematically short to allow(eb) for qubit manipulation.

**•Effect of strain on(e&eb) hyperfine-induced hole-spin decoherence in quantum dots Franziska Maier and Daniel Loss.Phys. Rev. B 85, 195323 (2012);**

•Theoretical consideration is given prominence to the effect(e&eb) of strain on the spin dynamics of a single heavy hole (HH) confined to a self-assembled quantum dot and interacting with the surrounding nuclei via hyperfine interaction. Confinement and strain hybridize(e&eb) the HH states, which show(eb) an exponential decay for a narrowed nuclear spin bath. For different strain configurations within the dot, the dependence of the spin decoherence time  $T_2$  on external parameters is shifted and the nonmonotonic dependence of the peak is altered. Application of external strain yields(eb) considerable shifts in the dependence of  $T_2$  on external parameters. It is found that external strain affects (e&eb)mostly the effective hyperfine coupling strength of the conduction band (CB), indicating(eb) that the CB admixture of the hybridized HH states plays a(e&eb) crucial role in the sensitivity of  $T_2$  on strain.

**•High threshold error correction for the surface code (James R. Wootton and Daniel Loss).**

•An algorithm is presented for error correction in the surface code quantum memory. This is shown to correct depolarizing noise up to a threshold error rate of 18.5%, exceeding previous results and coming close to the upper bound of 18.9%. The time complexity of the algorithm is found to be sub-exponential, offering a significant speed-up over brute force methods and allowing efficient error correction for codes of realistic sizes.

**Kevin A. van Hoogdalem and Daniel Loss.Phys. Rev. B 85, 054413 (2012);**

Motivated by potential applications in spintronics, the study concentrates on frequency *dependent* spin transport in noninteracting one-dimensional spin chains. A system is proposed that behaves as a capacitor for the spin degree of freedom. It consists of a spin chain with two impurities a distance  $d$  apart. We find that at low energy (frequency) the impurities flow to strong coupling, thereby effectively cutting (e) the chain into three parts, with the middle island containing (e) a discrete number of spin excitations. At finite frequency spin transport through the system increases. Authors find a strong dependence of the finite frequency characteristics both on the anisotropy of the spin chain and the applied magnetic field. A method is proposed to measure the finite-frequency conductance in this system.

**Electric-Field Induced Majorana Fermions in Armchair Carbon Nanotubes Jelena Klinovaja, Suhas Gangadharaiah, and Daniel Loss.Phys. Rev. Lett. 108, 196804 (2012)**

Authors' consider theoretically an armchair carbon nanotube (CNT) in the presence of an electric field and in contact with an s-wave superconductor. It is shown that the proximity effect (e&eb) opens (eb)up superconducting gaps in the CNT of different strengths for the exterior and interior branches of the two Dirac points. For strong proximity induced (eb) superconductivity the interior gap can be of the p-wave type, while the exterior gap can be tuned (e&eb) by the electric field to be of the s-wave type. Such a setup supports a single Majorana bound state at each end of the CNT. In the case of weak proximity induced superconductivity, the gaps in both branches are of the p-wave type. However, the temperature can be chosen in such a way that the smallest gap is effectively closed. Using renormalization group techniques authors' show that the Majorana bound states exist (eb) even after taking into (e) account electron-electron interactions.

**Thin-Film Magnetization Dynamics on the Surface of a Topological Insulator Yaroslav Tserkovnyak (UCLA) and Daniel Loss.Phys. Rev. Lett. 108, 187201 (2012);**

Authors theoretically study the magnetization dynamics of a thin ferromagnetic film exchange-coupled with a surface of a strong three-dimensional topological insulator. Focus is on the role of electronic zero modes associated with domain walls (DW's) and other topological textures in the magnetic film. Thermodynamically reciprocal hydrodynamic equations of motion are derived for the DW responding to electronic spin torques, on the one hand, and fictitious electromotive forces in the electronic chiral mode fomented by the DW, on the other. An experimental realization illustrating this physics is proposed based on a ferromagnetic strip, which cuts the topological insulator surface into two gapless regions. In the presence of a ferromagnetic DW, a chiral mode transverse to the magnetic strip acts as a dissipative interconnect, which is itself a dynamic object that controls (and, inversely, responds to) the magnetization dynamics.

**Singlet-triplet splitting in double quantum dots due to spin orbit and hyperfine interactions Dimitrije Stepanenko, Mark Rudner (Harvard), Bertrand I. Halperin (Harvard), and Daniel Loss.Phys. Rev. B 85, 075416 (2012);**

Authors' analyze the low-energy spectrum of a two-electron double quantum dot under a potential bias in the presence of an external magnetic field. Focus is on the regime of spin (e)blockade, taking into account the spin-orbit interaction (e&eb) and hyperfine coupling(e&eb) of electron and nuclear spins. Starting from a model for two interacting electrons in a double dot, authors' derive an effective two-level Hamiltonian in the vicinity of an avoided crossing between singlet and triplet levels, which are(e&eb) coupled by the spin-orbit and hyperfine interactions. Also, authors' evaluate the level splitting at the anticrossing, and show that it depends on a variety of parameters including the spin-orbit coupling strength, the orientation of the external magnetic field relative to an internal spin-orbit axis, the potential detuning of the dots, and the difference between hyperfine fields in the two dots. They provide a formula for the splitting in terms of the spin-orbit length, the hyperfine fields in the two dots, and the double dot parameters such as tunnel coupling and Coulomb energy. This formula should prove useful for extracting spin-orbit parameters from transport or charge sensing experiments in such systems. Authors identify a parameter regime where the spin-orbit and hyperfine terms can become of comparable strength, and discuss how this regime might be reached.

**Incoherent dynamics in the toric code subject to disorder**Beat Roethlisberger, James R. Wootton, Robert M. Heath (Leeds), Jiannis K. Pachos (Leeds), and Daniel Loss.Phys. Rev. A 85, 022313 (2012).

Authors' numerically study the *effects(e&eb) of two* forms of quenched disorder on the anyons of the toric code. Firstly, a new class of codes based on random lattices of stabilizer operators is presented, and shown to be superior to the standard square lattice toric code for certain forms of biased noise. It is further argued that these codes are close to optimal, in that they tightly reach the upper bound of error thresholds beyond which no correctable CSS codes can exist. Additionally, they study the classical motion of anyons in toric codes with randomly distributed onsite potentials. In the presence of repulsive long-range interaction between the anyons, a surprising increase with disorder strength of the lifetime of encoded states is reported and explained by an entirely incoherent mechanism. Finally, the coherent transport of the anyons in the presence of both forms of disorder is investigated, and a significant suppression of the anyon motion is found.

**Rashba spin orbit interaction in a quantum wire superlattice**(Gunnar Thorgilsson (Reykjavik), J. Carlos Egues (Sao Carlos), Daniel Loss, and Sigurdur I. Erlingsson (Reykjavik).Phys. Rev. B 85, 045306 (2012); Phys. Rev. B 85, 039904(E) (2012);

In this work authors' study the effects of a longitudinal periodic potential on a parabolic quantum wire defined in a two-dimensional electron gas with Rashba spin-orbit interaction. For an infinite wire superlattice we find, by direct diagonalization, that the energy gaps are shifted away from the usual Bragg planes due to the Rashba spin-orbit interaction. Interestingly, results show that the location of the band gaps in energy can be controlled via the strength of the Rashba spin-orbit interaction. Authors' have also calculated the charge conductance through a periodic potential of a finite length via the non-equilibrium Green's function method combined with the Landauer formalism. We find dips in the conductance that correspond well to the energy gaps of the infinite wire superlattice. From the infinite wire energy dispersion, is derived an equation relating the location of the conductance dips as a function of the (gate controllable) Fermi energy to the Rashba spin-orbit coupling strength. It is proposed that the strength of the Rashba spin-orbit interaction can be extracted via a charge conductance measurement.

**Localized end states in density modulated quantum wires and rings**Suhas Gangadharaiah, Luka Trifunovic, and Daniel Loss.Phys. Rev. Lett. 108, 136803 (2012);

Authors' study finite quantum wires and rings in the presence of a charge-density wave gap induced by a periodic modulation of the chemical potential. We show that the Tamm-Shockley bound states emerging at the ends of the wire are stable against weak disorder and interactions, for discrete open chains and for continuum systems. The low-energy physics can be mapped onto the Jackiw-Rebbi equations *describing* massive Dirac fermions and bound end states. They treat *interactions via* the continuum model and show that they increase the charge gap and further localize the end states. The electrons placed in the two localized states on the opposite ends of the wire can interact via exchange interactions and this setup can be used as a double quantum dot hosting spin qubits. The existence of these states could be experimentally detected through the presence of an unusual  $4\pi$  Aharonov-Bohm periodicity in the spectrum and persistent current as a function of the external flux.

**Long-distance spin-spin coupling via floating gates**Koji Sato (UCLA), Daniel Loss, and Yaroslav Tserkovnyak (UCLA).

Authors' theoretically study tunneling of Cooper pairs from an s-wave superconductor into two semiconductor quantum wires with strong spin-orbit interaction under magnetic field, which approximate helical Luttinger liquids. The entanglement of electrons within a Cooper pair can be detected by the electric current cross correlations in the wires. By *controlling the* relative orientation of the wires, either lithographically or mechanically, on the substrate, the *current correlations* can be tuned, as dictated by the initial spin entanglement. This proposal of a spin-to-charge readout of quantum correlations is alternative to a recently proposed utilization of the quantum spin Hall insulator. Subtle Coriolis force, earth's differential heating, Poincare Kelvin Functional are some of the factors that get affected by the phenomenon.

**Strong Spin-Orbit Interaction and Helical Hole States in Ge/Si Nanowires**Carbon nanotubes in electric and magnetic fields.Jelena Klinovaja, Manuel J. Schmidt, Bernd Braunecker, and Daniel Loss.Phys. Rev. B 84, 085452 (2011);

Authors' derive an effective low-energy theory for metallic (armchair and nonarmchair) single-wall nanotubes in the presence of an electric field perpendicular to the nanotube axis, and in the presence of magnetic fields, taking into account spin-orbit interactions and screening effects on the basis of a microscopic tight-binding model. The *interplay* between electric field and spin-orbit interaction allows us to *tune* armchair nanotubes into a helical conductor in both Dirac valleys. Metallic nonarmchair nanotubes are gapped by the surface curvature, yet helical conduction modes can be restored in one of

the valleys by a magnetic field along the nanotube axis. Furthermore, authors' discuss electric dipole spin resonance in carbon nanotubes, and find that the Rabi frequency shows a pronounced dependence on the momentum along the nanotube.

### **Loss of interference and the transition from quantum to classical**

The utility of decoherence lies in its application to the analysis of probabilities, before and after environmental interaction, and in particular to the vanishing of quantum interference terms after decoherence has occurred. If we ask what is the probability of observing the system making a transition or quantum leap from  $\psi$  to  $\phi$  **before**  $\psi$  has interacted with its environment, then application of the Born probability rule states that the transition probability is the modulus squared of the scalar product of the two states:

$$prob_{before}(\psi \rightarrow \phi) = |\langle \psi | \phi \rangle|^2 = \left| \sum_i \psi_i^* \phi_i \right|^2 = \sum_i |\psi_i^* \phi_i|^2 + \sum_{ij; i \neq j} \psi_i^* \psi_j \phi_j^* \phi_i$$

where  $\psi_i = \langle i | \psi \rangle$ ,  $\psi_i^* = \langle \psi | i \rangle$  and  $\phi_i = \langle i | \phi \rangle$  etc.

Terms appear in the expansion of the transition probability above which involve  $i \neq j$ ; these can be thought of as representing *interference* between the different basis elements or quantum alternatives. This is a purely quantum effect and represents the non-additivity of the probabilities of quantum alternatives.

To calculate the probability of observing the system making a quantum leap from  $\psi$  to  $\phi$  **after**  $\psi$  has interacted with its environment, then application of the Born probability rule states we must sum over all the relevant possible states of the environment,  $E_i$ , before squaring the modulus:

$$prob_{after}(\psi \rightarrow \phi) = \sum_j |\langle after | \phi, \epsilon_j \rangle|^2 = \sum_j \left| \sum_i \psi_i^* \langle i, \epsilon_i | \phi, \epsilon_j \rangle \right|^2 = \sum_j \left| \sum_i \psi_i^* \langle i | \phi \rangle \langle \epsilon_i | \epsilon_j \rangle \right|^2$$

The internal summation vanishes when we apply the decoherence / einselection condition  $\langle \epsilon_i | \epsilon_j \rangle \approx \delta_{ij}$  and the formula simplifies to:

$$prob_{after}(\psi \rightarrow \phi) \approx \sum_j |\psi_j^* \langle j | \phi \rangle|^2 = \sum_i |\psi_i^* \phi_i|^2$$

If we compare this with the formula we derived before the environment introduced decoherence we can see that the effect of decoherence has been to move the summation sign  $\sum_i$  from inside of the modulus sign to outside. As a result all the cross-or quantum interference-terms:

$$\sum_{ij; i \neq j} \psi_i^* \psi_j \phi_j^* \phi_i$$

have vanished from the transition probability calculation. The decoherence has irreversibly converted quantum behaviour (additive probability amplitudes) to classical behaviour (additive probabilities). In terms of density matrices, the loss of interference effects corresponds to the diagonalization of the "environmentally traced over" density matrix.

### **Density matrix approach**

The effect of decoherence on density matrices is essentially the decay or rapid vanishing of the off-diagonal elements of the partial trace of the joint system's density matrix, i.e. the trace, with respect to *any* environmental basis, of the density matrix of the combined system *and* its environment. The decoherence irreversibly converts the "averaged" or "environmentally traced over" density matrix from a pure state to a reduced mixture; it is this that gives the *appearance* of wavefunction collapse. Again this is called "environmentally-induced-super selection", or einselection. The advantage of taking the partial trace is that this procedure is indifferent to the environmental basis chosen.

The density matrix approach has been combined with the Bohmian approach to yield a *reduced trajectory approach*, taking into account the system reduced density matrix and the influence of the environment.<sup>[7]</sup>

### **Operator-sum representation**

Consider a system S and environment (bath) B, which are closed and can be treated quantum mechanically. Let  $\mathcal{H}_S$  and  $\mathcal{H}_B$  be the system's and bath's Hilbert spaces, respectively. Then the Hamiltonian for the combined system is

$$\hat{H} = \hat{H}_S \otimes \hat{I}_B + \hat{I}_S \otimes \hat{H}_B + \hat{H}_I$$

where  $\hat{H}_S$ ,  $\hat{H}_B$  are the system and bath Hamiltonians, respectively, and  $\hat{H}_I$  is the interaction Hamiltonian between the system and bath, and  $\hat{I}_S$ ,  $\hat{I}_B$  are the identity operators on the system and bath Hilbert spaces, respectively. The time-evolution of the density operator of this closed system is unitary and, as such, is given by

$$\rho_{SB}(t) = \hat{U}(t) \rho_{SB}(0) \hat{U}^\dagger(t)$$

where the unitary operator is  $\hat{U} = e^{\frac{-i\hat{H}t}{\hbar}}$ . If the system and bath are not entangled initially, then we can write  $\rho_{SB} = \rho_S \otimes \rho_B$ . Therefore, the evolution of the system becomes

$$\rho_{SB}(t) = \hat{U}(t)[\rho_S(0) \otimes \rho_B(0)]\hat{U}^\dagger(t).$$

The system-bath interaction Hamiltonian can be written in a general form as

$$\hat{H}_I = \sum_i \hat{S}_i \otimes \hat{B}_i,$$

where  $\hat{S}_i \otimes \hat{B}_i$  is the operator acting on the combined system-bath Hilbert space, and  $\hat{S}_i, \hat{B}_i$  are the operators that act on the system and bath, respectively. This coupling of the system and bath is the cause of decoherence in the system alone. To see this, a partial trace is performed over the bath to give a description of the system alone:

$$\rho_S(t) = Tr_B[\hat{U}(t)[\rho_S(0) \otimes \rho_B(0)]\hat{U}^\dagger(t)].$$

$\rho_S(t)$  is called the *reduced density matrix* and gives information about the system only. If the bath is written in terms of its set of orthogonal basis kets, that is, if it has been initially diagonalized then Computing the partial trace with respect to this (computational) basis gives:

$$\rho_S(t) = \sum_l \hat{A}_l \rho_S(0) \hat{A}_l^\dagger$$

where  $\hat{A}_l, \hat{A}_l^\dagger$  are defined as the **Kraus operators** and are represented as

$$\hat{A}_l = \sqrt{a_j} \langle k | \hat{U} | j \rangle.$$

This is known as the **operator-sum representation (OSR)**. A condition on the Kraus operators can be obtained by using the fact that  $Tr(\rho_S(t)) = 1$ ; this then gives

$$\sum_l \hat{A}_l^\dagger \hat{A}_l = \hat{I}_S.$$

This restriction determines if decoherence will occur or not in the OSR. In particular, when there is more than one term present in the sum for  $\rho_S(t)$  then the dynamics of the system will be non-unitary and hence decoherence will take place.

### Semi group approach

A more general consideration for the existence of decoherence in a quantum system is given by the **master equation**, which determines how the density matrix of the *system alone* evolves in time. This uses the Schrödinger picture, where evolution of the *state* (represented by its density matrix) is considered. The master equation is:

$$\rho'_S(t) = \frac{-i}{\hbar} [\tilde{\mathbf{H}}_S, \rho_S(t)] + L_D [\rho_S(t)]$$

where  $\tilde{\mathbf{H}}_S = \mathbf{H}_S + \Delta$  is the system Hamiltonian,  $\mathbf{H}_S$ , along with a (possible) unitary contribution from the bath,  $\Delta$  and  $L_D$  is the **Lindblad decohering term**. The Lindblad decohering term is represented as

$$L_D [\rho_S(t)] = \frac{1}{2} \sum_{\alpha, \beta=1}^M b_{\alpha\beta} ( [\mathbf{F}_\alpha, \rho_S(t) \mathbf{F}_\beta^\dagger] + [\mathbf{F}_\alpha \rho_S(t), \mathbf{F}_\beta^\dagger] ).$$

In quantum mechanics, **quantum decoherence** is the loss of coherence or ordering of the phase angles between the components of a system in a quantum superposition. A consequence of this dephasing (eb) leads to classical or probabilistically additive behavior. Quantum decoherence gives the appearance of wave function collapse (the reduction of the physical possibilities into a single possibility as seen by an observer) and justifies the framework and intuition of classical physics as an acceptable approximation: decoherence is the mechanism by which the classical limit emerges (eb) out of a quantum starting point and it determines the location of the quantum-classical boundary. Decoherence occurs when a system interacts with its environment in a thermodynamically irreversible way. This prevents different elements in the quantum superposition of the system+environment's wavefunction from interfering with each other. Decoherence has been a subject of active research since the 1980s.

Decoherence can be viewed as the loss of information from a system into the environment (often modeled as a heat bath) since every system is loosely coupled with the energetic state of its surroundings. Viewed in isolation, the system's dynamics are non-unitary (although the combined system plus environment evolves in a unitary fashion).<sup>[3]</sup> Thus the

dynamics of the system alone are irreversible. As with any coupling, entanglements are generated between the system and environment, which have the effect of sharing quantum information with—or transferring it to—the surroundings.

Decoherence does not generate *actual* wave function collapse. It only provides an explanation for the *appearance* of the wavefunction collapse, as the quantum nature of the system "leaks" into the environment. That is, components of the wavefunction are decoupled from a coherent system, and acquire phases from their immediate surroundings. A total superposition of the global or universal wavefunction still exists (and remains coherent at the global level), but its ultimate fate remains an interpretational issue. Specifically, decoherence does not attempt to explain the measurement problem. Rather, decoherence provides an explanation for the transition of the system to a mixture of states that seem to correspond to those states observers perceive. Moreover, our observation tells us that this mixture looks like a proper quantum ensemble in a measurement situation, as we observe that measurements lead to the "realization" of precisely one state in the "ensemble".

Decoherence represents a challenge for the practical realization of quantum computers, since they are expected to rely heavily on the undisturbed evolution of quantum coherences. Simply put; they require that coherent states be preserved and that decoherence is managed, in order to actually perform quantum computation.

#### Mechanisms

To examine how decoherence operates, an "intuitive" model is presented. The model requires some familiarity with quantum theory basics. Analogies are made between visualisable classical phase spaces and Hilbert spaces. A more rigorous derivation in Dirac notation shows how decoherence destroys interference effects and the "quantum nature" of systems. Next, the density matrix approach is presented for perspective.

#### Phase space picture

An  $N$ -particle system can be represented in non-relativistic quantum mechanics by a wavefunction,  $\psi(x_1, x_2, \dots, x_N)$ . This has analogies with the classical phase space. A classical phase space contains a real-valued function in  $6N$  dimensions (each particle contributes 3 spatial coordinates and 3 momenta). Our "quantum" phase space conversely contains a complex-valued function in a  $3N$  dimensional space. The position and momenta do not commute but can still inherit much of the mathematical structure of a Hilbert space. Aside from these differences, however, the analogy holds.

Different previously-isolated, non-interacting systems occupy different phase spaces. Alternatively we can say they occupy different, lower-dimensional subspaces in the phase space of the joint system. The *effective* dimensionality of a system's phase space is the number of *degrees of freedom* present which—in non-relativistic models—is 6 times the number of a system's *free* particles. For a macroscopic system this will be a very large dimensionality. When two systems (and the environment would be a system) start to interact, though, their associated state vectors are no longer constrained to the subspaces. Instead the combined state vector time-evolves a path through the "larger volume", whose dimensionality is the sum of the dimensions of the two subspaces. A square (2-d surface) extended by just one dimension (a line) forms a cube. The cube has a greater volume, in some sense, than its component square and line axes. The extent two vectors interfere with each other is a measure of how "close" they are to each other (formally, their overlap or Hilbert space scalar product together) in the phase space. When a system couples to an external environment, the dimensionality of, and hence "volume" available to, the joint state vector increases enormously. Each environmental degree of freedom contributes an extra dimension.

The original system's wavefunction can be expanded arbitrarily as a sum of elements in a quantum superposition. Each expansion corresponds to a projection of the wave vector onto a basis. The bases can be chosen at will. Let us choose any expansion where the resulting elements interact with the environment in an element-specific way. Such elements will—with overwhelming probability—be rapidly separated from each other by their natural unitary time evolution along their own independent paths. After a very short interaction, there is almost no chance of any further interference. The process is effectively irreversible. The different elements effectively become "lost" from each other in the expanded phase space created by coupling with the environment; in phase space, this decoupling is monitored through the Wigner quasi-probability distribution. The original elements are said to have *decohered*. The environment has effectively selected out those expansions or decompositions of the original state vector that decohere (or lose phase coherence) with each other. This is called "environmentally-induced-super selection", or einselection The decohered elements of the system no longer exhibit quantum interference between each other, as in a double-slit experiment. Any elements that decohere from each other via environmental interactions are said to be quantum entangled with the environment. The converse is not true: not all entangled states are decohered from each other.

Any measuring device or apparatus acts as an environment since, at some stage along the measuring chain, it has to be large enough to be read by humans. It must possess a very large number of hidden degrees of freedom. In effect, the interactions may be considered to be quantum measurements. As a result of an interaction, the wave functions of the system and the measuring device become entangled with each other. Decoherence happens when different portions of the system's wavefunction become entangled in different ways with the measuring device. For two einselected elements of the entangled system's state to interfere, both the original system and the measuring in both elements device must significantly overlap, in the scalar product sense. If the measuring device has many degrees of freedom, it is *very* unlikely for this to happen.

As a consequence, the system behaves as a classical statistical ensemble of the different elements rather than as a single coherent quantum superposition of them. From the perspective of each ensemble member's measuring device, the system appears to have irreversibly collapsed onto a state with a precise value for the measured attributes, relative to that element.

**Dirac notation**

Using the Dirac notation, let the system initially be in the state  $|\psi\rangle$  where

$$|\psi\rangle = \sum_i |i\rangle \langle i|\psi\rangle$$

where the  $|i\rangle$ s form an einselected basis (environmentally induced selected eigen basis); and let the environment initially be in the state  $|\epsilon\rangle$ . The vector basis of the total combined system and environment can be formed by tensor multiplying the basis vectors of the subsystems together. Thus, before any interaction between the two subsystems, the joint state can be written as:

$$|before\rangle = \sum_i |i\rangle |\epsilon\rangle \langle i|\psi\rangle.$$

where  $|i\rangle |\epsilon\rangle$  is shorthand for the tensor product:  $|i\rangle \otimes |\epsilon\rangle$ . There are two extremes in the way the system can interact with its environment: either (1) the system loses its distinct identity and merges with the environment (e.g. photons in a cold, dark cavity get converted into molecular excitations within the cavity walls), or (2) the system is not disturbed at all, even though the environment is disturbed (e.g. the idealized non-disturbing measurement). In general an interaction is a mixture of these two extremes, which we shall examine:

**System absorbed by environment**

If the environment absorbs the system, each element of the total system's basis interacts with the environment such that:

$$|i\rangle |\epsilon\rangle \text{ evolves into } |\epsilon_i\rangle$$

and so

$$|before\rangle \text{ evolves into } |after\rangle = \sum_i |\epsilon_i\rangle \langle i|\psi\rangle$$

where the unitarity of time-evolution demands that the total state basis remains orthonormal and in particular their scalar or inner products with each other vanish, since  $\langle i|j\rangle = \delta_{ij}$ :

$$\langle \epsilon_i | \epsilon_j \rangle = \delta_{ij}$$

This orthonormality of the environment states is the defining characteristic required for einselection.

**System not disturbed by environment**

This is the idealized measurement or undisturbed system case in which each element of the basis interacts with the environment such that:

$$|i\rangle |\epsilon\rangle \text{ evolves into the product } |i, \epsilon_i\rangle = |i\rangle |\epsilon_i\rangle$$

i.e. the system disturbs the environment, but is itself *undisturbed* by the environment.

and so:

$$|before\rangle \text{ evolves into } |after\rangle = \sum_i |i, \epsilon_i\rangle \langle i|\psi\rangle$$

where, again, unitarity demands that:

$$\langle i, \epsilon_i | j, \epsilon_j \rangle = \langle i|j\rangle \langle \epsilon_i | \epsilon_j \rangle = \delta_{ij} \langle \epsilon_i | \epsilon_j \rangle = \delta_{ij}$$

and *additionally* decoherence requires, by virtue of the large number of hidden degrees of freedom in the environment, that

$$\langle \epsilon_i | \epsilon_j \rangle \approx \delta_{ij}$$

As before, this is the defining characteristic for decoherence to become einselection.<sup>1</sup>The approximation becomes more exact as the number of environmental degrees of freedom affected increases.

Note that if the system basis  $|i\rangle$  were not an einselected basis then the last condition is trivial since the disturbed environment is not a function of  $i$  and we have the trivial disturbed environment basis  $|\epsilon_j\rangle = |\epsilon'\rangle$ . This would correspond to the system basis being degenerate with respect to the environmentally-defined-measurement-observable. For a complex environmental interaction (which would be expected for a typical macro scale interaction) a non-einselected basis would be hard to define.

**Loss of interference and the transition from quantum to classical**

The utility of decoherence lies in its application to the analysis of probabilities, before and after environmental interaction, and in particular to the vanishing of interference terms after decoherence has occurred. If weak what is the probability of

observing the system making a transition or quantum leap from  $\psi$  to **before**  $\psi$  has interacted with its environment, then application of the Born probability rule states that the transition probability is the modulus squared of the scalar product of the two states:

$$prob_{before}(\psi \rightarrow \phi) = |\langle \psi | \phi \rangle|^2 = \left| \sum_i \psi_i^* \phi_i \right|^2 = \sum_i |\psi_i^* \phi_i|^2 + \sum_{ij; i \neq j} \psi_i^* \psi_j \phi_j^* \phi_i$$

where  $\psi_i = \langle i | \psi \rangle$ ,  $\psi_i^* = \langle \psi | i \rangle$  and  $\phi_i = \langle i | \phi \rangle$  etc.

Terms appear in the expansion of the transition probability above which involve  $i \neq j$ ; these can be thought of as representing *interference* between the different basis elements or quantum alternatives. This is a purely quantum effect and represents the non-additivity of the probabilities of quantum alternatives.

To calculate the probability of observing the system making a quantum leap from  $\psi$  to  $\phi$  **after**  $\psi$  has interacted with its environment, then application of the Born probability rule states we must sum over all the relevant possible states of the environment,  $E_i$ , before squaring the modulus:

$$prob_{after}(\psi \rightarrow \phi) = \sum_j |\langle after | \phi, \epsilon_j \rangle|^2 = \sum_j \left| \sum_i \psi_i^* \langle i, \epsilon_i | \phi, \epsilon_j \rangle \right|^2 = \sum_j \left| \sum_i \psi_i^* \langle i | \phi \rangle \langle \epsilon_i | \epsilon_j \rangle \right|^2$$

The internal summation vanishes when we apply the decoherence / einselection condition  $\langle \epsilon_i | \epsilon_j \rangle \approx \delta_{ij}$  and the formula simplifies to:

$$prob_{after}(\psi \rightarrow \phi) \approx \sum_j |\psi_j^* \langle j | \phi \rangle|^2 = \sum_i |\psi_i^* \phi_i|^2$$

If we compare this with the formula we derived before the environment introduced decoherence we can see that the effect of decoherence has been to move the summation sign  $\sum_i$  from inside of the modulus sign to outside. As a result all the cross-or quantum interference-terms:

$$\sum_{ij; i \neq j} \psi_i^* \psi_j \phi_j^* \phi_i$$

have vanished from the transition probability calculation. The decoherence has irreversibly converted quantum behaviour (additive probability amplitudes) to classical behaviour (additive probabilities)

In terms of density matrices, the loss of interference effects corresponds to the diagonalization of the "environmentally traced over" density matrix.

### Density matrix approach

The effect of decoherence on density matrices is essentially the decay or rapid vanishing of the off-diagonal elements of the partial trace of the joint system's density matrix, i.e. the trace, with respect to *any* environmental basis, of the density matrix of the combined system *and* its environment. The decoherence irreversibly converts the "averaged" or "environmentally traced over"<sup>1</sup> density matrix from a pure state to a reduced mixture; it is this that gives the *appearance* of wavefunction collapse. Again this is called "environmentally-induced-super selection", or einselection. The advantage of taking the partial trace is that this procedure is indifferent to the environmental basis chosen.

The density matrix approach has been combined with the Bohmian approach to yield a *reduced trajectory approach*, taking into account the system reduced density matrix and the influence of the environment.<sup>1</sup>

### Operator-sum representation

Consider a system S and environment (bath) B, which are closed and can be treated quantum mechanically. Let  $\mathcal{H}_S$  and  $\mathcal{H}_B$  be the systems and bath's Hilbert spaces, respectively. Then the Hamiltonian for the combined system is

$$\hat{H} = \hat{H}_S \otimes \hat{I}_B + \hat{I}_S \otimes \hat{H}_B + \hat{H}_I$$

where  $\hat{H}_S$ ,  $\hat{H}_B$  are the system and bath Hamiltonians, respectively, and  $\hat{H}_I$  is the interaction Hamiltonian between the system and bath, and  $\hat{I}_S$ ,  $\hat{I}_B$  are the identity operators on the system and bath Hilbert spaces, respectively. The time-evolution of the density operator of this closed system is unitary and, as such, is given by

$$\rho_{SB}(t) = \hat{U}(t) \rho_{SB}(0) \hat{U}^\dagger(t)$$

where the unitary operator is  $\hat{U} = e^{\frac{-i\hat{H}t}{\hbar}}$ . If the system and bath are not entangled initially, then we can write  $\rho_{SB} = \rho_S \otimes \rho_B$ . Therefore, the evolution of the system becomes

$$\rho_{SB}(t) = \hat{U}(t)[\rho_S(0) \otimes \rho_B(0)]\hat{U}^\dagger(t).$$

The system-bath interaction Hamiltonian can be written in a general form as

$$\hat{H}_I = \sum_i \hat{S}_i \otimes \hat{B}_i,$$

where  $\hat{S}_i \otimes \hat{B}_i$  is the operator acting on the combined system-bath Hilbert space, and  $\hat{S}_i, \hat{B}_i$  are the operators that act on the system and bath, respectively. This coupling of the system and bath is the cause of decoherence in the system alone. To see this, a partial trace is performed over the bath to give a description of the system alone:

$$\rho_S(t) = \text{Tr}_B[\hat{U}(t)[\rho_S(0) \otimes \rho_B(0)]\hat{U}^\dagger(t)].$$

$\rho_S(t)$  is called the *reduced density matrix* and gives information about the system only. If the bath is written in terms of its set of orthogonal basis kets, that is, if it has been

$$\rho_B(0) = \sum_j a_j |j\rangle \langle j|.$$

**Semi group approach**

A more general consideration for the existence of decoherence in a quantum system is given by the **master equation**, which determines how the density matrix of the *system alone* evolves in time. This uses the Schrödinger picture, where evolution of the *state* (represented by its density matrix) is considered. The master equation is:

$$\rho'_S(t) = \frac{-i}{\hbar} [\tilde{\mathbf{H}}_S, \rho_S(t)] + L_D [\rho_S(t)]$$

where  $\tilde{\mathbf{H}}_S = \mathbf{H}_S + \Delta$  is the system Hamiltonian,  $\mathbf{H}_S$ , along with a (possible) unitary contribution from the bath,  $\Delta$  and  $L_D$  is the **Lindblad decohering term**. They are basis operators for the M-dimensional space of bounded operators that act on the system Hilbert space  $\mathcal{H}_S$ -these are the **error generators**-and  $b_{\alpha\beta}$  represent the elements of a positive semi-definite Hermitian matrix-these matrix elements characterize the decohering processes and, as such, are called the **noise parameters**.<sup>[8]</sup> The semi group approach is particularly nice, because it distinguishes between the unitary and decohering(non-unitary) processes, which is not the case with the OSR. In particular, the non-unitary dynamics are represented by  $L_D$ , whereas the unitary dynamics of the state are represented by the usual Heisenberg commutator. Note that when  $L_D[\rho_S(t)] = 0$ , the dynamical evolution of the system is unitary. The conditions for the evolution of the system density matrix to be described by the master equation are:

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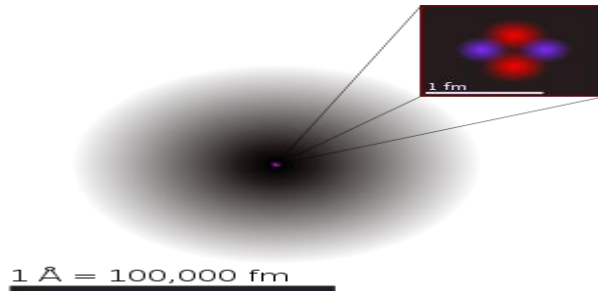
**QUANTUM MECHANICAL BEHAVIOUR  
 ATOMISM (DISAMBIGUATION) AND THEORY OF CLASSIFICATION OF QUANTUM MECHANICAL  
 BEHAVIOUR:**

Dirac equation as the basic formalism of quantum mechanics, representations of Dirac matrices, covariant realization of the Dirac equation, interpretation of negative energies, Foldy-Wouthuysen transformation, Klein's paradox, spherically symmetric interactions and a treatment of the relativistic hydrogen atom, etc., and also provides excellent additional treatments of a variety of other relevant topics. The monograph contains an extensive treatment of the Lorentz and Poincare groups and their representations. The author discusses in depth Lie algebraic and projective representations, covering groups, and Mackey's theory and Wigner's realization of induced representations. A careful classification of external fields with respect to their behavior under Poincare transformations is supplemented by a basic account of self-adjointness and spectral properties of Dirac operators. A state-of-the-art treatment of relativistic scattering theory based on a time-dependent approach originally due to Enss is presented. An excellent introduction to quantum electrodynamics in external fields is provided. Various appendices containing further details, notes on each chapter commenting on the history involved and



referring to original research papers and further developments in the literature, and a bibliography covering all relevant monographs and over 500 articles on the subject, complete this text.

## atom



An illustration of the helium atom, depicting the nucleus (pink) and the electron cloud distribution (black). The nucleus (upper right) in helium-4 is in reality spherically symmetric and closely resembles the electron cloud, although for more complicated nuclei this is not always the case. The black bar is one angstrom ( $10^{-10}$  m or 100 pm).

### Classification

Smallest recognized division of a chemical element

### Properties

Mass range: \* $1.67 \times 10^{-27}$  to  $4.52 \times 10^{-25}$  kg

Electric charge: \*zero (neutral), or ion charge

Diameter range: \*62 pm (He) to 520 pm (Cs) ([data page](#))

Components: \*Electrons and a compact nucleus of protons and neutrons

The **atom** is a basic unit of matter that consists of a dense central nucleus surrounded by a cloud of negatively charged electrons. The atomic nucleus contains a mix of positively charged protons and electrically neutral neutrons (except in the case of hydrogen-1, which is the only stable nuclide with no neutrons). The electrons of an atom are bound to the nucleus by the electromagnetic force. Likewise, a group of atoms can remain bound to each other, forming a molecule. An atom containing an equal number of protons and electrons is electrically neutral, otherwise it has a positive charge if there are fewer electrons (electron deficiency) or negative charge if there are more electrons (electron excess). A positively or negatively charged atom is known as an ion. An atom is classified according to the number of protons and neutrons in its nucleus: the number of protons determines the chemical element, and the number of neutrons determines the isotope of the element.

The name atom comes from the Greek ἄτομος (*atomos*, "indivisible") from ἀ- (*a-*, "not") and τέμνω (*temnō*, "I cut"),<sup>[2]</sup> which means uncuttable, or indivisible, something that cannot be divided further. The concept of an atom as an indivisible component of matter was first proposed by early Indian and Greek philosophers. In the 17th and 18th centuries, chemists provided a physical basis for this idea by showing that certain substances could not be further broken down by chemical methods. During the late 19th and early 20th centuries, physicists discovered subatomic components and structure inside the atom, thereby demonstrating that the 'atom' was divisible. The principles of quantum mechanics were used to successfully model the atom.

Atoms are minuscule objects with proportionately tiny masses. Atoms can only be observed individually using special instruments such as the scanning tunneling microscope. Over 99.94% of an atom's mass is concentrated in the nucleus,<sup>[note 1]</sup> with protons and neutrons having roughly equal mass. Each element has at least one isotope with an unstable nucleus that can undergo radioactive decay. This can result in a transmutation that changes the number of protons or neutrons in a nucleus.<sup>[6]</sup> Electrons that are bound to atoms possess a set of stable energy levels, or orbitals, and can undergo transitions between them by absorbing or emitting photons that match the energy differences between the levels. The electrons determine the chemical properties of an element, and strongly influence an atom's magnetic properties.

### History

#### Atomism

The concept that matter is composed of discrete units and cannot be divided into arbitrarily tiny quantities has been around for millennia, but these ideas were founded in abstract, philosophical reasoning rather than experimentation and empirical observation. The nature of atoms in philosophy varied considerably over time and between cultures and schools, and often had spiritual elements. Nevertheless, the basic idea of the atom was adopted by scientists thousands of years later because it elegantly explained new discoveries in the field of chemistry.

References to the concept of atoms date back to ancient Greece and India. In India, the Ājīvika, Jain, and Cārvāka schools of atomism may date back to the 6th century BCE. The Nyaya and Vaisheshika schools later developed theories on how atoms combined into more complex objects. In the West, the references to atoms emerged in the 5th century BCE with Leucippus, whose student, Democritus, systematized his views. In approximately 450 BCE, Democritus coined the term *atomos* (Greek: ἄτομος), which means "uncuttable" or "the smallest indivisible particle of matter". Although

the Indian and Greek concepts of the atom were based purely on philosophy, modern science has retained the name coined by Democritus

Corpuscularianism is the postulate, expounded in the 13th-century by the alchemist Pseudo-Geber (Geber), sometimes identified with Paul of Taranto, that all physical bodies possess an inner and outer layer of minute particles or corpuscles. Corpuscularianism is similar to the theory of atomism, except that where atoms were supposed to be indivisible, corpuscles could in principle be divided. In this manner, for example, it was theorized that mercury could penetrate into metals and modify their inner structure. Corpuscularianism stayed a dominant theory over the next several hundred years.

In 1661, natural philosopher Robert Boyle published *The Sceptical Chymist* in which he argued that matter was composed of various combinations of different "corpuscles" or atoms, rather than the classical elements of air, earth, fire and water.<sup>[13]</sup> During the 1670s Corpuscularianism was used by Isaac Newton in his development of the corpuscular theory of light. Further progress in the understanding of atoms did not occur until the science of chemistry began to develop. In 1789, French nobleman and scientific researcher Antoine Lavoisier discovered the law of conservation of mass and defined an element as a basic substance that could not be further broken down by the methods of chemistry.<sup>1</sup>

In 1805, English instructor and natural philosopher John Dalton used the concept of atoms to explain why elements always react in ratios of small whole numbers (the law of multiple proportions) and why certain gases dissolved better in water than others. He proposed that each element consists of atoms of a single, unique type, and that these atoms can join together to form chemical compounds.<sup>[16][17]</sup> Dalton is considered the originator of modern atomic theory

Dalton's atomic hypothesis did not specify the size of atoms. Common sense indicated they must be very small, but nobody knew how small. Therefore it was a major landmark when in 1865 Johann Josef Loschmidt's measured the size of the molecules that make up air.

An additional line of reasoning in support of particle theory (and by extension atomic theory) began in 1827 when botanist Robert Brown used a microscope to look at dust grains floating in water and discovered that they moved about erratically—a phenomenon that became known as "Brownian motion". J. Desaulx suggested in 1877 that the phenomenon was caused by the thermal motion of water molecules, and in 1905 Albert Einstein produced the first mathematical analysis of the motion. French physicist Jean Perrin used Einstein's work to experimentally determine the mass and dimensions of atoms, thereby conclusively verifying Dalton's atomic theory

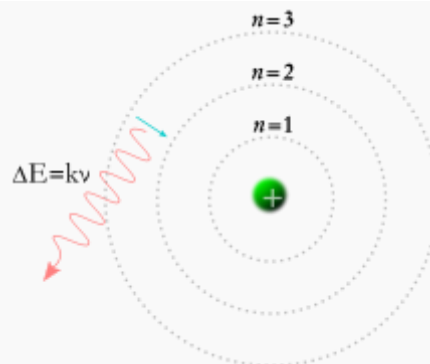
In 1869, building upon earlier discoveries by such scientists as Lavoisier, Dmitri Mendeleev published the first functional periodic table.<sup>[23]</sup> The table itself is a visual representation of the periodic law, which states that certain chemical properties of elements repeat periodically when arranged by atomic number.

### Subcomponents and quantum theory

The physicist J. J. Thomson, through his work on cathode rays in 1897, discovered the electron, and concluded that they were a component of every atom. Thus he overturned the belief that atoms are the indivisible, ultimate particles of matter. Thomson postulated that the low mass, negatively charged electrons were distributed throughout the atom, possibly rotating in rings, with their charge balanced by the presence of a uniform sea of positive charge. This later became known as the plum pudding model.

In 1909, Hans Geiger and Ernest Marsden, under the direction of physicist Ernest Rutherford, bombarded a sheet of gold foil with alpha rays—by then known to be positively charged helium atoms—and discovered that a small percentage of these particles were deflected through much larger angles than was predicted using Thomson's proposal. Rutherford interpreted the gold foil experiment as suggesting that the positive charge of a heavy gold atom and most of its mass was concentrated in a nucleus at the center of the atom—the Rutherford model

While experimenting with the products of radioactive decay, in 1913 radio chemist Frederick Soddy discovered that there appeared to be more than one type of atom at each position on the periodic table. The term isotope was coined by Margaret Todd as a suitable name for different atoms that belong to the same element. J.J. Thomson created a technique for separating atom types through his work on ionized gases, which subsequently led to the discovery of stable isotopes.



A Bohr model of the hydrogen atom, showing an electron jumping between fixed orbits and emitting a photon of energy with a specific frequency

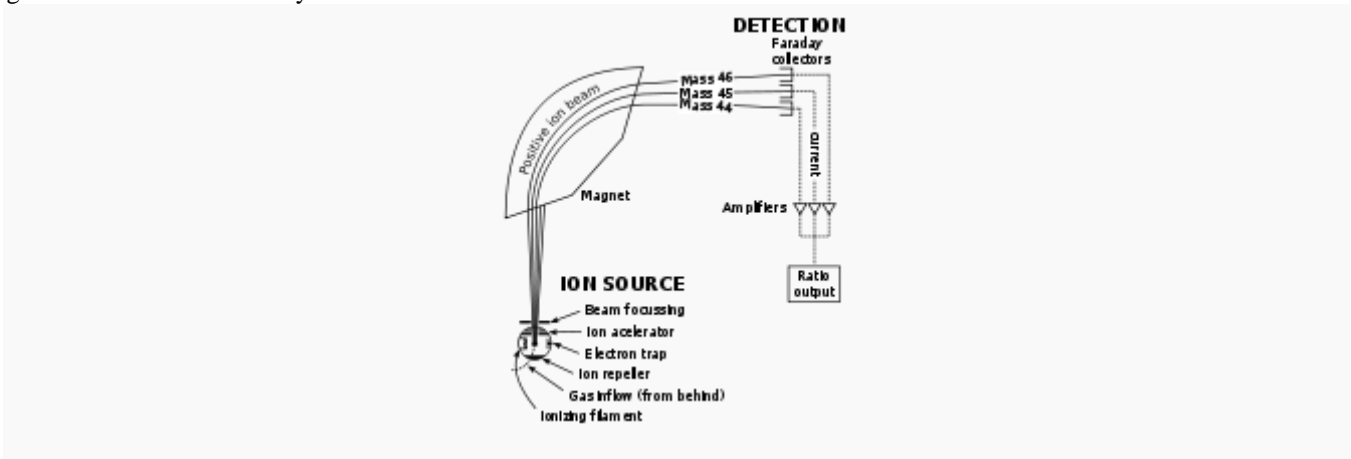
Meanwhile, in 1913, physicist Niels Bohr suggested that the electrons were confined into clearly defined, quantized orbits, and could jump between these, but could not freely spiral inward or outward in intermediate states. An electron must absorb or emit specific amounts of energy to transition between these fixed orbits. When the light from a heated material was passed through a prism, it produced a multi-colored spectrum. The appearance of fixed lines in this spectrum was successfully explained by these orbital transitions.

Later in the same year Henry Moseley provided additional experimental evidence in favor of Niels Bohr's theory. These results refined Ernest's and Antonius Van den Broek's model, which proposed that the atom contains in its nucleus a number of positive nuclear that is equal to its (atomic) number in the periodic table. Until these experiments, atomic number was not known to be a physical and experimental quantity. That it is equal to the atomic nuclear charge remains the accepted atomic model today

Chemical bonds between atoms were now explained, by Gilbert Newton Lewis in 1916, as the interactions between their constituent electrons.<sup>21</sup> As the chemical properties of the elements were known to largely repeat themselves according to the periodic law, in 1919 the American chemist Irving Langmuir suggested that this could be explained if the electrons in an atom were connected or clustered in some manner. Groups of electrons were thought to occupy a set of electron shells about the nucleus

The Stern–Gerlach experiment of 1922 provided further evidence of the quantum nature of the atom. When a beam of silver atoms was passed through a specially shaped magnetic field, the beam was split based on the direction of an atom's angular momentum, or spin. As this direction is random, the beam could be expected to spread into a line. Instead, the beam was split into two parts, depending on whether the atomic spin was oriented up or down.

In 1924, Louis de Broglie proposed that all particles behave to an extent like waves. In 1926, Erwin Schrödinger used this idea to develop a mathematical model of the atom that described the electrons as three-dimensional waveforms rather than point particles. A consequence of using waveforms to describe particles is that it is mathematically impossible to obtain precise values for both the position and momentum of a particle at the same time; this became known as the uncertainty principle, formulated by Werner Heisenberg in 1926. In this concept, for a given accuracy in measuring a position one could only obtain a range of probable values for momentum, and vice versa. This model was able to explain observations of atomic behavior that previous models could not, such as certain structural and spectral patterns of atoms larger than hydrogen. Thus, the planetary model of the atom was discarded in favor of one that described atomic orbital zones around the nucleus where a given electron is most likely to be observed



The development of the mass spectrometer allowed the exact mass of atoms to be measured. The device uses a magnet to bend the trajectory of a beam of ions, and the amount of deflection is determined by the ratio of an atom's mass to its charge. The chemist Francis William Aston used this instrument to show that isotopes had different masses. The atomic mass of these isotopes varied by integer amounts, called the whole number rule. The explanation for these different isotopes awaited the discovery of the neutron, a neutral-charged particle with a mass similar to the proton, by the physicist James Chadwick in 1932. Isotopes were then explained as elements with the same number of protons, but different numbers of neutrons within the nucleus.

### **Fission, high-energy physics and condensed matter**

#### **NUCLEAR FISSION AND QUANTUM CHIPS:**

The following dissemination of the information on the News Paper report(See references for further details):

Current technology uses ultraviolet light to (e)create the fine features in computer chips in a process called photolithography, which involves projecting the image of a mask onto a light-sensitive material, then chemically etching the resulting pattern.

New nanolithography will be needed to continue advances in computer technology and to extend Moore's law, an unofficial rule stating that the number of transistors on integrated circuits, or chips, doubles about every 18 months.

"We can't make devices much smaller using conventional lithography, so we have to find ways of creating beams having more narrow wavelengths," said Ahmed Hassanein, the Paul L. Wattelet Professor of Nuclear Engineering and head of Purdue's School of Nuclear Engineering.

The new plasma-based lithography under development generates "extreme ultraviolet" light having a wavelength of 13.5 nanometers, less than one-tenth the size of current lithography, Hassanein said.

Nuclear engineers and scientists at Purdue and the U.S. Department of Energy's Argonne National Laboratory are working to improve the efficiency of two techniques for producing the plasma: One approach uses a laser and the other "discharge-produced" method uses an electric current.

"In either case, only about 1 to 2 percent of the energy spent is converted into plasma," Hassanein said. "That conversion efficiency means you'd need greater than 100 kilowatts of power for this lithography, which poses all sorts of engineering problems. We are involved in optimizing conversion efficiency - reducing the energy requirements - and solving various design problems for the next-generation lithography."

Findings are detailed in a research paper scheduled to appear in the October-December 2009 issue of the *Journal of Micro/Nanolithography, MEMS, and MOEMS*. The paper was written by Hassanein, senior research scientist Valeryi Sizyuk, computer analyst Tatyana Sizyuk, and research assistant professor Sivanandan Harilal, all in the School of Nuclear Engineering.

Critical to the research is a computer simulation, called HEIGHTS -- for high-energy interaction with general heterogeneous target systems -- developed by Hassanein's team. Computations for a single HEIGHTS simulation using Argonne supercomputers can take several months to finish, said Hassanein, a former Argonne senior scientist who led work there to develop HEIGHTS.

The laser method creates plasma by heating xenon, tin or lithium. The plasma produces high-energy packets of light, photons, of extreme ultraviolet light.

Plasma is a partially ionized gas like material that conducts electricity. Because of this electrical conductivity, researchers are able to use magnetic fields to shape and control plasmas, forming beams, filaments and other structures. In experimental fusion reactors, magnetic fields are used to keep plasma-based nuclear fuel from touching the metal walls of the containment vessel, enabling the plasma to be heated to the extreme temperatures required to maintain fusion reactions.

HEIGHTS simulate the entire process of the plasma evolution: the laser interacting with the target, and the target evaporating, ionizing and turning into plasma. The simulation also shows what happens when the magnetic forces "pinch" the plasma cloud into a smaller diameter spot needed to generate the photons.

Findings in the paper detail the laser-produced plasma beams, showing that simulations match data from laboratory experiments recently built at Purdue, Hassanein said. One design challenge stems from the fact that lenses absorb the photons that make up light, meaning they cannot be used to focus the beam. Instead, mirrors are used in the design. However, plasma condenses on the mirrors, reducing their reflectivity and limiting the efficiency of the process.

"We are trying to help find innovative ways of producing these photons, optimizing the production and mitigating the effects of the plasma on the mirrors," Hassanein said. "So we are trying to improve the entire system."

The simulation tool combines computations in plasma physics, radiation transport, atomic physics, plasma-material interactions and magneto hydrodynamics, or what happens when a target is heated, melts and turns into a plasma.

In 1938, the German chemist Otto Hahn, a student of Rutherford, directed neutrons onto uranium atoms expecting to get transuranium elements. Instead, his chemical experiments showed barium as a product. A year later, Lise Meitner and her nephew Frisch verified that Hahn's result were the first experimental *nuclear fission*.<sup>1</sup> In 1944, Hahn received the Nobel prize in chemistry. Despite Hahn's efforts, the contributions of Meitner and Frisch were not recognized

In the 1950s, the development of improved particle accelerators and particle detectors allowed scientists to study the impacts of atoms moving at high energies.<sup>[44]</sup> Neutrons and protons were found to be hadrons, or composites of smaller particles called quarks. Standard models of nuclear physics were developed that successfully explained the properties of the nucleus in terms of these sub-atomic particles and the forces that govern their interactions

Components

## Subatomic particles

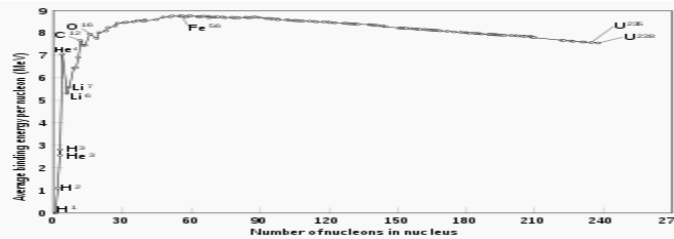
Though the word *atom* originally denoted a particle that cannot be cut into smaller particles, in modern scientific usage the atom is composed of various subatomic particles. The constituent particles of an atom are the electron, the proton and the neutron. However, the hydrogen-1 atom has no neutrons and a positive hydrogen ion has no electrons.

The electron is by far the least massive of these particles at  $9.11 \times 10^{-31}$  kg, with a negative electrical charge and a size that is too small to be measured using available techniques.<sup>1</sup> Protons have a positive charge and a mass 1,836 times that of the electron, at  $1.6726 \times 10^{-27}$  kg, although this can be reduced by changes to the energy binding the proton into an atom. Neutrons have no electrical charge and have a free mass of 1,839 times the mass of electrons, or  $1.6929 \times 10^{-27}$  kg. Neutrons and protons have comparable dimensions—on the order of  $2.5 \times 10^{-15}$  m—although the 'surface' of these particles is not sharply defined.

In the Standard Model of physics, both protons and neutrons are composed of elementary particles called quarks. The quark belongs to the fermion group of particles, and is one of the two basic constituents of matter—the other being the lepton, of

which the electron is an example. There are six types of quarks, each having a fractional electric charge of either  $+\frac{2}{3}$  or  $-\frac{1}{3}$ . Protons are composed of two up quarks and one down quark, while a neutron consists of one up quark and two down quarks. This distinction accounts for the difference in mass and charge between the two particles. The quarks are held together by the strong nuclear force, which is mediated by gluons. The gluon is a member of the family of gauge bosons, which are elementary particles that mediate physical forces.

## Nucleus



The binding energy needed for a nucleon to escape the nucleus, for various isotopes

All the bound protons and neutrons in an atom make up a tiny atomic nucleus, and are collectively called nucleons. The radius of a nucleus is approximately equal to  $1.07 \sqrt[3]{A} \text{ fm}$ , where  $A$  is the total number of nucleons<sup>1</sup>. This is much smaller than the radius of the atom, which is on the order of  $10^5 \text{ fm}$ . The nucleons are bound together by a short-ranged attractive potential called the residual strong force. At distances smaller than 2.5 fm this force is much more powerful than the electrostatic force that causes positively charged protons to repel each other.<sup>[52]</sup>

Atoms of the same element have the same number of protons, called the atomic number. Within a single element, the number of neutrons may vary, determining the isotope of that element. The total number of protons and neutrons determine the nuclide. The number of neutrons relative to the protons determines the stability of the nucleus, with certain isotopes undergoing radioactive decay.

The neutron and the proton are different types of fermions. The Pauli exclusion principle is a mechanical effect that prohibits identical fermions, such as multiple protons, from occupying the same quantum physical state at the same time. Thus every proton in the nucleus must occupy a different state, with its own energy level, and the same rule applies to all of the neutrons. This prohibition does not apply to a proton and neutron occupying the same quantum state

For atoms with low atomic numbers, a nucleus that has a different number of protons than neutrons can potentially drop to a lower energy state through a radioactive decay that causes the number of protons and neutrons to more closely match. As a result, atoms with roughly matching numbers of protons and neutrons are more stable against decay. However, with increasing atomic number, the mutual repulsion of the protons requires an increasing proportion of neutrons to maintain the stability of the nucleus, which modifies this trend. Thus, there are no stable nuclei with equal proton and neutron numbers above atomic number  $Z = 20$  (calcium); and as  $Z$  increases toward the heaviest nuclei, the ratio of neutrons per proton required for stability increases to about 1.5.<sup>[54]</sup>

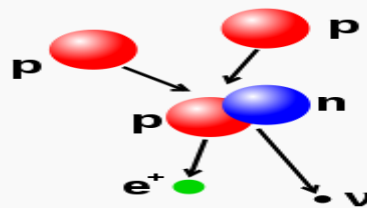


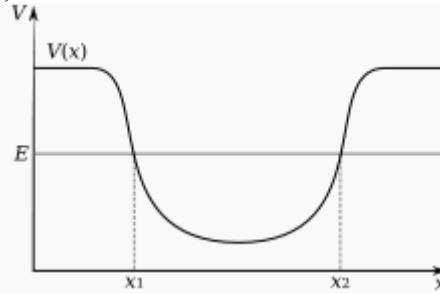
Illustration of a nuclear fusion process that forms a deuterium nucleus, consisting of a proton and a neutron, from two protons. A positron ( $e^+$ )—an antimatter electron—is emitted along with an electron neutrino.

The number of protons and neutrons in the atomic nucleus can be modified, although this can require very high energies because of the strong force. Nuclear fusion occurs when multiple atomic particles join to form a heavier nucleus, such as through the energetic collision of two nuclei. For example, at the core of the Sun protons require energies of 3–10 keV to overcome their mutual repulsion—the coulomb barrier—and fuse together into a single nucleus. Nuclear fission is the opposite process, causing a nucleus to split into two smaller nuclei—usually through radioactive decay. The nucleus can also be modified through bombardment by high energy subatomic particles or photons. If this modifies the number of protons in a nucleus, the atom changes to a different chemical element.<sup>1</sup> If the mass of the nucleus following a fusion reaction is less than the sum of the masses of the separate particles, then the difference between these two values can be emitted as a type of usable energy (such as a gamma ray, or the kinetic energy of a beta particle), as described by Albert Einstein's mass-energy equivalence formula,  $E = mc^2$ , where  $m$  is the mass loss and  $c$  is the speed of light. This deficit is part of the binding energy of the new nucleus, and it is the non-recoverable loss of the energy that causes the fused particles to remain together in a state that requires this energy to separate.<sup>[58]</sup>

The fusion of two nuclei that create larger nuclei with lower atomic numbers than iron and nickel—a total nucleon number of about 60—is usually an exothermic process that releases more energy than is required to bring them together.<sup>[59]</sup> It is this

energy-releasing process that makes nuclear fusion in stars a self-sustaining reaction. For heavier nuclei, the binding energy per nucleon in the nucleus begins to decrease. That means fusion processes producing nuclei that have atomic numbers higher than about 26, and atomic masses higher than about 60, is an endothermic process. These more massive nuclei can not undergo an energy-producing fusion reaction that can sustain the hydrostatic equilibrium of a star.<sup>[54]</sup>

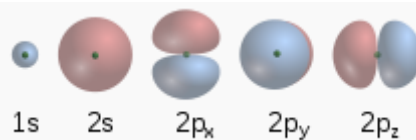
### Electron cloud, Electron Configuration, Atomic Orbital:



A potential well, showing, according to classical mechanics, the minimum energy  $V(x)$  needed to reach each position  $x$ . Classically, a particle with energy  $E$  is constrained to a range of positions between  $x_1$  and  $x_2$ .

The electrons in an atom are attracted to the protons in the nucleus by the electromagnetic force. This force binds the electrons inside an electrostatic potential well surrounding the smaller nucleus, which means that an external source of energy is needed for the electron to escape. The closer an electron is to the nucleus, the greater the attractive force. Hence electrons bound near the center of the potential well require more energy to escape than those at greater separations.

Electrons, like other particles, have properties of both a particle and a wave. The electron cloud is a region inside the potential well where each electron forms a type of three-dimensional standing wave—a wave form that does not move relative to the nucleus. This behavior is defined by an atomic orbital, a mathematical function that characterises the probability that an electron appears to be at a particular location when its position is measured.<sup>[60]</sup> Only a discrete (or quantized) set of these orbitals exist around the nucleus, as other possible wave patterns rapidly decay into a more stable form.<sup>[61]</sup> Orbitals can have one or more ring or node structures, and they differ from each other in size, shape and orientation.<sup>1</sup>



Wave functions of the first five atomic orbitals. The three 2p orbitals each display a single angular node that has an orientation and a minimum at the center.

Each atomic orbital corresponds to a particular energy level of the electron. The electron can change its state to a higher energy level by absorbing a photon with sufficient energy to boost it into the new quantum state. Likewise, through spontaneous emission, an electron in a higher energy state can drop to a lower energy state while radiating the excess energy as a photon. These characteristic energy values, defined by the differences in the energies of the quantum states, are responsible for atomic spectral lines.<sup>[61]</sup>

The amount of energy needed to remove or add an electron—the electron binding energy—is far less than the binding energy of nucleons. For example, it requires only 13.6 eV to strip a ground-state electron from a hydrogen atom,<sup>[63]</sup> compared to 2.23 million eV for splitting a deuterium nucleus.<sup>[64]</sup> Atoms are electrically neutral if they have an equal number of protons and electrons. Atoms that have either a deficit or a surplus of electrons are called ions. Electrons that are farthest from the nucleus may be transferred to other nearby atoms or shared between atoms. By this mechanism, atoms are able to bond into molecules and other types of chemical compounds like ionic and covalent network crystals.<sup>[65]</sup>

Properties

### Nuclear properties-Stable Isotopes, List of Nuclides and List of Elements by Isotopes:

By definition, any two atoms with an identical number of protons in their nuclei belong to the same chemical element. Atoms with equal numbers of protons but a different number of neutrons are different isotopes of the same element. For example, all hydrogen atoms admit exactly one proton, but isotopes exist with no neutrons (hydrogen-1, by far the most common form,<sup>[66]</sup> also called protium), one neutron (deuterium), two neutrons (tritium) and more than two neutrons. The known elements form a set of atomic numbers, from the single proton element hydrogen up to the 118-proton element ununoctium.<sup>[67]</sup> All known isotopes of elements with atomic numbers greater than 82 are radioactive.<sup>[68][69]</sup>

About 339 nuclides occur naturally on Earth,<sup>[70]</sup> of which 255 (about 75%) have not been observed to decay, and are referred to as "stable isotopes". However, only 90 of these nuclides are stable to all decay, even in theory. Another 165 (bringing the total to 255) have not been observed to decay, even though in theory it is energetically possible. These are also formally classified as "stable". An additional 33 radioactive nuclides have half-lives longer than 80 million years, and are long-lived

enough to be present from the birth of the solar system. This collection of 288 nuclides are known as primordial nuclides. Finally, an additional 51 short-lived nuclides are known to occur naturally, as daughter products of primordial nuclide decay (such as radium from uranium), or else as products of natural energetic processes on Earth, such as cosmic ray bombardment (for example, carbon-14).<sup>[71][note 2]</sup>

For 80 of the chemical elements, at least one stable isotope exists. As a rule, there is only a handful of stable isotopes for each of these elements, the average being 3.2 stable isotopes per element. Twenty-six elements have only a single stable isotope, while the largest number of stable isotopes observed for any element is ten, for the element tin. Elements 43, 61, and all elements numbered 83 or higher have no stable isotopes.<sup>[72][page needed]</sup>

Stability of isotopes is affected by the ratio of protons to neutrons, and also by the presence of certain "magic numbers" of neutrons or protons that represent closed and filled quantum shells. These quantum shells correspond to a set of energy levels within the shell model of the nucleus; filled shells, such as the filled shell of 50 protons for tin, confers unusual stability on the nuclide. Of the 255 known stable nuclides, only four have both an odd number of protons *and* odd number of neutrons: hydrogen-2 (deuterium), lithium-6, boron-10 and nitrogen-14. Also, only four naturally occurring, radioactive odd-odd nuclides have a half-life over a billion years: potassium-40, vanadium-50, lanthanum-138 and tantalum-180m. Most odd-odd nuclei are highly unstable with respect to beta decay, because the decay products are even-even, and are therefore more strongly bound, due to nuclear pairing effects.<sup>[72][page needed]</sup>

### Mass Number Of an Atom:

The large majority of an atom's mass comes from the protons and neutrons that make it up. The total number of these particles (called "nucleons") in a given atom is called the mass number. The mass number is a simple whole number, and has units of "nucleons." An example of use of a mass number is "carbon-12," which has 12 nucleons (six protons and six neutrons).

The actual mass of an atom at rest is often expressed using the unified atomic mass unit (u), which is also called a dalton (Da). This unit is defined as a twelfth of the mass of a free neutral atom of carbon-12, which is approximately  $1.66 \times 10^{-27}$  kg.<sup>[73]</sup> Hydrogen-1, the lightest isotope of hydrogen and the atom with the lowest mass, has an atomic weight of 1.007825 u.<sup>[74]</sup> The value of this number is called the atomic mass. A given atom has an atomic mass approximately equal (within 1%) to its mass number times the mass of the atomic mass unit. However, this number will not be an exact whole number except in the case of carbon-12 (see below)<sup>[75]</sup> The heaviest stable atom is lead-208,<sup>[68]</sup> with a mass of 207.9766521 u.<sup>[76]</sup>

As even the most massive atoms are far too light to work with directly, chemists instead use the unit of moles. One mole of atoms of any element always has the same number of atoms (about  $6.022 \times 10^{23}$ ). This number was chosen so that if an element has an atomic mass of 1 u, a mole of atoms of that element has a mass close to one gram. Because of the definition of the unified atomic mass unit, each carbon-12 atom has an atomic mass of exactly 12 u, and so a mole of carbon-12 atoms weighs exactly 0.012 kg.<sup>[73][page needed]</sup>

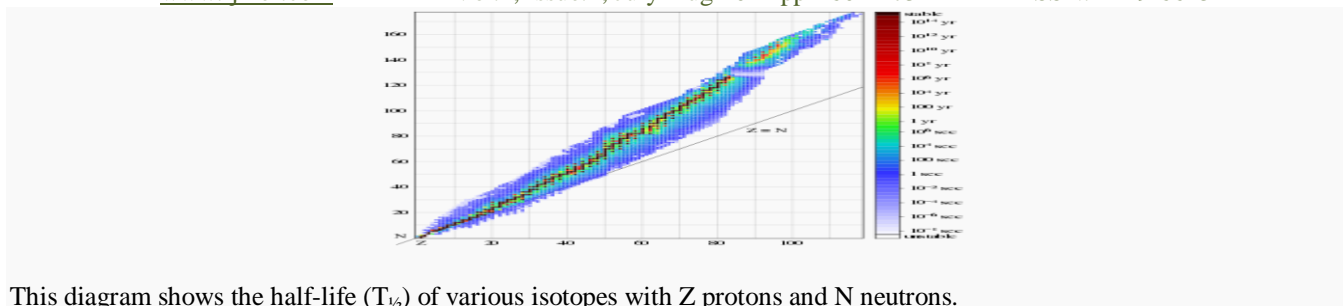
### Shape and size-The Atomic Radius:

Atoms lack a well-defined outer boundary, so their dimensions are usually described in terms of an atomic radius. This is a measure of the distance out to which the electron cloud extends from the nucleus. However, this assumes the atom to exhibit a spherical shape, which is only obeyed for atoms in vacuum or free space. Atomic radii may be derived from the distances between two nuclei when the two atoms are joined in a chemical bond. The radius varies with the location of an atom on the atomic chart, the type of chemical bond, the number of neighboring atoms (coordination number) and a quantum mechanical property known as spin.<sup>[77]</sup> On the periodic table of the elements, atom size tends to increase when moving down columns, but decrease when moving across rows (left to right).<sup>[78]</sup> Consequently, the smallest atom is helium with a radius of 32 pm, while one of the largest is caesium at 225 pm.<sup>[79]</sup>

When subjected to external fields, like an electrical field, the shape of an atom may deviate from that of a sphere. The deformation depends on the field magnitude and the orbital type of outer shell electrons, as shown by group-theoretical considerations. Aspherical deviations might be elicited for instance in crystals, where large crystal-electrical fields may occur at low-symmetry lattice sites. Significant ellipsoidal deformations have recently been shown to occur for sulfur ions in pyrite-type compounds.<sup>[81]</sup>

Atomic dimensions are thousands of times smaller than the wavelengths of light (400–700 nm) so they can not be viewed using an optical microscope. However, individual atoms can be observed using a scanning tunneling microscope. To visualize the minuteness of the atom, consider that a typical human hair is about 1 million carbon atoms in width.<sup>[82]</sup> A single drop of water contains about 2 sextillion ( $2 \times 10^{21}$ ) atoms of oxygen, and twice the number of hydrogen atoms.<sup>[83]</sup> A single carat diamond with a mass of  $2 \times 10^{-4}$  kg contains about 10 sextillion ( $10^{22}$ ) atoms of carbon.<sup>[note 3]</sup> If an apple were magnified to the size of the Earth, then the atoms in the apple would be approximately the size of the original apple.<sup>[84]</sup>

### Radioactive decay



This diagram shows the half-life ( $T_{1/2}$ ) of various isotopes with Z protons and N neutrons.

Every element has one or more isotopes that have unstable nuclei that are subject to radioactive decay, causing the nucleus to emit particles or electromagnetic radiation. Radioactivity can occur when the radius of a nucleus is large compared with the radius of the strong force, which only acts over distances on the order of 1 fm.

The most common forms of radioactive decay are Alpha is caused when the nucleus emits an alpha particle, which is a helium nucleus consisting of two protons and two neutrons. The result of the emission is a new element with a lower atomic number. Beta is regulated by the weak force, and results from a transformation of a neutron into a proton, or a proton into a neutron. The first is accompanied by the emission of an electron and an antineutrino, while the second causes the emission of a positron and a neutrino. The electron or positron emissions are called beta particles. Beta decay either increases or decreases the atomic number of the nucleus by one Gamma results from a change in the energy level of the nucleus to a lower state, resulting in the emission of electromagnetic radiation. This can occur following the emission of an alpha or a beta particle from radioactive decay. Other more rare types of radioactive decay include ejection of neutrons or protons or clusters of nucleons from a nucleus, or more than one beta particle, or result (through internal conversion) in production of high-speed electrons that are not beta rays, and high-energy photons that are not gamma rays.

Each radioactive isotope has a characteristic decay time period—the half-life—that is determined by the amount of time needed for half of a sample to decay. This is an exponential decay process that steadily decreases the proportion of the remaining isotope by 50% every half-life. Hence after two half-lives have passed only 25% of the isotope is present, and so forth

#### ELECTRON MAGNETIC DIPOLE MOMENT AND QUANTUM MECHANICAL BEHAVIOUR(entanglement):

Elementary particles possess an intrinsic quantum mechanical property known as spin. This is analogous to the angular momentum of an object that is spinning around its center of mass, although strictly speaking these particles are believed to be point-like and cannot be said to be rotating. Spin is measured in units of the reduced Planck constant ( $\hbar$ ), with electrons, protons and neutrons all having spin  $\frac{1}{2} \hbar$ , or "spin- $\frac{1}{2}$ ". In an atom, electrons in motion around the nucleus possess orbital angular momentum in addition to their spin, while the nucleus itself possesses angular momentum due to its nuclear spin

The magnetic field produced by an atom—its magnetic moment—is determined by these various forms of angular momentum, just as a rotating charged object classically produces a magnetic field. However, the most dominant contribution comes from spin. Due to the nature of electrons to obey the Pauli exclusion principle, in which no two electrons may be found in the same quantum state, bound electrons pair up with each other, with one member of each pair in a spin up state and the other in the opposite, spin down state. Thus these spins cancel each other out, reducing the total magnetic dipole moment to zero in some atoms with even number of electrons.

In ferromagnetic elements such as iron, an odd number of electrons lead to an unpaired electron and a net overall magnetic moment. The orbitals of neighboring atoms overlap and a lower energy state is achieved when the spins of unpaired electrons are aligned with each other, a process known as an exchange interaction. When the magnetic moments of ferromagnetic atoms are lined up, the material can produce a measurable macroscopic field. Paramagnetic materials have atoms with magnetic moments that line up in random directions when no magnetic field is present, but the magnetic moments of the individual atoms line up in the presence of a field.

The nucleus of an atom can also have a net spin. Normally these nuclei are aligned in random directions because of thermal equilibrium. However, for certain elements (such as xenon-129) it is possible to polarize a significant proportion of the nuclear spin states so that they are aligned in the same direction—a condition called hyper polarization. This has important applications in magnetic resonance imaging.

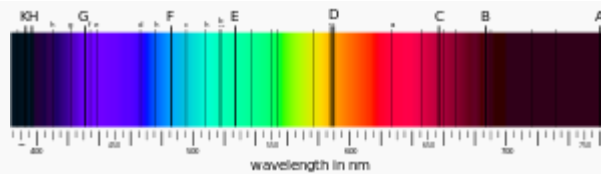
#### ENERGY LEVELS AND QUANTUM SPECTRAL LINE:

When an electron is bound to an atom, it has a potential energy that is inversely proportional to its distance from the nucleus. This is measured by the amount of energy needed to unbind the electron from the atom, and is usually given in units of electron volts (eV). In the quantum mechanical model, a bound electron can only occupy a set of states centered on the nucleus, and each state corresponds to a specific energy level. The lowest energy state of a bound electron is called the ground state, while an electron at a higher energy level is in an excited state.

For an electron to transition between two different states, it must absorb or emit a photon at an energy matching the difference in the potential energy of those levels. The energy of an emitted photon is proportional to its frequency, so these



specific energy levels appear as distinct bands in the electromagnetic spectrum. Each element has a characteristic spectrum that can depend on the nuclear charge, sub shells filled by electrons, the electromagnetic interactions between the electrons and other factors



### An example of absorption lines in a spectrum

When a continuous spectrum of energy is passed through a gas or plasma, some of the photons are absorbed by atoms, causing electrons to change their energy level. Those excited electrons that remain bound to their atom spontaneously emit this energy as a photon, traveling in a random direction, and so drop back to lower energy levels. Thus the atoms behave like a filter that forms a series of dark absorption bands in the energy output. (An observer viewing the atoms from a view that does not include the continuous spectrum in the background, instead sees a series of emission lines from the photons emitted by the atoms.) Spectroscopic measurements of the strength and width of spectral lines allow the composition and physical properties of a substance to be determined.

Close examination of the spectral lines reveals that some display a fine structure splitting. This occurs because of spin-orbit coupling, which is an interaction between the spin and motion of the outermost electron. When an atom is in an external magnetic field, spectral lines become split into three or more components; a phenomenon called the Zeeman effect. This is caused by the interaction of the magnetic field with the magnetic moment of the atom and its electrons. Some atoms can have multiple electron configurations with the same energy level, which thus appear as a single spectral line. The interaction of the magnetic field with the atom shifts these electron configurations to slightly different energy levels, resulting in multiple spectral lines. The presence of an external electric field can cause a comparable splitting and shifting of spectral lines by modifying the electron energy levels, a phenomenon called the Stark effect.

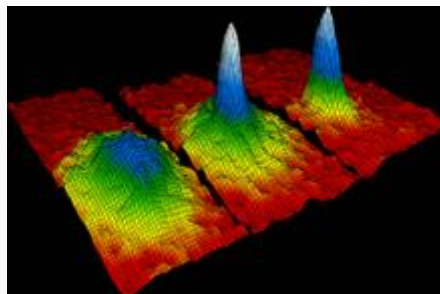
If a bound electron is in an excited state, an interacting photon with the proper energy can cause stimulated emission of a photon with a matching energy level. For this to occur, the electron must drop to a lower energy state that has an energy difference matching the energy of the interacting photon. The emitted photon and the interacting photon then move off in parallel and with matching phases. That is, the wave patterns of the two photons are synchronized. This physical property is used to make lasers, which can emit a coherent beam of light energy in a narrow frequency band

### Valence and bonding behavior

The outermost electron shell of an atom in its uncombined state is known as the valence shell, and the electrons in that shell are called valence electrons. The number of valence electrons determines the bonding behavior with other atoms. Atoms tend to chemically react with each other in a manner that fills (or empties) their outer valence shells. For example, a transfer of a single electron between atoms is a useful approximation for bonds that form between atoms with one-electron more than a filled shell, and others that are one-electron short of a full shell, such as occurs in the compound sodium chloride and other chemical ionic salts. However, many elements display multiple valences, or tendencies to share differing numbers of electrons in different compounds. Thus, chemical bonding between these elements takes many forms of electron-sharing that are more than simple electron transfers. Examples include the element carbon and the organic compounds

The chemical elements are often displayed in a periodic table that is laid out to display recurring chemical properties, and elements with the same number of valence electrons form a group that is aligned in the same column of the table. (The horizontal rows correspond to the filling of a quantum shell of electrons.) The elements at the far right of the table have their outer shell completely filled with electrons, which results in chemically inert elements known as the noble gases.

### STATE OF MATTER AND PHASE OF MATTER FOR CLASSIFICATION OF QUANTUM MECHANICAL BEHAVIOUR:



Snapshots illustrating the formation of a Bose–Einstein condensate

Quantities of atoms are found in different states of matter that depend on the physical conditions, such as temperature and pressure. By varying the conditions, materials can transition between solids, liquids, gases and

plasmas Within a state, a material can also exist in different phases. An example of this is solid carbon, which can exist as graphite or diamond.

At temperatures close to absolute zero, atoms can form a Bose–Einstein condensate, at which point quantum mechanical effects, which are normally only observed at the atomic scale, become apparent on a macroscopic scale This super-cooled collection of atoms then behaves as a single super atom, which may allow fundamental checks of quantum mechanical behavior.

The scanning tunneling microscope is a device for viewing surfaces at the atomic level. It uses the quantum tunneling phenomenon, which allows particles to pass through a barrier that would normally be insurmountable. Electrons tunnel through the vacuum between two planar metal electrodes, on each of which is an adsorbed atom, providing a tunneling-current density that can be measured. Scanning one atom (taken as the tip) as it moves past the other (the sample) permits plotting of tip displacement versus lateral separation for a constant current. The calculation shows the extent to which scanning-tunneling-microscope images of an individual atom are visible. It confirms that for low bias, the microscope images the space-averaged dimensions of the electron orbitals across closely packed energy levels—the Fermi level local density of states

An atom can be ionized by removing one of its electrons. The electric charge causes the trajectory of an atom to bend when it passes through a magnetic field. The radius by which the trajectory of a moving ion is turned by the magnetic field is determined by the mass of the atom. The mass spectrometer uses this principle to measure the mass-to-charge ratio of ions. If a sample contains multiple isotopes, the mass spectrometer can determine the proportion of each isotope in the sample by measuring the intensity of the different beams of ions. Techniques to vaporize atoms include inductively coupled plasma atomic emission spectroscopy and inductively coupled plasma mass spectrometry, both of which use a plasma to vaporize samples for analysis

A more area-selective method is electron energy loss spectroscopy, which measures the energy loss of an electron beam within a transmission when it interacts with a portion of a sample. The atom-probe tomograph has sub-nanometer resolution in 3-D and can chemically identify individual atoms using time-of-flight mass spectrometry

Spectra of excited states can be used to analyze the atomic composition of distant stars. Specific light wavelengths contained in the observed light from stars can be separated out and related to the quantized transitions in free gas atoms. These colors can be replicated using a gas-discharge lamp containing the same element Helium was discovered in this way in the spectrum of the Sun 23 years before it was found on Earth.

Origin and current state

Atoms form about 4% of the total energy density of the observable universe, with an average density of about  $0.25 \text{ atoms/m}^3$ . Within a galaxy such as the Milky Way, atoms have a much higher concentration, with the density of matter in the interstellar medium (ISM) ranging from  $10^5$  to  $10^9 \text{ atoms/m}^3$ . The Sun is believed to be inside the Local Bubble, a region of highly ionized gas, so the density in the solar neighborhood is only about  $10^3 \text{ atoms/m}^3$  Stars form from dense clouds in the ISM, and the evolutionary processes of stars result in the steady enrichment of the ISM with elements more massive than hydrogen and helium. Up to 95% of the Milky Way's atoms are concentrated inside stars and the total mass of atoms forms about 10% of the mass of the galaxy. (The remainder of the mass is an unknown dark matter.)

### Nucleosynthesis

Stable protons and electrons appeared one second after the Big Bang. During the following three minutes, Big Bang nucleosynthesis produced most of the helium, lithium, and deuterium in the universe, and perhaps some of the beryllium and boron. The first atoms (complete with bound electrons) were theoretically created 380,000 years after the Big Bang—an epoch called recombination, when the expanding universe cooled enough to allow electrons to become attached to nuclei. Since the Big Bang, which produced no carbon, atomic nuclei have been combined in stars through the process of nuclear fusion to produce more of the element helium, and (via the triple alpha process) the sequence of elements from carbon up to iron. Isotopes such as lithium-6, as well as some beryllium and boron are generated in space through cosmic ray spallation. This occurs when a high-energy proton strikes an atomic nucleus, causing large numbers of nucleons to be ejected. Elements heavier than iron were produced in supernovae through the r-process and in AGB stars through the s-process, both of which involve the capture of neutrons by atomic nuclei. Elements such as lead formed largely through the radioactive decay of heavier elements.<sup>1</sup>

### Earth

Most of the atoms that make up the Earth and its inhabitants were present in their current form in the nebula that collapsed out of a molecular cloud to form the Solar System. The rest are the result of radioactive decay, and their relative proportion can be used to determine the age of the Earth through radiometric dating. Most of the helium in the crust of the Earth (about 99% of the helium from gas wells, as shown by its lower abundance of helium-3) is a product of alpha decay.

There are a few trace atoms on Earth that were not present at the beginning (i.e., not "primordial"), nor are results of radioactive decay. Carbon-14 is continuously generated by cosmic rays in the atmosphere. Some atoms on Earth have been artificially generated either deliberately or as by-products of nuclear reactors or explosions Of the transuranic elements—those with atomic numbers greater than 92—only plutonium and neptunium occur naturally on Earth. Transuranic elements have radioactive lifetimes shorter than the current age of the Earth and thus identifiable quantities of these elements have

long since decayed, with the exception of traces of plutonium-244 possibly deposited by cosmic dust. Natural deposits of plutonium and neptunium are produced by neutron capture in uranium ore

The Earth contains approximately  $1.33 \times 10^{50}$  atoms. In the planet's atmosphere, small numbers of independent atoms of noble gases exist, such as argon and neon. The remaining 99% of the atmosphere is bound in the form of molecules, including carbon Dioxide and diatomic oxygen and nitrogen. At the surface of the Earth, atoms combine to form various compounds, including water, salt, silicates and oxides. Atoms can also combine to create materials that do not consist of discrete molecules, including crystals and liquid or solid metals. This atomic matter forms networked arrangements that lack the particular type of small-scale interrupted order associated with molecular matter

### Rare and theoretical forms

While isotopes with atomic numbers higher than lead (82) are known to be radioactive, an "island of stability" has been proposed for some elements with atomic numbers above 103. These super heavy elements may have a nucleus that is relatively stable against radioactive decay. The most likely candidate for a stable super heavy atom, unbihexium, has 126 protons and 184 neutrons.

Each particle of matter has a corresponding antimatter particle with the opposite electrical charge. Thus, the positron is a positively charged antielectron and the antiproton is a negatively charged equivalent of a proton. When a matter and corresponding antimatter particle meet, they annihilate each other. Because of this, along with an imbalance between the number of matter and antimatter particles, the latter are rare in the universe. (The first causes of this imbalance are not yet fully understood, although the baryogenesis theories may offer an explanation.) As a result, no antimatter atoms have been discovered in nature. However, in 1996, antihydrogen, the antimatter counterpart of hydrogen, was synthesized at the CERN laboratory in Geneva. Other exotic atoms have been created by replacing one of the protons, neutrons or electrons with other particles that have the same charge. For example, an electron can be replaced by a more massive muon, forming a muonic atom. These types of atoms can be used to test the fundamental predictions of physics

## QUANTUM MECHANICAL BEHAVIOUR: SYSTEMAL IMPLICATIONS:

### ASSUMPTIONS:

- a) QUANTUM MECHANICAL BEHAVIOURS are classified into three categories;
  - 1) Category 1 **OF QUANTUM MECHANICAL BEHAVIOUR CONCOMITANT TO QUANTUM COMPUTATION CLASSIFICATION IN CATEGORY 1**
  - 2) Category 2 (second interval) **OF QUANTUM MECHANICAL BEHAVIOUR CONCOMITANT TO QUANTUM COMPUTATION CLASSIFICATION IN CATEGORY 2**
  - 3) Category 3 **OF QUANTUM MECHANICAL BEHAVIOUR CONCOMITANT TO QUANTUM COMPUTATION CLASSIFICATION IN CATEGORY 3**

In this connection, it is to be noted that there is no sacrosanct time scale as far as the above pattern of classification is concerned. Any operationally feasible scale with an eye on the classification of **QUANTUM MECHANICAL BEHAVIOUR CONCOMITANT TO QUANTUM COMPUTATION CLASSIFICATION IN Categories 1, 2, and 3** would be in the fitness of things. For category 3. "Over and above" nomenclature could be used to encompass a wider range of CATEGORICAL CONSTITUENTS. Similarly, a "less than" scale for category 1 can be used. The speed of growth of **QUANTUM MECHANICAL BEHAVIOUR CONCOMITANT TO QUANTUM COMPUTATION CLASSIFICATION IN CATEGORY 1** proportional to the total amount of **QUANTUM MECHANICAL BEHAVIOUR CONCOMITANT TO QUANTUM COMPUTATION CLASSIFICATION IN CATEGORY 2** In essence the accentuation coefficient in the model is representative of the constant of proportionality between **QUANTUM MECHANICAL BEHAVIOUR CONCOMITANT TO QUANTUM COMPUTATION CLASSIFICATION IN CATEGORY** under category 1 and category 2. This assumptions is made to foreclose the necessity of addition of one more variable, that would render the systemic equations unsolvable

### DISSIPATION OF QUANTUM MECHANICAL BEHAVIOUR:

The dissipation in all the three categories is attributable to the following two phenomenon : **Aging phenomenon:** The aging process leads to transference of the **QUANTUM MECHANICAL BEHAVIOUR CONCOMITANT TO QUANTUM COMPUTATION CLASSIFICATION IN CATEGORY 1** to the next category, **Depletion phenomenon:** Complete destruction of say quantum entanglement For detailed exposition see essay at the end.\*

### NOTATION :

$G_{36}$  : **QUANTUM MECHANICAL BEHAVIOUR CONCOMITANT TO QUANTUM COMPUTATION CLASSIFICATION IN CATEGORY**

$G_{37}$  : **OF QUANTUM MECHANICAL BEHAVIOUR CONCOMITANT TO QUANTUM COMPUTATION CLASSIFICATION IN CATEGORY 2**

$G_{38}$  : **QUANTUM MECHANICAL BEHAVIOUR CONCOMITANT TO QUANTUM COMPUTATION CLASSIFICATION IN CATEGORY 3**

$(a_{36})^{(7)}, (a_{37})^{(7)}, (a_{38})^{(7)}$  : Accentuation coefficients

$(a'_{36})^{(7)}, (a'_{37})^{(7)}, (a'_{38})^{(7)}$  : Dissipation coefficients\*

### FORMULATION OF THE SYSTEM :

In the light of the assumptions stated in the foregoing, we infer the following:-

The growth speed in category 1 is the sum of a accentuation term  $(a_{36})^{(7)}G_{37}$  and a dissipation term  $-(a'_{36})^{(7)}G_{36}$ , the amount of dissipation taken to be proportional to the **QUANTUM MECHANICAL BEHAVIOUR CONCOMITANT TO QUANTUM COMPUTATION CLASSIFICATION IN CATEGORY**

(a) The growth speed in category 2 is the sum of two parts  $(a_{37})^{(7)}G_{36}$  and  $-(a'_{37})^{(7)}G_{37}$  the inflow from the category 1 dependent on the total amount standing in that category.

The growth speed in category 3 is equivalent to  $(a_{38})^{(7)}G_{37}$  and  $-(a'_{38})^{(7)}G_{38}$  dissipation ascribed only to depletion phenomenon.\*

\*  
\*

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)}G_{37} - (a'_{36})^{(7)}G_{36} \quad *1$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)}G_{36} - (a'_{37})^{(7)}G_{37} \quad *2$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)}G_{37} - (a'_{38})^{(7)}G_{38} \quad *3$$

$$(a_i)^{(7)} > 0, \quad i = 36, 37, 38 \quad *4$$

$$(a'_i)^{(7)} > 0, \quad i = 36, 37, 38 \quad *5$$

$$(a_{37})^{(7)} < (a'_{36})^{(7)} \quad *6$$

$$(a_{38})^{(7)} < (a'_{37})^{(7)} \quad *7$$

We can rewrite \*

$$\frac{dG_{36}}{(a_{36})^{(7)}G_{37} - (a'_{36})^{(7)}G_{36}} = dt \quad *8$$

$$\frac{dG_{37}}{(a_{37})^{(7)}G_{36} - (a'_{37})^{(7)}G_{37}} = dt \quad *9$$

Or we write a single equation as \*

$$\frac{dG_{36}}{(a_{36})^{(7)}G_{37} - (a'_{36})^{(7)}G_{36}} = \frac{dG_{37}}{(a_{37})^{(7)}G_{36} - (a'_{37})^{(7)}G_{37}} = \frac{dG_{38}}{(a_{38})^{(7)}G_{37} - (a'_{38})^{(7)}G_{38}} = dt \quad *10$$

The equality of the ratios in equation (10) remains unchanged in the event of multiplication of numerator and denominator by a constant factor.\*

For constant multiples  $\alpha, \beta, \gamma$  all positive we can write equation (10) as\*

$$\frac{\alpha dG_{36}}{\alpha((a_{36})^{(7)}G_{37} - (a'_{36})^{(7)}G_{36})} = \frac{\beta dG_{37}}{\beta((a_{37})^{(7)}G_{36} - (a'_{37})^{(7)}G_{37})} = \frac{\gamma dG_{38}}{\gamma((a_{38})^{(7)}G_{37} - (a'_{38})^{(7)}G_{38})} = dt \quad *14$$

The general solution of the **SYSTEM OF QUANTUM MECHANICAL BEHAVIOUR CONCOMITANT TO QUANTUM COMPUTATION CLASSIFICATION IN CATEGORY** can be written in the form \*

$\alpha_i G_i + \beta_i G_i + \gamma_i G_i = C_i e_i^{\lambda_i t}$  Where  $i = 36, 37, 38$  and  $C_{36}, C_{37}, C_{38}$  are arbitrary constant coefficients.\*

**STABILITY ANALYSIS :**

### **STABILITY ANALYSIS OF QUANTUM MECHANICAL SYSTEMS:**

**THERE ARE MANY ILLUSTRATIONS AND EXAMPLES UNDER THIS HEAD THAT CAN BE DISCUSSED WHERE THE QUANTUM ENTANGLEMENT STABILITY OR INSTABILITY PERSISTS OR NOT AND ITS WIDE RANGING AMPLITUDINAL RAMIFICATIONS THEREOF:**

#### **Quantum entanglement at Nano Scale:**

More often than not, issues of Quantum Entanglement theory, quantum molecular entanglement in nano-scale, quantum coherence in delocalized bond structures and quantum entanglement in Nanoscale dot- systems are interlinked to each other. One of most peculiar properties of Quantum Physics is focused on the Entanglement that get the possibility to built up special quantum shared states based on delocalized electron's field. In fact Entanglement permits **to (e&eb)change** the degree of localization of quantum/wave particles ;in fact also during a spatial separation of pair wise electrons quantum entanglement **generate(eb) a new** partial localized conjugate-systems of bonding atoms. The entanglement activity **can (eb)evolve in** strength and in coherence of simultaneity properties of mixed delocalized states and/or in the successive decay to localized single states in function of some noises ( temperature and other interferences) that **dis-entangled the stability** in the time-scale of the simultaneity co-existence of entangled states . To investigate on the properties innovation of entanglement effects good experimental information **can be (eb)obtained** looking at the **spectrum of emission induced by lasers and measured in Femto-seconds** (Femto-chemistry \*). In fact this fast-method of investigation can give information observed exactly what happens at the molecular level during a chemical reaction. So that ultrafast molecular dynamics in future can permit to deeply understand **the effect(e&eb) of** entangled hybridization of electron's field ( in some way similar to the metallic bond) caused by the over position of electron orbitals **create(eb) the** molecular bonding in the nano-scale dimension.

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#### **Stability of atomic clocks based (e)on entangled atoms**

Under this head, many authors have shown that the stability and instability of quantum evolution are analyzed in the interaction of a two-level atom with a quantized-field mode in an ideal cavity with allowance for photon recoil, which is the basic model of cavity QED. It is shown that the Jaynes-Cammings quantum dynamics **can be unstable** in the *regime of the*

random walk of the atom in the quantized field of a standing wave in the absence of any interaction with the environment. This **instability is manifested** in large fluctuations of the quantum entropy, which correlate with a classical-chaos measure, the maximum Lyapunov exponent, and in the exponential sensitivity of the fidelity of the quantum states **of the strongly coupled** atom-field system to small variations of resonance detuning. Numerical experiments reveal the **sensitivity of the atomic population inversion to the initial conditions** and to **correlation between** the quantum and classical degrees of freedom of the atom.

### **Stability in atomic clocks:**

**Effect(e&eb) of** realistic noise sources for an atomic clock consisting of a local oscillator that is actively **locked to a** spin-squeezed (entangled) ensemble of  $N$  atoms. Use of entangled states **can lead (eb) to an** improvement of the long-term stability of the clock when the measurement **is(e) limited by** decoherence associated with instability of the local oscillator combined with fluctuations in the atomic ensemble's Bloch vector. Atomic states with a moderate degree of entanglement **yield (eb) the** maximal clock stability, resulting in an improvement that scales as  $N^{1/6}$  compared to the atomic shot noise level. Quantum entanglement is the basis for many of the proposed applications of quantum information science. The experimental implementation of these ideas is challenging since entangled states are easily **destroyed(e) (by** decoherence. To evaluate the potential usefulness of entanglement it is therefore essential to include a realistic description of noise in experiments of interest. Although decoherence is commonly analyzed in the context of simple models, practical sources of noise often **possess(e) a** non-trivial frequency spectrum, and enter through a variety of different physical processes. **Effect(e&eb) of** realistic decoherence processes and noise sources in an atomic clock that is actively **locked(e&eb) to a** spin-squeezed (entangled) ensemble of atoms. The performance of an atomic clock can be characterized by **its frequency accuracy and stability**. Accuracy refers to the frequency offset from the ideal value, whereas stability **describes** the fluctuations around, and drift away from the average frequency. To improve the **long-term clock stability**, it has been suggested **to (e)use** entangled atomic ensembles, in the **presence of** realistic decoherence and noise. In practice, an atomic clock operates **by (e&eb)locking the** frequency of a local oscillator (L.O.) **to the** transition frequency between two levels in an atom. This locking is achieved by a spectroscopic measurement determining the L.O. frequency offset  $\delta\omega$  from the atomic resonance, followed by a feedback Mechanism which steers the L.O. frequency so as to null the mean frequency offset. The problem of frequency control thus **combines** elements of **quantum parameter estimation theory** and **control of stochastic systems via feedback**. The spectroscopic measurement of the atomic transition frequency is typically **achieved** through Ramsey spectroscopy, in which the atoms are illuminated by two short, near-resonant pulses from the local oscillator, **separated by a** long period of free evolution, referred to as the Ramsey time  $T$ . During the free evolution the atomic state and the L.O. acquire a relative phase difference  $\delta\phi = \delta\omega T$ , which is subsequently **determined** by a projection measurement. If a long time  $T$  is used, then Ramsey spectroscopy provides a very sensitive measurement of the L.O. frequency offset  $\delta\omega$ . Situation IS relevant to trapped particles, such as atoms in an optical lattice or trapped ions. In this situation, the optimal value of  $T$  **is(eb) determined by** atomic decoherence (**caused by** imperfections in the experimental setup) which **therefore(eb) determines the** ultimate performance of the clock. Using a simple noise model it was shown that entanglement **provides(eb) little gain** in spectroscopic sensitivity in the presence of atomic decoherence. In essence, random fluctuations in the phase of the atomic coherence **cause(eb) a** rapid smearing of the error contour. In practice, the Stability of atomic clocks is often **limited(e) primarily by** fluctuations of the L.O. Atomic dephasing **and(e) the use of** entangled states **can (eb)lead to a** significant improvement in clocks. Pracy, Disclaimer, Terms & Conditions, and Copyright Inf

### **Large scale effective Theory for cosmological bounces( See for details Martin Bojowald)\***

An exactly solvable bounce model in loop quantum cosmology is identified which serves as a perturbative basis for realistic bounce scenarios. Its bouncing solutions are derived analytically, demonstrating why recent numerical simulations **robustly led to** smooth bounces under the assumption **of semi classicality**. Several **effects(e&eb)**, easily **included in a** perturbative analysis, can **however(e&eb) change this** smoothness. The effective theory is not only applicable to such situations where a numerical technique become highly involved but also allows one to discuss conceptual issues. For instance, consequences of the notoriously difficult physical inner product can be implemented at the effective level. This indicates that even physical predictions from full quantum gravity **can be (e)obtained from** perturbative effective equations.

Using Lagrange–Poincare stability analysis and some catastrophe theory classification of singularities, we analyze the two-slit experiments of quantum physics. It is shown that assuming micro-spacetime to be a **Fuzzy Kähler-like manifold  $K(\epsilon^{(c)})$**  with an **inbuilt** wave–particle duality, **one of the two slits is always(e&eb) unstable**. Consequently, the faintest interference **with the experiment** is sufficient to **break the** symmetry of “equilibrium” and **leads (eb) to what is perceived** on the other side of the quantum-classical interface as a wave collapse.

**Black strings and p-branes are (e&eb) unstable (Ruth Gregory)**

Investigation is on the evolution of small perturbations around black strings and branes which are low energy solutions of string theory. For simplicity we focus attention on the zero charge case and show that there are unstable modes for a range of time frequency and wavelength in the extra  $10-D$  dimensions. These perturbations can be stabilized if the extra dimensions are compactified to a scale smaller than the minimum wavelength for which instability occurs and thus will not affect large astrophysical black holes in four dimensions. We comment on the implications of this result for the cosmic censorship hypothesis.

\*

Supposing  $G_i(0) = G_i^0(0) > 0$ , and denoting by  $\lambda_i$  the characteristic roots of the system, it easily results that

1. If  $(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} > 0$  all the components of the solution, i.e all the three parts of the **OF QUANTUM MECHANICAL BEHAVIOUR CONCOMITANT TO QUANTUM COMPUTATION CLASSIFICATION IN CATEGORY 1** tend to zero, and the solution is stable with respect to the initial data.

2. If  $(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$  and  $(\lambda_{37} + (a'_{36})^{(7)})G_{36}^0 - (a_{36})^{(7)}G_{37}^0 \neq 0, (\lambda_{37} < 0)$ , the first two components of the solution tend to infinity as  $t \rightarrow \infty$ , and  $G_{38} \rightarrow 0$ , i.e. The category 1 and category 2 parts grows to infinity, whereas the third part category 3 tends to zero.

3. If  $(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$  and  $(\lambda_{37} + (a'_{36})^{(7)})G_{36}^0 - (a_{36})^{(7)}G_{37}^0 = 0$  Then all the three parts tend to zero, but the solution is not stable i.e. at a small variation of the initial values of  $G_i$ , the corresponding solution tends to infinity.\*

\*

From the above stability analysis we infer the following:

1.The adjustment process is stable in the sense that the system converges to equilibrium.2.The approach to equilibrium is a steady one, and there exists progressively diminishing oscillations around the equilibrium point3.Conditions 1 and 2 are independent of the size and direction of initial disturbance7.The actual shape of the time path system is determined by efficiency parameter, the strength of the response of the portfolio in question, and the initial disturbance5.Result 3 warns us that we need to make an exhaustive study of the behavior of any case in which generalization derived from the model do not hold6.Growth studies as the one in the extant context are related to the systemic growth paths with full employment of resources that are available in question,

Traveling wave analysis of semiconductor lasers: modulation responses, mode stability and quantum mechanical treatment of noise spectra is an example of the system in which stability analysis could be conducted. A traveling wave analysis of a general class of semiconductor lasers, which includes multisession DFB/DBR lasers and gain-coupled DFB lasers. The analysis leads to new semi analytic expressions for the small-signal IM and FM modulation responses, the intensity and FM noise spectra, and the line width. The expressions are given in terms of solutions to four coupled linear homogeneous differential equations and can easily be evaluated numerically. Derivation of a stability parameter  $\sigma$ , for which  $\sigma < 0$  indicates that the model is unstable with respect to small-scale fluctuations. The noise spectra are derived from semiclassical calculations as well as from calculations based on quantized fields, and there are limitations of the semiclassical approach. The formalism of the quantum mechanical treatment has a built-in relationship between the relative intensity noise and the noise of the injection current. This relationship is discussed and illustrated by numerical examples by various authors extensively in literature.

Carlton M. Caves and G. J. Milburn have studied Quantum-mechanical model for continuous position measurements, which is another example of the Quantum Mechanical Behaviour and Dissipation where Stability Analysis could be carried out..They present an idealized model for a sequence of position measurements, and then take an appropriate limit in which the measurements become continuous. The measurements lead to fluctuations without systematic dissipation, and they rapidly destroy off-diagonal terms in the position basis; thus the pointer basis is position. A modification of the model incorporates systematic dissipation via a feedback mechanism; in the modified model there is no decay of off-diagonal coherence in the position basis.

Igamberdiev A.U. is another author who has studied the Prigogine's dissipative structures and has analyzed the stability processes thereof. In Quantum mechanical properties of biosystems: A framework for complexity, structural stability, and transformations is provided by the author. Internal quantum non-demolition measurements are inherent for biological organization and determine the essential features of living systems. Low energy dissipation in these measurements provided by slow conformational relaxation of biomacromolecular complexes (regarded as measuring devices) is the main precondition of enzyme operation and information transfer determining the steady non-equilibrium state of biosystems. The presence of an internal formal description inside a biosystems, expressed in genetic structures (developmental program), is a consequence of its quantum properties. Incompleteness of this formal description provides the possibility of the generation of new functional relations and interconnections inside the system. This is a logical precondition of an evolutionary process. The quantum mechanical uncertainty that underlies the appearance of bifurcations is considered to be the main physical foundation of complication and irreversible transformation of biosystems. It provides a framework for temporal and spatial characteristics of the biosystems. The logic of such a framework derives from Aristotle, as in his philosophy we find the analysis of fundamental irreversibility and self determination of the living processes.

Quantum properties of biosystems structures was studied by, Pattee (1968) and Rosen (1977) concluded that its logic should reveal an internal resemblance to the logic of quantum mechanical measurement, in which a non-formal process of mapping physical events into symbols takes place. Biological molecular complexes are operational structures put in correspondence

with other molecules and processes. These considerations give a theoretical framework for a description of the complexity, **structural stability and transformations of living systems**. Quantum measurement is connected with low energy dissipation in the case where the relaxation period of a macroscopic oscillator ( $t^*$ ) is many times larger than the time interval of measurement ( $t'$ ). Minimal energy dissipation in quantum measurement is calculated. \*

### **QUANTUM COMPUTER:**

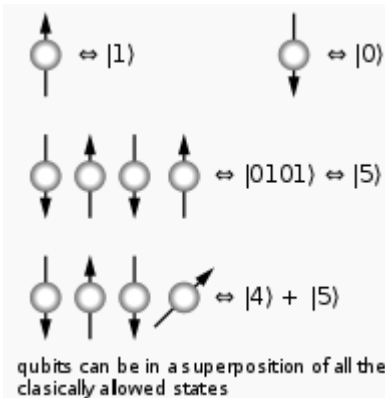
**Quantum computer** is a device for computation that makes **direct use of** quantum mechanical phenomena, such as **superposition and entanglement**, to perform operations on data. Quantum computers are different from digital computers based on transistors. Whereas digital computers require data to be encoded into binary digits (bits), quantum computation **utilizes** quantum properties to represent data and perform operations on these data. A theoretical model is the **Quantum Turing machine**, also known as the universal quantum computer. Quantum computers share theoretical similarities with non-deterministic and probabilistic computers, like the ability to be in more than one state simultaneously. The field of quantum computing was first introduced by Richard Feynman in 1982.

Although quantum computing is still in its infancy, experiments have been carried out in which quantum computational operations were executed on a very small number of qubits (quantum bits). Both practical and theoretical research continues, and many national government and military funding agencies support quantum computing research to develop quantum computers for both civilian and national security purposes, such as cryptanalysis.

Large-scale quantum computers could be able to solve certain problems much faster than any classical computer by using the best currently known algorithms, like integer factorization using Shor's algorithm or the simulation of quantum many-body systems. There exist quantum algorithms, such as Simon's algorithm, which run faster than any possible probabilistic classical algorithm. Given unlimited resources, a classical computer can simulate an arbitrary quantum algorithm so quantum **computation does not violate** the Church-Turing thesis. However, in practice infinite resources are never available and the computational basis of 500 qubits, for example, would already be too large to be represented on a classical computer because it would require  $2^{500}$  complex values to be stored. (For comparison, a terabyte of digital information stores only  $2^{43}$  discrete on/off values) Nielsen and Chuang point out that "Trying to store all these complex numbers would not be possible on any conceivable classical computer."

Bits vs. qubits

A quantum computer with a given number of qubits is fundamentally different from a classical computer composed of the same number of classical bits. For example, to represent the state of an  $n$ -qubit system on a classical computer would require the storage of  $2^n$  complex coefficients. Although this fact may seem to indicate that qubits **can hold** exponentially more information than their classical counterparts, care must be taken not to **overlook the fact** that the qubits are only in a probabilistic **superposition of all** of their states. This means that when the final state of the qubits is measured, they will only be found in one of the possible configurations they were in before measurement. Moreover, it is incorrect to think of the qubits as only being in one particular state before measurement since the fact that they were in **a superposition** of states before the measurement was made directly affects the possible outcomes of the computation.



Qubits are made up of controlled particles and the means of control (e.g. devices that trap particles and switch them from one state to another). For example: Consider first a classical computer that operates on a three-bit register. The state of the computer at any time is a probability distribution over the  $2^3 = 8$  different three-bit strings 000, 001, 010, 011, 100, 101, 110, 111. If it is a deterministic computer, then it is in exactly one of these states with probability 1. However, if it is a probabilistic computer, then there is a possibility of it being in any *one* of a number of different states. We can **describe this** probabilistic state by eight nonnegative numbers  $A, B, C, D, E, F, G, H$  (where  $A$  = probability computer is in state 000,  $B$  = probability computer is in state 001, etc.). There is **a restriction** that these probabilities sum to 1.

The state of a three-qubit quantum computer is similarly described by an eight-dimensional vector  $(a, b, c, d, e, f, g, h)$ , called a **ket**. However, instead of adding to one, the sum of the *squares* of the coefficient magnitudes,  $|a|^2 + |b|^2 + \dots + |h|^2$ , must equal one. Moreover, the coefficients are complex numbers. Since the probability amplitudes of the states are represented with complex numbers, the phase between any two states is a meaningful parameter, which is a key difference between quantum computing and probabilistic classical computing.<sup>[8]</sup>

If you measure the three qubits, you will observe a three-bit string. The probability of measuring a given string is the squared magnitude of that string's coefficient (i.e., the probability of measuring 000 =  $|a|^2$ , the probability of measuring 001 =  $|b|^2$ , etc.). Thus, measuring a quantum state described by complex coefficients ( $a, b, \dots, h$ ) gives the classical probability distribution ( $|a|^2, |b|^2, \dots, |h|^2$ ) and we say that the quantum state "**collapses**" to a classical state **as a result of making** the measurement.

Note that an eight-dimensional vector can be specified in many different ways depending on what basis is chosen for the space. The basis of bit strings (e.g., 000, 001, and 111) is known as the computational basis. Other possible bases are unit-length, orthogonal vectors and the eigenvectors of the Pauli-x operator. **Ket notation** is often used to make the choice of basis explicit. For example, the state ( $a, b, c, d, e, f, g, h$ ) in the computational basis can be written as:

$$a|000\rangle + b|001\rangle + c|010\rangle + d|011\rangle + e|100\rangle + f|101\rangle + g|110\rangle + h|111\rangle$$

where, e.g.,  $|010\rangle = (0, 0, 1, 0, 0, 0, 0, 0)$

#### Assumptions:

Category 1 **QUANTUM COMPUTER (INFORMATION) CORRESPONDING TO THE QUANTUM MECHANICAL BEHAVIOUR STRATIFIED IN THE CORRESPONDING CATEGORY 1**

Category 2 **QUANTUM COMPUTER (INFORMATION) CORRESPONDING TO THE QUANTUM MECHANICAL BEHAVIOUR STRATIFIED IN THE CORRESPONDING CATEGORY 2**

Category 3 **QUANTUM COMPUTER (INFORMATION) CORRESPONDING TO THE QUANTUM MECHANICAL BEHAVIOUR STRATIFIED IN THE CORRESPONDING CATEGORY 3\***

The speed of growth of **QUANTUM COMPUTER (INFORMATION) CORRESPONDING TO THE QUANTUM MECHANICAL BEHAVIOUR STRATIFIED IN THE CORRESPONDING CATEGORY** is a linear function of the in category 2 at the time of reckoning. As before the accentuation coefficient that characterizes the speed of growth in category 1 is the proportionality factor between balance in category 1 and category 2. The dissipation coefficient in the growth model is attributable to two factors passage of time and DETRITION DUE TO Quantum Information (For examples please see Hawking Radiation essays and others at the end written in the form of epilogue to substantiate the factors that are used in the paper. Nevertheless, one instantaneous example that comes to mind is the following:

Quantum engineering via dissipation is an example of dissipation of Quantum Information. Due to the ongoing miniaturization of devices, one of the central challenges of the 21st century's technology will be to handle **quantum effects** at the Nanoscale. A first fundamental paradigm shift happened in the mid '90s when it was realized that quantum effects, which from the traditional point of view put fundamental limits on the possible miniaturization, could be exploited to do information theoretic tasks impossible with classical devices. The main obstacle in building such quantum devices however is the **occurrence of** decoherence, by which coherence within the quantum device gets **degraded due** to the **coupling with** the environment. In second proposal, a second paradigm shift was resorted to, by demonstrating that one can actually **take advantage of** decoherence if engineered in a smart way. The central focus will be the study of quantum processes driven by **dissipation**, and investigations whether quantum coherence and the associated applications can actually be **driven(eb) by** decoherence. The main tools that we plan to use to achieve that goal **originate from** the theory of quantum entanglement. The timing of this innovative project is actually perfect as the field of entanglement theory is just mature enough to pursue the ambitious goals stated in this proposal.

The main objectives of this proposal are:

(1) To set up a rigorous mathematical framework for studying fixed points and convergence rates of dissipative processes; (2) To investigate how highly entangled quantum states arising in strongly correlated quantum systems or in a quantum information theoretic context can be created by dissipative processes; (3) To study quantum devices powered by dissipation such as quantum memories and quantum Metropolis devices; (4) To use such devices to come up with novel ways for implementing quantum computation in the presence of decoherence (Decoherence dissipates Quantum entanglement); (5) To study non-equilibrium phase transitions driven by dissipation and associated to that new possible phases of matter.

J. Hassel, H. Seppa, P. Helisto studied the RSFQ devices with selective dissipation for quantum information processing. They study the possibility to use frequency dependent damping in RSFQ circuits as means to **reduce** dissipation and **consequent** decoherence in RSFQ/qubit circuits. They also show that stable RSFQ operation can be achieved **by shunting** the **Josephson junctions** with an  $\text{\$R\$}$  circuit instead of a plain resistor. A presentation of derived criteria for the stability of such an arrangement, and discuss the **effect** on decoherence and the optimization issues. We also design a simple flux generator aimed at manipulating flux qubits.

Robust edge states and non-Abelian excitations are another example of the Dissipation of Quantum information that occurs in systems be it natural or human made, The trademark of topological states of matter, with promising applications such as 'topologically protected' quantum memory and computing. So far, topological phases have been exclusively discussed in a Hamiltonian context. Here we show that such phases and the associated topological protection and phenomena also emerge in open quantum systems with engineered dissipation. The specific system studied here is a quantum wire of spinless atomic fermions in an optical lattice coupled to a bath. The key feature of the dissipative dynamics described by a Lindblad master equation is the existence of Majorana edge modes, representing a non-local decoherence-free subspace. The isolation of the



edge states is enforced by a dissipative gap in the  $p$ -wave paired bulk of the wire. We describe dissipative non-Abelian braiding operations within the Majorana subspace, and illustrate the insensitivity to imperfections. Topological protection is granted by a non-trivial winding number of the system density matrix

In the following we discuss some salient points and cardinal aspects of the Quantum Dissipation which is an important phenomenon in Quantum Computers:

### Motivation

The goal of *dissipative quantum mechanics* or 'quantum dissipation theory' is to formulate microscopic theories of irreversible behaviour of quantum systems. Simply speaking, one would like to understand processes like, e.g., *friction* or 'damping' on a microscopic level. This requires at least two things: 'friction' means that physical objects interact with each other, i.e., we need to talk about interactions. Furthermore, this occurs as a function of time for systems which are usually out of equilibrium, i.e., we need to talk about dynamics.

A further, more ambitious goal is to better understand the relation between microscopic and macroscopic theories, e.g., the relation between mechanics (classical or quantum) and statistical mechanics (again classical or quantum).

Already in classical (Newtonian) mechanics, the description of irreversible behaviour is a non-trivial problem. One can often introduce dissipation into microscopic equations by adding phenomenological terms, such as the velocity-dependent

damping term  $\gamma \dot{x}(t)$  ( $\gamma > 0$ ) in the damped (forced) harmonic oscillator,

$$\ddot{x}(t) + \gamma \dot{x}(t) + \omega^2 x(t) = f(t).$$

1)

In this example, one of the goals would be to derive this equation and to actually calculate  $\gamma$  from an underlying microscopic theory.

Other examples (some of these are very tough, some not so tough problems):

- What is the spontaneous photon emission rate of an atom in vacuum?
- What is the electrical resistance of a (small or large) piece of metal at very low temperatures?
- How does a Laser work?
- What is the typical time after which a given realisation of a qubit (a quantum two-level system as realised in, e.g., a linear ion trap, the charge or magnetic flux in superconducting junctions, the electron charge or spin in semiconductor quantum dots, the nuclear spin etc.) fails to operate in the desired manner?

### Origin of Dissipation, System-Bath Theories

The most successful approach to quantum dissipation has been the use of *System-Bath Theories*, which will be the main topic of this chapter. The main idea is the following:

**STEP 1:** we divide the 'world' into two parts: a) the part we are really interested in (for example, all the conduction band electrons in a piece of metal), and b) the part we are not so much interested in (for example, all the rest of the metal). This splitting obviously is a choice that depends on what we want to describe/calculate

**STEP 2:** Call these two parts 'system' and 'reservoir', identify the interaction between system and reservoir, and then derive an effective theory for the Example

Single oscillator ('system') with angular frequency  $\omega_0$ , mass  $M$ , position  $x$ , coupled to  $N \gg 1$  oscillators ('reservoir') with angular frequencies  $\omega_i$ , masses  $m_i$ , position  $x_i$ , coupling of the form  $c_i x_i x$  via position coordinates.

The coupling leads to an effective dynamics of the system oscillator governed by the sum of many eigenmodes with eigenfrequencies. This sum is determined by the coupling constants  $c_i$ . For finite  $N$ , this is just a problem of coupled

oscillators, and the motion of the system oscillator must therefore be periodic with a (large) period  $T$ . The time  $T$  after which the entire system returns back to its initial starting point is called *Poincaré* time.

$$t \ll T \quad p$$

The key point now is: 1. For times  $t \ll T$ , the effective dynamics of the system ( $x$  and  $p$  of the system oscillator) very much resembles the dynamics we would expect from a *damped* system: a sum of many oscillatory terms with 'nearly

$$t \ll T$$

random' coefficients decays as a function of time  $t$ . 2. In most known cases,  $T$  is very, very large ('larger than the age of the universe'). This means that one can safely neglect the periodic 'Poincaré return' of the system.

### Formal Splitting

The basic idea in microscopic theories of dissipation is a decomposition of a total system into a system  $S$  and a reservoir part  $R$  or  $B$ , 'bath'. The (Hamiltonian) dynamics of the total system is reversible, but the dynamics of the system  $S$  is

$$t \ll T$$

effectively not reversible for times  $t \ll T$ .

In this lecture, we formulate these ideas for quantum systems. The Hilbert space of the total system is defined by the tensor product

$$\mathcal{H}_{\text{total}} = \mathcal{H}_S \otimes \mathcal{H}_B. \quad ***(2)$$

The Hamiltonian of the total system is defined as

$$H_{\text{total}} \equiv H \equiv H_S + H_{SB} + H_B \quad ***(3)$$

Here and in the following, we will mostly discuss *time-independent* Hamiltonians. Time-dependent

$$H = H(t)$$

Hamiltonians can be treated

### Overview

1. 'Simple' Systems with *few* degrees of freedom: typically quantum optics systems, atoms, few-level systems, cavity modes.

- Weak coupling approximation: Master Equation (Born and Markov Approximation)
- Damped harmonic oscillator.
- Solution methods: phase-space methods ( $P$ -representation etc.).
- Solution methods: quantum trajectories.
- Correlation Functions, Quantum Regression Theorem.
- Beyond weak coupling approximation: Feynman-Vernon influence functional (path integral) theories; R. P. Feynman, F. L. Vernon, Ann. Phys. (N.Y.) **24**, 118 (1963). Non-exponential decay laws at low temperatures.
- Exact solution of damped harmonic oscillator.
- Spin-Boson Problem, Two-Level System.
- Non-Markovian versus Markovian, Lindblad versus non-Lindblad.
- 'Non-standard' methods.

2. Systems with *many* degrees of freedom: typically condensed matter systems, electrons + phonons (particle statistics).

- Quasiclassical kinetic theories, Boltzmann equation.
- Quantum Many-Body Non-Equilibrium Methods. (Keldysh) Greens Function Methods, quantum Boltzmann equation.

In this chapter, we will concentrate on 1. ('Simple' Systems with *few* degrees of freedom). Also, not discussed in detail in this lecture are

- Nakajima-Zwanzig theories, Mori projection operator theories. These give a more formal approach towards system-bath theories.
- 'Early approaches' such as Wigner-Weisskopf theory of spontaneous emission.
- ...

Generally speaking, quantum dissipation can be regarded as a subfield of non-equilibrium quantum statistics/ non-equilibrium many-body theory. d as well but require additional techniques (e.g., Floquet theory for period time-dependence; adiabatic theorems for slow time-dependence).

**Interaction Picture**

We define an interaction picture by writing

$$H \equiv H_0 + V, \quad H_0 \equiv H_S + H_B, \quad V \equiv H_{SB} \tag{4}$$

with the Hamiltonian  $H_0$  describing the time evolution of the uncoupled system and bath, and the perturbation  $V$  describing the interaction  $H_{SB}$ .

We define  $\chi(t)$  as the total density matrix (system + bath) which obeys the Liouville-von-Neumann equation ,

$$\frac{d}{dt}\chi(t) = -i[H, \chi(t)] \rightsquigarrow \chi(t) = e^{-iHt}\chi(t=0)e^{iHt}, \tag{5}$$

where we start with the initial condition  $\chi(t=0)$  at time  $t=0$ . In the interaction picture,

$$\tilde{\chi}(t) \equiv e^{iH_0t}\chi(t)e^{-iH_0t} \tag{6}$$

$$\tilde{A}(t) \equiv e^{iH_0t}Ae^{-iH_0t}. \tag{7}$$

The equation of motion for the density operator in the interaction picture becomes

$$\begin{aligned} \frac{d}{dt}\tilde{\chi}(t) &= i[H_0, \tilde{\chi}(t)] + e^{iH_0t}\frac{d}{dt}\chi(t)e^{-iH_0t} \\ &= i[H_0, \tilde{\chi}(t)] - ie^{iH_0t}[H, \chi(t)]e^{-iH_0t} \\ &= i[H_0, \tilde{\chi}(t)] - ie^{iH_0t}[H_0 + V, \chi(t)]e^{-iH_0t} \\ &= i[H_0, \tilde{\chi}(t)] - i[H_0 + \tilde{V}(t), \tilde{\chi}(t)] \\ &= -i[\tilde{V}(t), \tilde{\chi}(t)]. \end{aligned} \tag{8}$$

In integral form, this can be written as

$$\tilde{\chi}(t) = \chi(t=0) - i \int_0^t dt' [\tilde{V}(t'), \tilde{\chi}(t')] \tag{9}$$

which we insert into Eq. (7.8) to obtain

$$\frac{d}{dt} \tilde{\chi}(t) = -i [\tilde{V}(t), \chi(t=0)] - \int_0^t dt' [\tilde{V}(t), [\tilde{V}(t'), \tilde{\chi}(t')]] \tag{10}$$

**Up t Effective Density Matrix of the System**

$t > 0$

We wish to obtain an equation of motion for the *effective density matrix of the system* at time

$$\rho(t) \equiv \text{Tr}_B [\chi(t)] \tag{11}$$

$A_S$

This object is sufficient to calculate expectation values of *system operators* :

$$\begin{aligned} \langle A_S \rangle_t &\equiv \text{Tr}_{\text{total}} [\chi(t) A_S] \\ &= \text{Tr}_S [\text{Tr}_B \chi(t)] A_S = \text{Tr}_S [\rho(t) A_S] \end{aligned} \tag{12}$$

Now use

$$\begin{aligned} \text{Tr}_B [\tilde{\chi}(t)] &= \text{Tr}_B e^{iH_0 t} \chi(t) e^{-iH_0 t} \\ &= e^{iH_S t} (\text{Tr}_B e^{iH_B t} \chi(t) e^{-iH_B t}) e^{-iH_S t} = e^{iH_S t} \rho(t) e^{-iH_S t} \\ &\equiv \tilde{\rho}(t) \end{aligned} \tag{13}$$

Note that the interaction picture  $\rho(t) \leftrightarrow \tilde{\rho}(t)$  involves only the free *System Hamiltonian*  $H_S$  and not  $H_0$ ,

$$\tilde{\rho}(t) \equiv e^{iH_S t} \rho(t) e^{-iH_S t} \tag{14}$$

Using

$$\tilde{A}_S(t) \equiv e^{iH_0 t} A_S e^{-iH_0 t} = e^{iH_S t} A_S e^{-iH_S t} \tag{15}$$

for system operators, one has

$$\langle A_S \rangle_t = \text{Tr}_S [\tilde{\rho}(t) \tilde{A}_S(t)] = \text{Tr}_S [\rho(t) A_S(t)] \tag{16}$$

**Assumption (factorising initial condition):**

$$\chi(t=0) = R_0 \otimes \rho(t=0) \tag{18}$$

$$R_0 \equiv \text{Tr}_S[\chi(t=0)], \quad \rho(t=0) \equiv \text{Tr}_B[\chi(t=0)].$$

This *factorisation assumption* is key to most of the results that follow. Its validity has been discussed and criticised in the past (see Weiss book for further references). Some of the issues are:

- Does the factorisation assumption only affect transient or also the long-time behaviour of the density matrix?
- Are there exactly solvable models where these issues can be clarified?

A theoretical formulation of time-evolution for arbitrary initial condition is sometimes possible: 'preparation function' (exact solution of dissipative quantum oscillator; Grabert, Ingold et al); generalisation of many-body Keldysh GF (three-by-three matrix instead of two-by-two matrix, M. Wagner).

**Born Approximation**

In the interaction picture,

$$\tilde{\chi}(t') = R_0 \otimes \tilde{\rho}(t=0) \text{ to zeroth order in } V. \tag{19}$$

The Born approximation in the equation of motion Eq.(7.17) consists in

$$\tilde{\chi}(t') \approx R_0 \otimes \tilde{\rho}(t') \tag{20}$$

Born approximation

This means one assumes that for all times  $t' > 0$ , the total density matrix remains a product of the initial bath density

matrix  $R_0$  and the system density matrix  $\tilde{\rho}(t')$ . Intuitively, one argues that this is justified when the bath is 'very large'

and the coupling  $H_{SB}$  'weak', so that the back-action of the system onto the bath can be neglected. In practice, one usually assumes a thermal equilibrium for the bath,

$$R_0 = \frac{e^{-\beta H_B}}{\text{Tr} e^{-\beta H_B}}, \tag{21}$$

thermal equilibrium bath,

where  $\beta = 1/k_B T$  with  $T$  the bath equilibrium temperature.

**Remark:** A more detailed analysis of the Born approximation and alternative approximations can be done within the framework of the *Projection Operator formalism*.

Within the Born approximation, with Eq. (7.20), (7.18), and (7.17), one obtains a *closed integro-differential equation* for the

reduced density operator  $\tilde{\rho}(t)$  of the system in the interaction picture,

$$\frac{d}{dt} \tilde{\rho}(t) = -i \text{Tr}_B [\tilde{V}(t), R_0 \otimes \tilde{\rho}(t=0)] - \int_0^t dt' \text{Tr}_B [\tilde{V}(t), [\tilde{V}(t'), R_0 \otimes \tilde{\rho}(t')]].$$

\*\*\* (22)

**Remark:** Eq.(7.22) is exact up to second order in the perturbation  $V$ : set  $\tilde{\rho}(t') = \rho(0)$  on the r.h.s. of Eq.(7.22).

Since  $\tilde{\rho}(t')$  in the double commutator on the r.h.s. of Eq.(7.22) depends on  $V$ , Eq.(7.22) is to infinite order in  $V$  though not exact. Diagrammatically this corresponds to a summation of an infinite series of diagrams. It is non-trivial to make this statement more precise, but roughly speaking these diagrams contain certain vertex corrections as can be seen from the fact that  $\rho(t)$  is a density matrix and not a wave function.

**Motivation: telegraphic fluorescence (driven spontaneous emission) of single atoms**

Example single V-systems: two upper levels 1 (fast spontaneous emission) and 2 (slow spontaneous emission), one lower level 0 driven by two lasers. Transition  $0 \rightarrow 2$  traps the system in 2 for a long time. Resonance fluorescence intensity  $I(t)$  therefore exhibits jumps: 'telegraphic fluorescence' with random switching between bright and dark periods. Aim: calculate distribution of dark periods.

Length  $T_D$  of dark period can be simply calculated from the density matrix element  $\rho_{22}$

$$T_D^{-1} = \dot{\rho}_{22}(t=0), \quad \rho_{22} = 0, \quad \text{*(138)}$$

where the derivative is calculated from the underlying equation of motion (Master equation). However, the calculation of other, more complicated quantities related to the description of telegraphic fluorescence turns out to be technically complicated within the Master equation formalism. Example: 'exclusive probability'  $P_0(t)$  that, after an emission at time  $t = 0$ , no other photon has been emitted in the time interval  $[0, t]$ .

- Some people raise 'objections' against the traditional Master equation approach: the density operator describes *ensembles* of quantum systems and is therefore inappropriate to describe *single* quantum systems such as a single ion in an ion trap. However, these objections are unjustified; as long as one sticks with the probabilistic interpretation of Quantum Mechanics, the density operator description is perfectly valid for a *single* quantum system.
- 'Single quantum systems' can not only be realised in ion traps, but also in 'artificial atoms' and 'artificial molecules' (solid state based quantum dots, superconducting charge or flux qubits). These will be discussed in a later chapter.

**NOTATION :**  
 $T_{36}$  : QUANTUM COMPUTER(INFORMATION) CORRESPONDING TO THE QUANTUM MECHANICAL BEHAVIOUR STRATIFIED IN THE CORRESPONDING CATEGORY 1  
 $T_{37}$  : QUANTUM COMPUTER(INFORMATION) CORRESPONDING TO THE QUANTUM MECHANICAL BEHAVIOUR STRATIFIED IN THE CORRESPONDING CATEGORY2  
 $T_{38}$  : QUANTUM COMPUTER (INFORMATION) CORRESPONDING TO THE QUANTUM MECHANICAL BEHAVIOUR STRATIFIED IN THE CORRESPONDING CATEGORY, namely category 3.  
 $(b_{36})^{(7)}, (b_{37})^{(7)}, (b_{38})^{(7)}$  : Accentuation coefficients  
 $(b'_{36})^{(7)}, (b'_{37})^{(7)}, (b'_{38})^{(7)}$  : Dissipation coefficients\*

**FORMULATION OF THE SYSTEM :FOR DETAILED EXPLANATION OF THE LOSS OF QUANTUM INFORMATION AND THE CASES AND CONDITIONALITIES UNDER WHICH IT OCCURS PLEASE SEE THE INTRODUCTORY NOTE AS WELL AS THE EPILOGUE WHEREIN IS EXPOSITION OF SUCH CASE ID GIVEN IN DETAIL**

Under the above assumptions, we derive the following :

a) The growth speed in category 1 is the sum of two parts:

A term  $+(b_{36})^{(7)}T_{37}$  proportional to the **QUANTUM COMPUTER(INFORMATION) CORRESPONDING TO THE QUANTUM MECHANICAL BEHAVIOUR STRATIFIED IN THE CORRESPONDING CATEGORY** in category 2

A term  $-(b'_{36})^{(7)}T_{36}$  representing the quantum of balance dissipated from category 1 .

1. The growth speed in category 2 is the sum of two parts:

A term  $+(b_{37})^{(7)}T_{36}$  constitutive of the amount of inflow from the category 1 of **QUANTUM COMPUTER(INFORMATION) CORRESPONDING TO THE QUANTUM MECHANICAL BEHAVIOUR STRATIFIED IN THE CORRESPONDING CATEGORY**

A term  $-(b'_{37})^{(7)}T_{37}$  the dissipation factor of **QUANTUM COMPUTER(INFORMATION) CORRESPONDING TO THE QUANTUM MECHANICAL BEHAVIOUR STRATIFIED IN THE CORRESPONDING CATEGORY**

\*  
\*

### GOVERNING EQUATIONS: QUANTUM COMPUTER(INFORMATION)

Following are the differential equations that govern the growth in the terrestrial organisms portfolio\*

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)}T_{37} - (b'_{36})^{(7)}T_{36} \quad *12$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)}T_{36} - (b'_{37})^{(7)}T_{37} \quad *13$$

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - (b'_{38})^{(7)}T_{38} \quad *14$$

$$(b_i)^{(7)} > 0, \quad i = 36, 37, 38 \quad *15$$

$$(b'_i)^{(7)} > 0, \quad i = 36, 37, 38 \quad *16$$

$$(b_{37})^{(7)} < (b'_{36})^{(7)} \quad *17$$

$$(b_{38})^{(7)} < (b'_{37})^{(7)} \quad *18$$

Following the same procedure outlined in the previous section , the general solution of the governing equations is

$$\alpha'_i T_i + \beta'_i T_j + \gamma'_i T_i = C'_i e_i^{\lambda' i t}, \quad i = 36, 37, 38 \quad \text{where } C'_{36}, C'_{37}, C'_{38} \text{ are arbitrary constant coefficients and } \alpha_{36}, \alpha_{37}, \alpha_{38}, \gamma_{36}, \gamma_{37}, \gamma_{38} \text{ corresponding multipliers to the characteristic roots of the System}$$

\*

### QUANTUM COMPUTER AND QUANTUM MECHANICAL BEHAVIOUR- A DUAL SYSTEM ANALYSIS

Various present and future specialized applications of magnets require monodisperse, small magnetic particles, and the discovery of molecules that can function as Nanoscale magnets was an important development in this regard<sup>1,2,3</sup>. These molecules act as single-domain magnetic particles that, below their blocking temperature, exhibit magnetization hysteresis, a classical property of macroscopic magnets. Such 'single-molecule magnets' (SMMs)<sup>4</sup> straddle the interface between classical and quantum mechanical behaviour because they also display quantum tunnelling of magnetization<sup>5,6</sup> and quantum phase interference<sup>7</sup>. Quantum tunnelling of magnetization can be advantageous for some potential applications of SMMs, for example, in providing the quantum superposition of states required for quantum computing<sup>8</sup>. However, it is a disadvantage in other applications, such as information storage, where it would lead to information loss. Thus it is important to both understand and control the quantum properties of SMMs. Here we report a supramolecular SMM dimer in which antiferromagnetic coupling between the two components results in quantum behaviour different from that of the individual SMMs. Our experimental observations and theoretical analysis suggest a means of tuning the quantum tunnelling of magnetization in SMMs. This system may also prove useful for studying quantum tunnelling of relevance to mesoscopic antiferromagnets.

#### **he Possibilities of Quantum Computing**

The special properties of qubits will allow quantum computers to work on millions of computations at once, while desktop PCs can typically handle minimal simultaneous computations. For example, a single 250-qubit state contains more bits of information than there are atoms in the universe.

These properties will have wide-spread implications foremost for the field of data encryption where quantum computers could factor very large numbers like those used to decode and encode sensitive information.

"The quantum computing work we are doing shows it is no longer just a brute force physics experiment. It's time to start creating systems based on this science that will take computing to a new frontier," says IBM scientist Matthias Steffen,

manager of the IBM Research team that's focused on developing quantum computing systems to a point where it can be applied to real-world problems.

Other potential applications for quantum computing may include searching databases of unstructured information, performing a range of optimization tasks and solving previously unsolvable mathematical problems.

### How Quantum Computing Works

The most basic piece of information that a typical computer understands is a bit. Much like a light that can be switched on or off, a bit can have only one of two values: "1" or "0". For qubits, they can hold a value of "1" or "0" as well as both values at the same time. Described as superposition, this is what allows quantum computers to perform millions of calculations at once.

One of the great challenges for scientists seeking to harness the power of quantum computing is controlling or removing quantum decoherence – the creation of errors in calculations caused by interference from factors such as heat, electromagnetic radiation, and materials defects. To deal with this problem, scientists have been experimenting for years to discover ways of reducing the number of errors and of lengthening the time periods over which the qubits retain their quantum mechanical properties. When this time is sufficiently long, error correction schemes become effective making it possible to perform long and complex calculations.

There are many viable systems that can potentially lead to a functional quantum computer. IBM is focusing on using superconducting qubits that will allow a more facile transition to scale up and manufacturing.

IBM has recently been experimenting with a unique "three dimensional" superconducting qubit (3D qubit), an approach that was initiated at Yale University. Among the results, the IBM team has used a 3D qubit to extend the amount of time that the qubits retain their quantum states up to 100 microseconds – an improvement of 2 to 4 times upon previously reported records. This value reaches just past the minimum threshold to enable effective error correction schemes and suggests that scientists can begin to focus on broader engineering aspects for scalability.

3D" superconducting qubit device where a qubit (about 1mm in length) is suspended in the center of the cavity on a small Sapphire chip. The cavity is formed by closing the two halves, and measurements are done by passing microwave signals to the connectors. Despite the apparent large feature size (the cavity is about 1.5 inches wide) for this single qubit demonstration, the team believes it is possible to scale such a system to hundreds or thousands of qubits.

A picture of IBM's "3D" superconducting qubit device where a qubit (about 1mm in length) is suspended in the center of the cavity on a small Sapphire chip. The cavity is formed by closing the two halves, and measurements are done by passing microwave signals to the connectors. Despite the apparent large feature size (the cavity is about 1.5 inches wide) for this single qubit demonstration, the team believes it is possible to scale such a system to hundreds or thousands of qubits.

In separate experiments, the group at IBM also demonstrated a more traditional "two-dimensional" qubit (2D qubit) device and implemented a two-qubit logic operation – a controlled-NOT (CNOT) operation, which is a fundamental building block of a larger quantum computing system. Their operation showed a 95 percent success rate, enabled in part due to the long coherence time of nearly 10 microseconds. These numbers are on the cusp of effective error correction schemes and greatly facilitate future multi-qubit experiments.

*Imagine a computer whose memory is exponentially larger than its apparent physical size; a computer that can manipulate an exponential set of inputs simultaneously; a computer that computes in the twilight zone of Hilbert space. You would be thinking of a quantum computer. Relatively few and simple concepts from quantum mechanics are needed to make quantum computers a possibility. The subtlety has been in learning to manipulate these concepts. Is such a computer an inevitability or will it be too difficult to build?*

Quantum Mechanics can be used to improve computation. Our challenge: solving an exponentially difficult problem for a conventional computer---that of factoring a large number. As a prelude, there are standard tools of computation, universal gates and machines. These ideas are then applied first to classical, dissipation less computers and then to quantum computers. A schematic model of a quantum computer is described as well as some of the subtleties in its programming. The Shor algorithm for efficiently factoring numbers on a quantum computer is presented in two parts: the quantum procedure within the algorithm and the classical algorithm that calls the quantum procedure. The mathematical structure in factoring which makes the Shor algorithm possible has been studied widely by many authors.. .

Let us start by describing the problem at hand: factoring a number  $N$  into its prime factors (e.g., the number **51688** may be

$$2^3 \times 7 \times 13 \times 71$$

decomposed as ). A convenient way to quantify how quickly a particular algorithm may solve a problem is to ask how the number of steps to complete the algorithm scales with the size of the "input" the algorithm is fed.

$$\log N$$

For the factoring problem, this input is just the number  $N$  we wish to factor; hence the length of the input is . (The base of the logarithm is determined by our numbering system. Thus a base of **2** gives the length in binary; a base of **10** in decimal.) 'Reasonable' algorithms are ones which scale as some small-degree polynomial in the input size (with a degree of perhaps **2** or **3**).



On conventional computers the best known factoring algorithm runs

$$O(\exp[(64/9)^{1/3}(\ln N)^{1/3}(\ln \ln N)^{2/3}])$$

in  $\log N$  steps. This algorithm, therefore, scales exponentially with the

input size  $\log N$ .

For instance, in 1994 a 129 digit number (known as RSA129) was successfully factored using this algorithm on approximately 1600 workstations scattered around the world; the entire factorization took eight months. Using this to estimate the prefactor of the above exponential scaling, we find that it would take roughly 800,000 years to factor a 250 digit number with the same computer power; similarly, a 1000 digit number would require  $10^{25}$  years (significantly longer than the age of the universe). The difficulty of factoring large numbers is crucial for public-key cryptosystems, such as ones used by banks. There, such codes rely on the difficulty of factoring numbers with around 250 digits.

$$O((\log N)^{2+\epsilon})$$

Recently, an algorithm was developed for factoring numbers on a quantum computer which runs in steps where  $\epsilon$  is small. This is roughly quadratic in the input size, so factoring a 1000 digit number with such an algorithm would require only a few million steps. The implication is that public key cryptosystems based on factoring may be breakable.

To give you an idea of how this exponential improvement might be possible, we review an elementary quantum mechanical experiment that demonstrates where such power may lie hidden. The two-slit experiment is prototypic for observing quantum mechanical behavior: A source emits photons, electrons or other particles that arrive at a pair of slits. These particles undergo unitary evolution and finally measurement. We see an interference pattern, with both slits open, which wholly vanishes if either slit is covered. In some sense, the particles pass through both slits in parallel. If such unitary evolution were to represent a calculation (or an operation within a calculation) then the quantum system would be performing computations in parallel. Quantum parallelism comes for free. The output of this system would be given by the constructive interference among the parallel computations.

The traditional management skills of planning, organizing, directing and controlling are inadequate in the fast-paced, constantly changing, highly complex world of twenty-first century organizations. Concepts from quantum mechanics and chaos theory as metaphors for a new management skill set can enable managers to actualize more of their leadership potential. The seven quantum skills are ancient and futuristic, scientific and spiritual, simple and complex. Together they form a model that balances the traditional left-brain management skills with new skills that more fully utilize both hemispheres of the brain. As managers master these skills, they transcend the limitations of mechanistic, deterministic, reductionistic thinking and become authentic change masters, changing themselves and their organizations at depth.

### **Quantum computing (For details see Andrew Steane A. M. Steane, Reports on Progress in Physics, vol 61, pp 117-173 (1998)).**

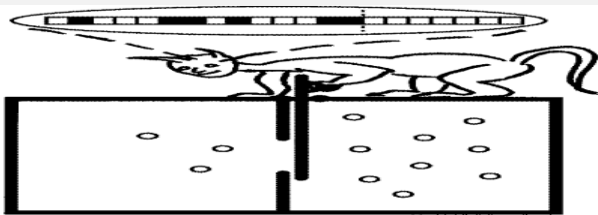
The subject of quantum computing brings together ideas from classical information theory, computer science, and quantum physics. This review aims to summarize not just quantum computing, but the whole subject of quantum information theory. It turns out that information theory and quantum mechanics fit together very well. In order to explain their relationship, the review begins with an introduction to classical information theory and computer science, including Shannon's theorem, error correcting codes, Turing machines and computational complexity. The principles of quantum mechanics are then outlined, and the EPR experiment described. The EPR-Bell correlations and quantum entanglement in general, form the essential new ingredient which distinguishes quantum from classical information theory, and, arguably, quantum from classical physics. Basic quantum information ideas are described, including key distribution, teleportation, data compression, quantum error correction, the universal quantum computer and quantum algorithms. The common theme of all these ideas is the use of quantum entanglement as a computational resource. Experimental methods for small quantum processors are briefly sketched, concentrating on ion traps, high Q cavities, and NMR. The review concludes with an outline of the main features of quantum information physics, and avenues for future research.

The science of physics seeks to ask, and find precise answers to, basic questions about why nature is as it is. Historically, the fundamental principles of physics have been concerned with questions such as "what are things made of?" and "why do things move as they do?" In his *Principia*, Newton gave very wide-ranging answers to some of these questions. By showing that the same mathematical equations could describe the motions of everyday objects and of planets, he showed that an everyday object such as a tea pot is made of essentially the *same sort of stuff* as a planet: the motions of both can be described in terms of their mass and the forces acting on them. Nowadays we would say that both move in such a way as to conserve energy and momentum. In this way, physics allows us to abstract from nature concepts such as energy or momentum which always obey fixed equations, although the same energy might be expressed in many different ways: for example, an electron in the large electron-positron collider at CERN, Geneva, can have the same kinetic energy as a slug on a lettuce leaf.

Another thing which can be expressed in many different ways is *information*. For example, the two statements "the quantum computer is very interesting" and "l'ordinateur quantique est tres interessant" have something in common, although they share no words. The thing they have in common is their *information* content. Essentially the same information could be expressed in many other ways, for example by substituting numbers for letters in a scheme such as  $a \rightarrow 97$ ,  $b \rightarrow 98$ ,  $c \rightarrow 99$  and so on, in which case the English version of the above statement becomes 116 104 101 32 113 117 97 110 116 117 109... . It is very significant that information can be expressed in different ways without losing its essential nature, since this leads to the possibility of the automatic manipulation of information: a machine need only be able to manipulate quite simple things like integers in order to do surprisingly powerful information processing, from document preparation to differential calculus, even to translating between human languages. We are familiar with this now, because of the ubiquitous computer, but even fifty years ago such a widespread significance of automated information processing was not foreseen.

However, there is one thing that all ways of expressing information must have in common: they all use real physical things to do the job. Spoken words are conveyed by air pressure fluctuations, written ones by arrangements of ink molecules on paper, even thoughts depend on neurons (Landauer 1991). The rallying cry of the information physicist is "no information without physical representation!" Conversely, the fact that information is insensitive to exactly how it is expressed, and can be freely translated from one form to another, makes it an obvious candidate for a fundamentally important role in physics, like energy and momentum and other such abstractions. However, until the second half of this century, the precise mathematical treatment of information, especially information processing, was undiscovered, so the significance of information in physics was only hinted at in concepts such as entropy in thermodynamics. It now appears that information may have a much deeper significance. Historically, much of fundamental physics has been concerned with discovering the fundamental particles of nature and the equations which describe their motions and interactions. It now appears that a different programme may be equally important: to discover the ways that nature allows, and prevents, *information* to be expressed and manipulated, rather than particles to move. For example, the best way to state exactly what can and cannot travel faster than light is to identify information as the speed-limited entity. In quantum mechanics, it is highly significant that the state vector must not contain, whether explicitly or implicitly, more information than can meaningfully be associated with a given system. Among other things this produces the wavefunction symmetry requirements which lead to Bose Einstein and Fermi Dirac statistics, the periodic structure of atoms, and so on.

Historically, the concept of information in physics does not have a clear-cut origin. An important thread can be traced if we consider the paradox of Maxwell's demon of 1871 (Fig. 1) (see also Brillouin 1956). Recall that Maxwell's demon is a creature that opens and closes a trap door between two compartments of a chamber containing gas, and pursues the subversive policy of only opening the door when fast molecules approach it from the right, or slow ones from the left. In this way the demon establishes a temperature difference between the two compartments without doing any work, in violation of the second law of thermodynamics, and consequently permitting a host of contradictions.



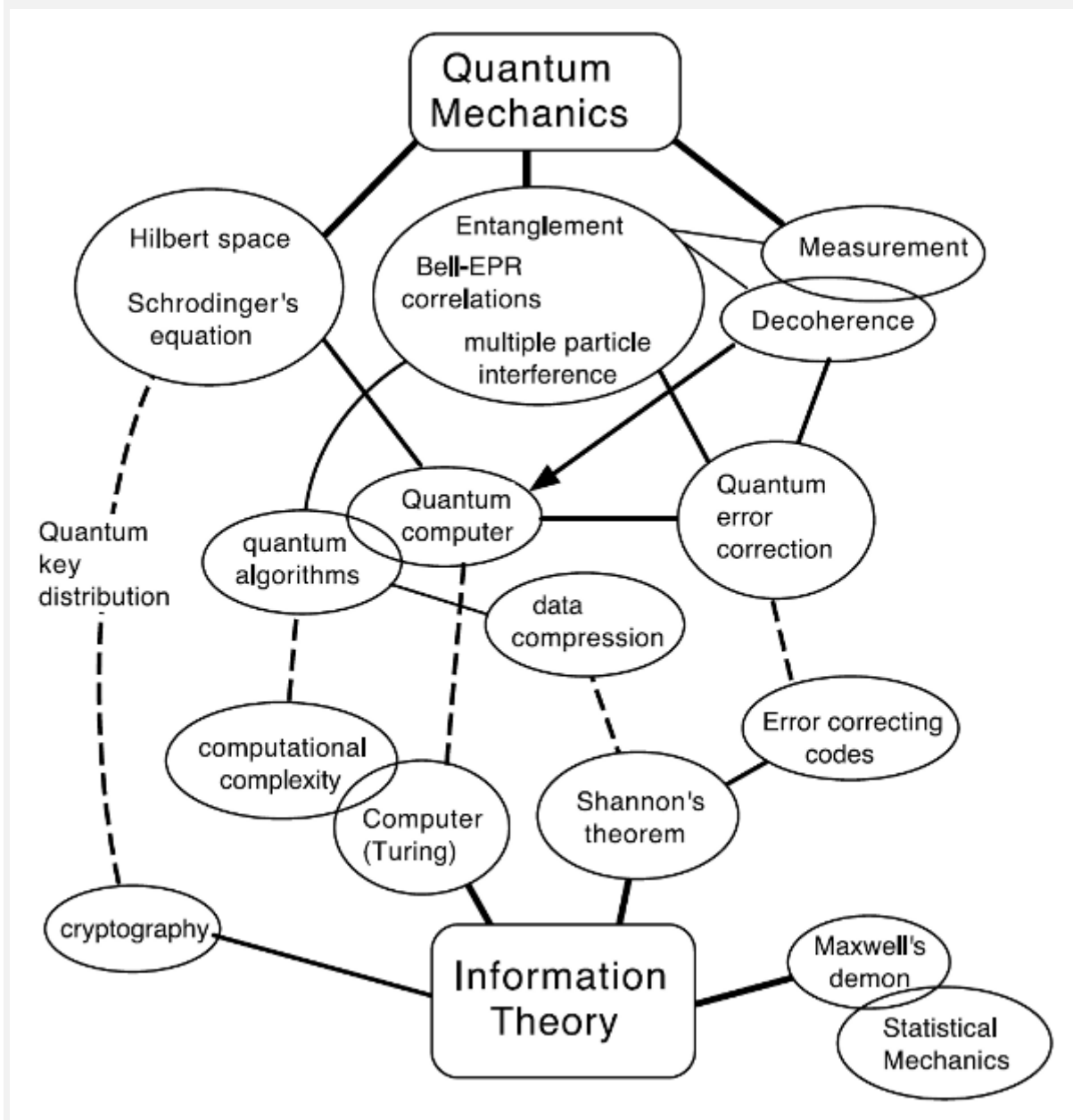
**Fig. 1.** Maxwell's demon. In this illustration the demon sets up a pressure difference by only raising the partition when more gas molecules approach it from the left than from the right. This can be done in a completely reversible manner, as long as the demon's memory stores the random results of its observations of the molecules. The demon's memory thus gets hotter. The irreversible step is not the acquisition of information, but the loss of information if the demon later clears its memory.

A number of attempts were made to exorcise Maxwell's demon (see Bennett 1987), such as arguments that the demon cannot gather information without doing work, or without disturbing (and thus heating) the gas, both of which are untrue. Some were tempted to propose that the 2nd law of thermodynamics could indeed be violated by the actions of an "intelligent being." It was not until 1929 that Leo Szilard made progress by reducing the problem to its essential components, in which the demon need merely identify whether a single molecule is to the right or left of a sliding partition, and its action allows a simple heat engine, called Szilard's engine, to be run. Szilard still had not solved the problem, since his analysis was unclear about whether or not the act of measurement, whereby the demon learns whether the molecule is to the left or the right, must involve an increase in entropy.

A definitive and clear answer was not forthcoming, surprisingly, until a further fifty years had passed. In the intermediate years digital computers were developed, and the physical implications of information gathering and processing were

carefully considered. The thermodynamic costs of elementary information manipulations were analyzed by Landauer and others during the 1960s (Landauer 1961, Keyes and Landauer 1970), and those of general computations by Bennett, Fredkin, Toffoli and others during the 1970s (Bennett 1973, Toffoli 1980, Fredkin and Toffoli 1982). It was found that almost anything can in principle be done in a reversible manner, i.e. with no entropy cost at all (Bennett and Landauer 1985). Bennett (1982) made explicit the relation between this work and Maxwell's paradox by proposing that the demon can indeed learn where the molecule is in Szilard's engine without doing any work or increasing any entropy in the environment, and so obtain useful work during one stroke of the engine. However, the information about the molecule's location must then be present in the demon's memory (Fig. 1). As more and more strokes are performed, more and more information gathers in the demon's memory. To complete a thermodynamic cycle, the demon must *erase* its memory, and it is during this erasure operation that we identify an increase in entropy in the environment, as required by the 2nd law. This completes the essential physics of Maxwell's demon; further subtleties are discussed by Zurek (1989), Caves (1990), and Caves, Unruh and Zurek (1990).

The thread we just followed was instructive, but to provide a complete history of ideas relevant to quantum computing is a formidable task. Our subject brings together what are arguably two of the greatest revolutions in twentieth-century science, namely quantum mechanics and information science (including computer science). The relationship between these two giants is illustrated in Figure 2.



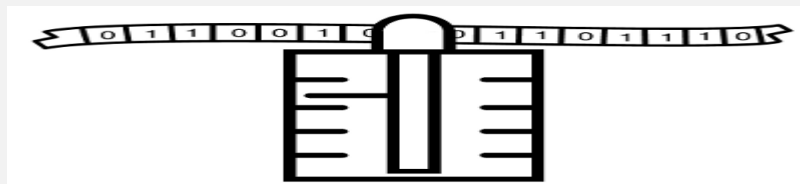
**Fig. 2.** Relationship between quantum mechanics and information theory. This diagram is not intended to be a definitive statement, the placing of entries being to some extent subjective, but it indicates many of the connections discussed in the article.

Classical information theory is founded on the definition of information. A warning is in order here. Whereas the theory tries to capture much of the normal meaning of the term 'information', it can no more do justice to the full richness of that term in everyday language than particle physics can encapsulate the everyday meaning of 'charm'. 'Information' for us will be an abstract term. Much of information theory dates back to seminal work of Shannon in the 1940's (Slepian 1974). The observation that information can be translated from one form to another is encapsulated and quantified in Shannon's noiseless coding theorem (1948), which quantifies the resources needed to store or transmit a given body of information. Shannon also considered the fundamentally important problem of communication in the presence of noise, and established Shannon's main theorem which is the central result of classical information theory. Error-free communication even in the presence of noise is achieved by means of 'error-correcting codes', and their study is a branch of mathematics in its own right. Indeed, the journal *IEEE Transactions on Information Theory* is almost totally taken up with the discovery and analysis of error-correction by coding. Pioneering work in this area was done by Golay (1949) and Hamming (1950).

The foundations of computer science were formulated at roughly the same time as Shannon's information theory, and this is no coincidence. The father of computer science is arguably Alan Turing (1912-1954), and its prophet is Charles Babbage (1791-1871). Babbage conceived of most of the essential elements of a modern computer, though in his day there was not the technology available to implement his ideas. A century passed before Babbage's Analytical Engine was improved upon when Turing described the Universal Turing Machine in the mid 1930s. Turing's genius (see Hodges 1983) was to clarify exactly what a calculating machine might be capable of, and to emphasise the role of programming, i.e. software, even more than Babbage had done. The giants on whose shoulders Turing stood in order to get a better view were chiefly the mathematicians David Hilbert and Kurt Gödel. Hilbert had emphasized between the 1890s and 1930s the importance of asking fundamental questions about the nature of mathematics. Instead of asking "is this mathematical proposition true?" Hilbert wanted to ask "is it the case that every mathematical proposition can in principle be proved or disproved?" This was unknown, but Hilbert's feeling, and that of most mathematicians, was that mathematics was indeed complete, so that conjectures such as Goldbach's (that every even number can be written as the sum of two primes) could be proved or disproved somehow, although the logical steps might be as yet undiscovered.

**Gödel destroyed this hope by establishing the existence of mathematical propositions which were undecidable, meaning that they could be neither proved nor disproved. The next interesting question was whether it would be easy to identify such propositions. Progress in mathematics had** always relied on the use of creative imagination, yet with hindsight mathematical proofs appear to be automatic, each step following inevitably from the one before. Hilbert asked whether this 'inevitable' quality could be captured by a 'mechanical' process. In other words, was there a universal mathematical method, which would establish the truth or otherwise of every mathematical assertion? After Gödel, Hilbert's problem was re-phrased into that of establishing decidability rather than truth, and this is what Turing sought to address.

In the words of Newman, Turing's bold innovation was to introduce 'paper tape' into symbolic logic. In the search for an automatic process by which mathematical questions could be decided, Turing envisaged a thoroughly mechanical device, in fact a kind of glorified typewriter (Fig. 3). The importance of the Turing machine (Turing 1936) arises from the fact that it is sufficiently complicated to address highly sophisticated mathematical questions, but sufficiently simple to be subject to detailed analysis. Turing used his machine as a theoretical construct to show that the assumed existence of a mechanical means to establish decidability leads to a contradiction. In other words, he was initially concerned with quite abstract mathematics rather than practical computation. However, by seriously establishing the idea of automating abstract mathematical proofs rather than merely arithmetic, Turing greatly stimulated the development of general purpose information processing. This was in the days when a "computer" was a person doing mathematics.



**Fig. 3.** The Turing Machine. This is a conceptual mechanical device which can be shown to be capable of efficiently simulating all classical computational methods. The machine has a finite set of internal states, and a fixed design. It reads one binary symbol at a time, supplied on a tape. The machine's action on reading a given symbol  $s$  depends only on that symbol and the internal state  $G$ . The action consists in overwriting a new symbol  $s'$  on the current tape location, changing state to  $G'$ , and moving the tape one place in direction  $d$  (left or right). The internal construction of the machine can therefore be specified by a finite fixed list of rules of the form  $(s, G \rightarrow s', G', d)$ . One special internal state is the 'halt' state: once in this state the machine ceases further activity. An input 'programme' on the tape is transformed by the machine into an output result printed on the tape.

Modern computers are neither Turing machines nor Babbage engines, though they are based on broadly similar principles, and their computational power is equivalent (in a technical sense) to that of a Turing machine. I will not trace their development here, since although this is a wonderful story, it would take too long to do justice to the many people involved.

Let us just remark that all of this development represents a great improvement in speed and size, but does not involve any change in the essential idea of what a computer is, or how it operates. Quantum mechanics raises the possibility of such a change, however.

Quantum mechanics is the mathematical structure which embraces, in principle, the whole of physics. We will not be directly concerned with gravity, high velocities, or exotic elementary particles, so the standard non-relativistic quantum mechanics will suffice. The significant feature of quantum theory for our purpose is not the precise details of the equations of motion, but the fact that they treat quantum amplitudes, or state vectors in a Hilbert space, rather than classical variables. It is this that allows new types of information and computing.

There is a parallel between Hilbert's questions about mathematics and the questions we seek to pose in quantum information theory. Before Hilbert, almost all mathematical work had been concerned with establishing or refuting particular hypotheses, but Hilbert wanted to ask what general type of hypothesis was even amenable to mathematical proof. Similarly, most research in quantum physics has been concerned with studying the evolution of specific physical systems, but we want to ask what general type of evolution is even conceivable under quantum mechanical rules.

The first deep insight into quantum information theory came with Bell's 1964 analysis of the paradoxical thought-experiment proposed by Einstein, Podolsky and Rosen (EPR) in 1935. Bell's inequality draws attention to the importance of correlations between separated quantum systems which have interacted (directly or indirectly) in the past, but which no longer influence one another. In essence his argument shows that the degree of correlation which can be present in such systems exceeds that which could be predicted on the basis of *any* law of physics which describes particles in terms of classical variables rather than quantum states. Bell's argument was clarified by Bohm (1951, also Bohm and Aharonov 1957) and by Clauser, Holt, Horne and Shimony (1969), and experimental tests were carried out in the 1970s (see Clauser and Shimony (1978) and references therein). Improvements in such experiments are largely concerned with preventing the possibility of any interaction between the separated quantum systems, and a significant step forward was made in the experiment of Aspect, Dalibard and Roger (1982), (see also Aspect 1991) since in their work any purported interaction would have either to travel faster than light, or possess other almost equally implausible qualities.

The next link between quantum mechanics and information theory came about when it was realized that simple properties of quantum systems, such as the unavoidable disturbance involved in measurement, could be put to practical use, in *quantum cryptography* (Wiesner 1983, Bennett *et al.* 1982, Bennett and Brassard 1984; for a recent review see Brassard and Crepeau 1996). Quantum cryptography covers several ideas, of which the most firmly established is quantum key distribution. This is an ingenious method in which transmitted quantum states are used to perform a very particular communication task: to establish at two separated locations a pair of identical, but otherwise random, sequences of binary digits, without allowing any third party to learn the sequence. This is very useful because such a random sequence can be used as a cryptographic key to permit secure communication. The significant feature is that the principles of quantum mechanics guarantee a type of conservation of quantum information, so that if the necessary quantum information arrives at the parties wishing to establish a random key, they can be sure it has not gone elsewhere, such as to a spy. Thus the whole problem of compromised keys, which fills the annals of espionage, is avoided by taking advantage of the structure of the natural world.

While quantum cryptography was being analyzed and demonstrated, the quantum computer was undergoing a quiet birth. Since quantum mechanics underlies the behaviour of all systems, including those we call classical ("even a screwdriver is quantum mechanical", Landauer (1995)), it was not obvious how to conceive of a distinctively quantum mechanical computer, i.e. one which did not merely reproduce the action of a classical Turing machine. Obviously it is not sufficient merely to identify a quantum mechanical system whose evolution could be interpreted as a computation; one must prove a much stronger result than this. Conversely, we know that classical computers can simulate, by their computations, the evolution of any quantum system... with one reservation: no classical process will allow one to prepare separated systems whose correlations break the Bell inequality. It appears from this that the EPR-Bell correlations are the quintessential quantum-mechanical property (Feynman 1982).

In order to think about computation from a quantum-mechanical point of view, the first ideas involved converting the action of a Turing machine into an equivalent reversible process, and then inventing a Hamiltonian which would cause a quantum system to evolve in a way which mimicked a reversible Turing machine. This depended on the work of Bennett (1973; see also Lecerf 1963) who had shown that a universal classical computing machine (such as Turing's) could be made reversible while retaining its simplicity. Benioff (1980, 1982) and others proposed such Turing-like Hamiltonians in the early 1980s. Although Benioff's ideas did not allow the full analysis of quantum computation, they showed that unitary quantum evolution is at least as powerful computationally as a classical computer.

A different approach was taken by Feynman (1982, 1986) who considered the possibility not of universal computation, but of universal *simulation* -- i.e. a purpose-built quantum system which could simulate the *physical behaviour* of any other. Clearly, such a simulator would be a universal computer too, since any computer must be a physical system. Feynman gave arguments which suggested that quantum evolution could be used to compute certain problems more efficiently than any

classical computer, but his device was not sufficiently specified to be called a computer, since he assumed that any interaction between adjacent two-state systems could be 'ordered', without saying how.

In 1985 an important step forward was taken by Deutsch. Deutsch's proposal is widely considered to represent the first blueprint for a quantum computer, in that it is sufficiently specific and simple to allow real machines to be contemplated, but sufficiently versatile to be a universal quantum simulator, though both points are debatable. Deutsch's system is essentially a line of two-state systems, and looks more like a register machine than a Turing machine (both are universal classical computing machines). Deutsch proved that if the two-state systems could be made to evolve by means of a specific small set of simple operations, then *any* unitary evolution could be produced, and therefore the evolution could be made to simulate that of any physical system. He also discussed how to produce Turing-like behaviour using the same ideas.

Deutsch's simple operations are now called quantum 'gates', since they play a role analogous to that of binary logic gates in classical computers. Various authors have investigated the minimal class of gates which are sufficient for quantum computation.

The two questionable aspects of Deutsch's proposal are its efficiency and realisability. The question of efficiency is absolutely fundamental in computer science, and on it the concept of 'universality' turns. A *universal* computer is one that not only can reproduce (i.e. simulate) the action of any other, but can do so without running too slowly. The 'too slowly' here is defined in terms of the number of computational steps required: this number must not increase exponentially with the size of the input. Deutsch's simulator is not universal in this strict sense, though it was shown to be efficient for simulating a wide class of quantum systems by Lloyd (1996). However, Deutsch's work has established the concepts of quantum networks (Deutsch 1989) and quantum logic gates, which are extremely important in that they allow us to think clearly about quantum computation.

In the early 1990's several authors (Deutsch and Jozsa 1992, Berthiaume and Brassard 1992, Bernstein and Vazirani 1993) sought computational tasks which could be solved by a quantum computer more efficiently than *any* classical computer. Such a quantum algorithm would play a conceptual role similar to that of Bell's inequality, in defining something of the essential nature of quantum mechanics. Initially only very small differences in performance were found, in which quantum mechanics permitted an answer to be found with certainty, as long as the quantum system was noise-free, where a probabilistic classical computer could achieve an answer 'only' with high probability. An important advance was made by Simon (1994), who described an efficient quantum algorithm for a (somewhat abstract) problem for which no efficient solution was possible classically, even by probabilistic methods. This inspired Shor (1994) who astonished the community by describing an algorithm which was not only efficient on a quantum computer, but also addressed a central problem in computer science: that of factorizing large integers.

Shor discussed both factorization and discrete logarithms, making use of a quantum Fourier transform method discovered by Coppersmith (1994) and Deutsch. Further important quantum algorithms were discovered by Grover (1997) and Kitaev (1995).

Just as with classical computation and information theory, once theoretical ideas about computation had got under way, an effort was made to establish the essential nature of quantum information -- the task analogous to Shannon's work. The difficulty here can be seen by considering the simplest quantum system, a two-state system such as a spin half in a magnetic field. The quantum state of a spin is a continuous quantity defined by two real numbers, so in principle it can store an infinite amount of classical information. However, a measurement of a spin will only provide a single two-valued answer (spin up/spin down) -- there is no way to gain access to the infinite information which appears to be there, therefore it is incorrect to consider the information content in those terms. This is reminiscent of the renormalization problem in quantum electrodynamics. How much information can a two-state quantum system store, then? The answer, provided by Jozsa and Schumacher (1994) and Schumacher (1995), is *one two-state system's worth!* Of course Schumacher and Jozsa did more than propose this simple answer, rather they showed that the two-state system plays the role in quantum information theory analogous to that of the bit in classical information theory, in that the quantum information content of *any* quantum system can be meaningfully measured as the minimum number of two-state systems, now called quantum bits or qubits, which would be needed to store or transmit the system's state with high accuracy.

Let us return to the question of realisability of quantum computation. It is an elementary, but fundamentally important, observation that the quantum interference effects which permit algorithms such as Shor's are extremely fragile: the quantum computer is ultra-sensitive to experimental noise and impression. It is not true that early workers were unaware of this difficulty; rather their first aim was to establish whether a quantum computer had any fundamental significance at all. Armed with Shor's algorithm, it now appears that such a fundamental significance is established, by the following argument: either nature does allow a device to be run with sufficient precision to perform Shor's algorithm for large integers (greater than, say, a googol which is 1 followed by 100 zeroes) or there are fundamental natural limits to precision in real systems. Both eventualities represent an important insight into the laws of nature.

At this point, ideas of quantum information and quantum computing come together. For, a quantum computer can be made much less sensitive to noise by means of a new idea which comes directly from the marriage of quantum mechanics with classical information theory, namely *quantum error correction*. Although the phrase 'error correction' is a natural one and was used with reference to quantum computers prior to 1996, it was only in that year that two important papers, of Calderbank and Shor, and independently Steane, established a general framework whereby quantum information processing can be used to combat a very wide class of noise processes in a properly designed quantum system. Much progress has since been made in generalizing these ideas (Knill and Laflamme 1997, Ekert and Macchiavello 1996, Bennett *et al.* 1996b, Gottesman 1996, Calderbank *et al.* 1997). An important development was the demonstration by Shor (1996) and Kitaev (1996) that correction can be achieved even when the corrective operations are themselves imperfect. Such methods lead to a general concept of 'fault tolerant' computing, of which a helpful review is provided by Preskill (1997).

If, as seems almost certain, quantum computation will only work in conjunction with quantum error correction, it appears that the relationship between quantum information theory and quantum computers is even more intimate than that between Shannon's information theory and classical computers. Error correction does not in itself guarantee accurate quantum computation, since it cannot combat all types of noise, but the fact that it is possible at all is a significant development.

A computer which only exists on paper will not actually perform any computations, and in the end the only way to resolve the issue of feasibility in quantum computer science is to build a quantum computer. To this end, a number of authors proposed computer designs based on Deutsch's idea, but with the physical details more fully worked out (Teich *et al.* 1988, Lloyd 1993, Berman *et al.* 1994, DiVincenzo 1995b). The great challenge is to find a sufficiently complex system whose evolution is nevertheless both coherent (i.e. unitary) and controllable. It is not sufficient that only some aspects of a system should be quantum mechanical, as in solid-state 'quantum dots', or that there is an implicit assumption of unfeasible precision or cooling, which is often the case for proposals using solid-state devices. Cirac and Zoller (1995) proposed the use of a linear ion trap, which was a significant improvement in feasibility, since heroic efforts in the ion trapping community had already achieved the necessary precision and low temperature in experimental work, especially the group of Wineland who demonstrated cooling to the ground state of an ion trap in the same year (Diedrich *et al.* 1989, Monroe *et al.* 1995). More recently, Gershenfeld and Chuang (1997) and Cory *et al.* (1996,1997) have shown that nuclear magnetic resonance (NMR) techniques can be adapted to fulfill the requirements of quantum computation, making this approach also very promising. Other recent proposals of Privman *et al.* (1997) and Loss and DiVincenzo (1997) may also be feasible.

As things stand, no quantum computer has been built, nor looks likely to be built in the author's lifetime, if we measure it in terms of Shor's algorithm, and ask for factoring of large numbers. However, if we ask instead for a device in which quantum information ideas can be explored, then only a few quantum bits are required, and this will certainly be achieved in the near future. Simple two-bit operations have been carried out in many physics experiments, notably magnetic resonance, and work with three to ten qubits now seems feasible. Notable recent experiments in this regard are those of Brune *et al.* (1994), Monroe *et al.* (1995b), Turchette *et al.* (1995) and Mattle *et al.* (1996).

\*

We will denote

By  $T_i(t)$ ,  $i = 36,37,38$ , the three parts of the **QUANTUM COMPUTER(INFORMATION) CORRESPONDING TO THE QUANTUM MECHANICAL BEHAVIOUR STRATIFIED IN THE CORRESPONDING CATEGORY**

By  $(a_i'')^{(7)}(T_{37}, t)$  ( $T_{37} \geq 0, t \geq 0$ ), the contribution of the **QUANTUM COMPUTER(INFORMATION) CORRESPONDING TO THE QUANTUM MECHANICAL BEHAVIOUR STRATIFIED IN THE CORRESPONDING CATEGORY**

1) By  $(-b_i'')^{(7)}(G_{36}, G_{37}, G_{38}, t) = -(b_i'')^{(7)}((G_{39}), t)$ , the contribution of the **QUANTUM COMPUTER(INFORMATION) CORRESPONDING TO THE QUANTUM MECHANICAL BEHAVIOUR STRATIFIED IN THE CORRESPONDING CATEGORY**

#### **STABILITY AND INSTABILITY OF QUANTUM INFORMATION:**

##### **Stability Diagram of a Few-Electron Triple Dot**

Individual and coupled quantum dots containing one or two electrons have been realized and are regarded as components for future quantum information circuits. Mapping out has been done several authors (see references) out experimentally the stability diagram of the few electron triple dot system, the electron configuration map as a function of the external tuning parameters, and reveal experimentally for the first time the existence of quadruple points, a signature of the three dots being in resonance. In the vicinity of these quadruple points we observe a duplication of charge transfer transitions related to charge and spin reconfigurations triggered by changes in the total electron occupation number. These(e&eb) results are relevant for future quantum mechanical engineering applications within both quantum information and quantum cellular automata (QCA) architectures. A comparison between single quantum dots and real atoms confirms both analogous and dissimilar properties. Atomic-like shell structure and Hund's rules govern both systems. The very different energy scale of the artificial atom, however, manifests itself in novel interaction phenomena which have no analogue in real atoms, such as singlet triplet transitions and spin texture arrangements of electrons. The tunability of these devices makes them promising candidates for future quantum information applications as well as for fundamental studies of quantum molecular effects, and for exploiting nanospintronic functionalities

**Stability, Gain, and Robustness in Quantum Feedback Networks (For details see C. D'Helon and M.R. James)**

**Problem of stability for quantum feedback networks, of quantum optics how stability of quantum feedback networks can be guaranteed using only simple gain inequalities for network components and algebraic relationships determined by the network.** Quantum feedback networks are shown to be stable if the loop gain is less than one—this is an extension of the famous small gain theorem of classical control theory. Authors' illustrate the simplicity and power of the small gain approach with applications to important problems of robust stability and robust stabilization. Quantum feedback networks, stability, input-output stability, robustness, stabilization, quantum optics. Are all interrelated. Stable operation is a fundamental pre-requisite for the proper functioning of any technological system. Instability can cause some system variables to grow in magnitude without bound (or at least saturate or oscillate), with detrimental effects on performance and even damage. Consequently, methods for stability analysis and design have played an important role in the development of classical technologies. A significant early example was Watt's steam engine governor in the 1780's (subsequently analyzed by Maxwell in 1868), Indeed, one of the chief applications of feedback (but by no means the only application) is to stabilize systems that would otherwise be unstable. A striking example of this is the X29 plane, which has forward swept wings and requires the use of a stabilizing feedback control system. However, feedback per se does not guarantee stability: indeed, feedback can be "degenerative or regenerative—either stabilizing or destabilizing",. In particular, when interconnections of stable components include components with active elements, instability can occur (such as when a microphone is placed too close to a loudspeaker). An additional requirement of considerable practical importance is that stable operation be maintained in the presence of uncertainty (e.g., due to model error and approximation, altered operating conditions, etc.) and noise—this is a basic robustness requirement. Feedback is increasingly being used in the design of new technologies that include quantum components, In fact, a wide range of quantum technologies can be considered as networks of quantum and classical components which include cascade and feedback interconnections. Since these networks may include components that are active, e.g. optical amplifiers or classical amplifiers, questions of network stability are of considerable importance. Quantum input-output theory started developing in the 1980's, however general methods for stability analysis and design for quantum networks still do not apply to Theory of Quantum information

#### **SOME STABILITY AND QUINTESSENTIAL REVIEWS ON QUANTUM COMPUTATION:**

##### **Loschmidt echoes (Tomaz Prosen, Thomas H. Seligman, Marko Znidaric**

In this paper authors' investigate on the theoretical approach to quantum Loschmidt echoes, i.e. various properties of the so called echo dynamics -- the composition of forward and backward time evolutions generated by two slightly different Hamiltonians, such as the state autocorrelation function (fidelity) and the purity of a reduced density matrix traced over a subsystem (purity fidelity). Our main theoretical result is linear response formalism, expressing the fidelity and purity fidelity in terms of integrated time autocorrelation function of the generator of the perturbation. Surprisingly, this relation predicts that **the decay** of fidelity is the slower the faster the decay of correlations. In particular for a static (time-independent) perturbation, and for non-ergodic and non-mixing dynamics where asymptotic decay of correlations is absent, a qualitatively different and faster decay of fidelity is predicted on a time scale  $1/\delta$  as opposed to mixing dynamics where the fidelity is found to decay exponentially on a time-scale  $1/\delta^2$ , where  $\delta$  is a strength of perturbation. A detailed discussion of a semi-classical regime of small effective values of Planck constant is given where classical correlation functions can be used to predict quantum fidelity decay. Note that the correct and intuitively expected classical stability behavior is recovered in the classical limit, as the perturbation and classical limits do not commute. The theoretical results are demonstrated numerically for two models, the quantized kicked top and the multi-level Jaynes Cummings model. **Method can for example be applied to the stability analysis of quantum computation and quantum information processing.**

##### **Towards scalable quantum information processing and quantum simulation with trapped ions(See Diedrich Leibfried)**

Quantum information processing and experiments towards Quantum Information Processing (QIP) and Quantum Simulation (QS) with trapped ions are discussed. Most requirements for QIP and QS have been demonstrated in this system, with two big challenges remaining: Improving operation fidelity and scaling up to larger numbers of qubits. The architecture pursued by the Ion Stage Group at NIST is based on quantum information stored in long lived internal (hyperfine) states of the ions. We investigate the use of laser beams and microwave fields to induce both single-qubit rotations and multi-qubit gates mediated by the Coulomb interaction between ions. Moving ions through a multi-zone trap architecture allows for keeping the number of ions per zone small, while sympathetic cooling with a second ion species can remove energy and entropy from the system. After an introduction to these elements, I will discuss the current status of experiments and some future perspectives for QIP and QS as well as for other applications based on trapped ions.

##### **Quantum circuits: from concept to future applications (Armand C. R. Niederberger)**

Current experimental progress in quantum optics and nanophotonics is establishing a solid base for fascinating future applications. We may soon be able to create integrated circuits of nanophotonic components for ultra-low power and ultra-high speed optical switching. My theory seminar presents the methods with which we are currently studying photonic circuit models and discusses examples of circuits for classical photonic logic. Nanophotonics in general and the advantages of using optical interconnects over electronic interconnects in particular. Second, I present our high-level quantum hardware description language which links graphical circuit design tools with recent mathematical developments to describe open quantum optical networks, thus enabling scientists and engineers to simulate quantum circuits without having to deal with



the details of quantum optics. Third, author shows how to perform design optimization on nanophotonics circuits using adjoint calculus. This method is based on the use of Lagrange multipliers and drastically **reduces** the number of computations in parameter optimization and stability analysis.

#### **Does a closed quantum system equilibrate? (Paolo Zanardi)**

Author discusses the issue of whether and how we can make sense of the notion of **equilibration("convergence" to equilibrium)** for a large but finite quantum system with only internal degrees of freedom. (i.e., closed). It is illustrated that the results on equilibration of the Loschmidt echo in nearly-critical quantum many-body systems evolving unitarily.

#### **Quantum wavelength conversion and transmission in opto mechanical systems( Lin Tian)**

Optomechanical systems with strong light-matter interaction can be explored as an interface between photon modes of distinct wavelengths, e.g. an optical mode and a microwave mode. Authors' study transient and adiabatic schemes for cavity state conversion and for photon transmission in the optomechanical system. Author states that his results can be applied to various applications in optical quantum information processing, such as photon pulse generation and state manipulation, quantum repeaters, and conversion of information between different photon modes.

#### **Protecting quantum gates from control noise (Constantin Brif)**

External controls are necessary to enact quantum logic operations, and the inevitable **control** noise will **result in** gate errors in a realistic quantum circuit. Author investigates the robustness of quantum gates to random noise in an optimal control field, by utilizing properties of the quantum control landscape that relates the physical objective (in the present case, the quantum gate fidelity) to the applied controls. An approximate result obtained for the statistical expectation value of the gate fidelity in the weak noise regime is shown to be in excellent agreement with direct Monte Carlo sampling over noise process realizations for fidelity values relevant for practical quantum information processing. Using this approximate result, they demonstrate that maximizing the robustness to additive/multiplicative white noise **is equivalent to** minimizing the total control time/fluence. Also, a genetic optimization algorithm is **used to identify controls** with improved robustness to colored noise.

#### **Exciton-Polariton Quantum Emulators (Na Young Kim)**

Microcavity exciton-polaritons are hybrid light-matter quasi-particles **arising from** the mixed states between cavity photons and quantum well excitons. The inherent light matter duality **provides** experimental advantages: the stimulated scattering among interacting particles and the small effective mass ( $\sim 10e-8$  times the hydrogen atom) **form** coherent condensate states at high temperatures (e.g. 4 K in GaAs and room temperature in GaN materials). In addition, the dynamics of exciton-polaritons are accessed by capturing the leaked photons out of the cavity due to the short lifetime. **Utilizing** coherence and open-dissipative nature of exciton-polariton condensates, we engineer a two-dimensional (2D) polariton-lattice system for investigating exotic quantum phase order. Via micro-photoluminescence measurements in both real and momentum spaces, authors' have observed d-orbital condensate states, vortex-antivortex phase order, massless Dirac dispersions in 2D square, honeycomb, and triangular lattices respectively. These results **demonstrate** that the polariton-lattice systems will be promising solid-state quantum emulators in the quest for better understanding strongly correlated materials and in the development of novel optoelectronic devices.

#### **Benchmarking and Protecting Adiabatic Quantum Computation(Daniel Lidar)**

How many bits does it take to track an open quantum system?

In general if one obtains information about an open quantum system by measuring its environment, that measurement will alter the future evolution of the system. However in the Markovian case this back-action is negligible and one can "track" the system i.e. assign it a (stochastically evolving) pure state at all times **without disturbing** its (deterministic) average evolution. In general this stochastic evolution **creates a** trajectory passing through infinitely many different pure states, even for a finite dimensional quantum system. Hence an infinite classical memory would be required to track such evolution. Here author shows that, for any ergodic master equation, there should **exist a** monitoring scheme (which in general must be adaptive) on the environment that can confine the system state to jumping between finitely many states, so that only a finitely large classical memory is required

#### **Using symmetries to understand molecular devices and magnetic ad-atoms on substrates**

Realizing a quantum transistor **built of** molecules or quantum dots has been one of the most ambitious challenges in nanotechnology. Even though remarkable progress has been made, being able to gate and **control** nanometer scale objects, as well to **interconnect** them to achieve scalability remains extremely difficult. Most experiments concern a single quantum dot or molecule, and they are made at ultra low temperature **to avoid** decoherence and tunnelling. Author proposes to use canonical transformations **to design** quantum devices that are **protected** by symmetry, and therefore, may be operational at high temperatures. We illustrate the idea with an example of a quantum transistor controlled by a gate electrode in a three terminal setup. They consider the **effects of** interactions, and we find that the same principles can be applied to design a device that could operate as an electrically controlled spin qubit. Author shows that similar but more sophisticated principles can be used to improve our understanding **of the effects of magnetic ad-atoms on substrates, such as** graphene.

#### **Quantum Annealing with Superconducting Flux Qubits(Mark Jonson)**

D-Wave Systems has implemented a processor based on Quantum Annealing, an algorithm for finding the ground state of a system of interacting spins. The technology is built on a superconducting chip composed of flux qubits that enable a quantum annealing algorithm, and digital components that apply programmable on-chip flux biases. In this presentation, author reviews Quantum Annealing, and then give a brief overview of the processor architecture. He also discusses a method for observing the system dynamics during the annealing process for a sample eight spin problem instance, and describe how the temperature dependence of these **dynamics(eb)** provides a signature of Quantum Annealing.\*

#### **QUANTUM MECHANICAL BEHAVIOUR AND QUANTUM COMPUTER**

**GOVERNING EQUATIONS**

**Quantum gravity** (QG) is the field of theoretical physics which attempts to develop scientific models that unify quantum mechanics (describing three of the four known fundamental interactions) with general relativity (describing the fourth, gravity). It is hoped that development of such a theory would unify into a single mathematical framework all fundamental interactions and to describe all known observable interactions in the universe, at both subatomic and cosmological scales. Such a theory of quantum gravity would yield the same experimental results as ordinary quantum mechanics in conditions of weak gravity (gravitational potentials much less than  $c^2$ ) and the same results as Einsteinian general relativity in phenomena at scales much larger than individual molecules (action much larger than reduced Planck's constant), but moreover be able to predict the outcome of situations where both quantum effects and strong-field gravity are important (at the Planck scale, unless large extra dimension conjectures are correct). If the theory of quantum gravity also achieves a grand unification of the other known interactions, it is referred to as a theory of everything (TOE).

Motivation for quantizing gravity comes from the remarkable success of the quantum theories of the other three fundamental interactions, and from experimental evidence suggesting that gravity can be made to show quantum effects. Although some quantum gravity theories such as string theory and other unified field theories (or 'theories of everything') attempt to unify gravity with the other fundamental forces, others such as loop quantum gravity make no such attempt; they simply quantize the gravitational field while keeping it separate from the other forces.

Observed physical phenomena can be described well by quantum mechanics or general relativity, without needing both. This can be thought of as due to an extreme separation of mass scales at which they are important. Quantum effects are usually important only for the "very small", that is, for objects no larger than typical molecules. General relativistic effects, on the other hand, show up mainly for the "very large" bodies such as collapsed stars. (Planets' gravitational fields, as of 2011, are well-described by linearised except for Mercury's perihelion precession; so strong-field effects—any effects of gravity beyond lowest nonvanishing order in  $\phi/c^2$ —have not been observed even in the gravitational fields of planets and main sequence stars). There is a lack of experimental evidence relating to quantum gravity, and classical physics adequately describes the observed effects of gravity over a range of 50 orders of magnitude of mass, i.e., for masses of objects from about  $10^{-23}$  to  $10^{30}$  kg.

Much of the difficulty in meshing these theories at all energy scales comes from the different assumptions that these theories make on how the universe works. Quantum field theory depends on particle fields embedded in the flat space-time of special relativity. General relativity models gravity as a curvature within space-time that changes as a gravitational mass moves. Historically, the most obvious way of combining the two (such as treating gravity as simply another particle field) ran quickly into what is known as the renormalization problem. In the old-fashioned understanding of renormalization, gravity particles would attract each other and adding together all of the interactions results in many infinite values which cannot easily be cancelled out mathematically to yield sensible, finite results. This is in contrast with quantum electrodynamics where, while the series still do not converge, **the interactions** sometimes evaluate to infinite results, but those are few enough in number to **be removable via renormalization**.

**Effective field theories**

Quantum gravity can be treated as an effective field theory. Effective quantum field theories come with some high-energy cutoff, beyond which we do not expect that the theory provides a good description of nature. The "infinities" then become large but finite quantities proportional to this finite cutoff scale, and correspond to processes that involve very high energies near the fundamental cutoff. These quantities can then be absorbed into an infinite collection **of coupling constants**, and at energies **well below** the fundamental cutoff of the theory, to any desired precision; only a finite number of **these coupling constants** need to be measured in order to make legitimate quantum-mechanical predictions. This same logic works just as well for the highly successful theory of low-energy pions as for quantum gravity. Indeed, the first quantum-mechanical corrections to graviton-scattering and Newton's law of gravitation have been explicitly computed (although they are so astronomically small that we may never be able to measure them). In fact, gravity is in many ways a much better quantum field theory than the Standard Model, since it appears to be valid all the way up to its cutoff at the Planck scale. (By comparison, the Standard Model is expected to start to break down above its cutoff at the much smaller scale of around 1000 GeV)

While confirming that quantum mechanics and gravity are indeed consistent at reasonable energies, it is clear that near or above the fundamental cutoff of our effective quantum theory of gravity (the cutoff is generally assumed to be of the order of the Planck scale), a new model of nature will be needed. Specifically, the problem of combining quantum mechanics and gravity becomes an issue only at very high energies, and may well require a totally new kind of model.

**Quantum gravity theory for the highest energy scales**

The general approach to deriving a quantum gravity theory that is valid at even the highest energy scales is to assume that such a theory will be simple and elegant and, accordingly, to study symmetries and other clues offered by current theories that might suggest ways **to combine them** into a comprehensive, **unified theory**. One problem with this approach is that it is unknown whether quantum gravity will actually conform to a simple and elegant theory, as it should resolve the dual conundrums of special relativity with regard to the uniformity of acceleration and gravity, and general relativity with regard to spacetime curvature.

Such a theory is required in order to understand problems involving the combination of very high energy and very small dimensions of space, such as the behavior of black holes, and the origin of the universe.

**Quantum mechanics and general relativity****Gravity Probe B (GP-B) has measured spacetime curvature near Earth to test related models in application of Einstein's general theory of relativity.****The graviton**

At present, one of the deepest problems in theoretical physics is harmonizing the theory of general relativity, which describes gravitation, and applies to large-scale structures (stars, planets, galaxies), with quantum mechanics, which describes the other three fundamental forces acting on the atomic scale. This problem must be put in the proper context, however. In particular, contrary to the popular claim that quantum mechanics and general relativity are fundamentally incompatible, one can demonstrate that the structure of general relativity essentially follows inevitably from the quantum mechanics of interacting theoretical spin-2 massless particles (called gravitons).

While there is no concrete proof of the existence of gravitons, quantized theories of matter may necessitate their existence<sup>1</sup> Supporting this theory is the observation that all fundamental forces except gravity have one or more known messenger particles, leading researchers to believe that at least one most likely does exist; they have dubbed these hypothetical particles *gravitons*. Many of the accepted notions of a unified theory of physics since the 1970s, including string theory, superstring theory, M-theory, loop quantum gravity, all assume, and to some degree depend upon, the existence of the graviton. Many researchers view the detection of the graviton as vital to validating their work.

**The dilaton**

The dilaton made its first appearance in Kaluza–Klein theory, a five-dimensional theory that combined gravitation and electromagnetism. Generally, it appears in string theory. More recently, it has appeared in the lower-dimensional many-bodied gravity problem based on the field theoretic approach of Roman Jackiw. The impetus arose from the fact that complete analytical solutions for the metric of a covariant  $N$ -body system have proven elusive in General Relativity. To simplify the problem, the number of dimensions was lowered to  $(I+1)$  namely one spatial dimension and one temporal dimension. **This model problem, known as  $R=T$  theory (as opposed to the general  $G=T$  theory)** was amenable to exact solutions in terms of a generalization of the Lambert  $W$  function. It was also found that the field equation governing the dilaton (derived from differential geometry) was the Schrödinger equation and consequently amenable to quantization. Thus, one had a theory which combined gravity, quantization and even the electromagnetic interaction, promising ingredients of a fundamental physical theory. It is worth noting that the outcome revealed a previously unknown and already existing *natural link* between general relativity and quantum mechanics. However, this theory needs to be generalized in  $(2+1)$  or  $(3+1)$  dimensions although, in principle, the field equations are amenable to such generalization as shown with the inclusion of a one-graviton process and yielding the correct Newtonian limit in  $d$  dimensions if a dilaton is included. However, it is not yet clear what the full field equation will govern the dilaton in higher dimensions. This is further complicated by the fact that gravitons can propagate in  $(3+1)$  dimensions and consequently that would imply gravitons and dilatons exist in the real world. Moreover, detection of the dilaton is expected to be even more elusive than the graviton. However, since this approach allows for the combination of gravitational, electromagnetic and quantum effects, their coupling could potentially lead to a means of vindicating the theory, through cosmology and perhaps even *experimentally*.

**Nonrenormalizability of gravity**

General relativity, like electromagnetism, is a classical field theory. One might expect that, as with electromagnetism, there should be a corresponding quantum field theory. However, gravity is perturbatively nonrenormalizable. For a quantum field theory to be well-defined according to this understanding of the subject, it must be asymptotically free or asymptotically safe. The theory must be characterized by a choice of *finitely many* parameters, which could, in principle, be set by experiment. For example, in quantum electrodynamics, these parameters are the charge and mass of the electron, as measured at a particular energy scale. On the other hand, in quantizing gravity, there are *infinitely many independent parameters* (counterterm coefficients) needed to define the theory. For a given choice of those parameters, one could make sense of the theory, but since we can never do infinitely many experiments to fix the values of every parameter, we do not have a meaningful physical theory:

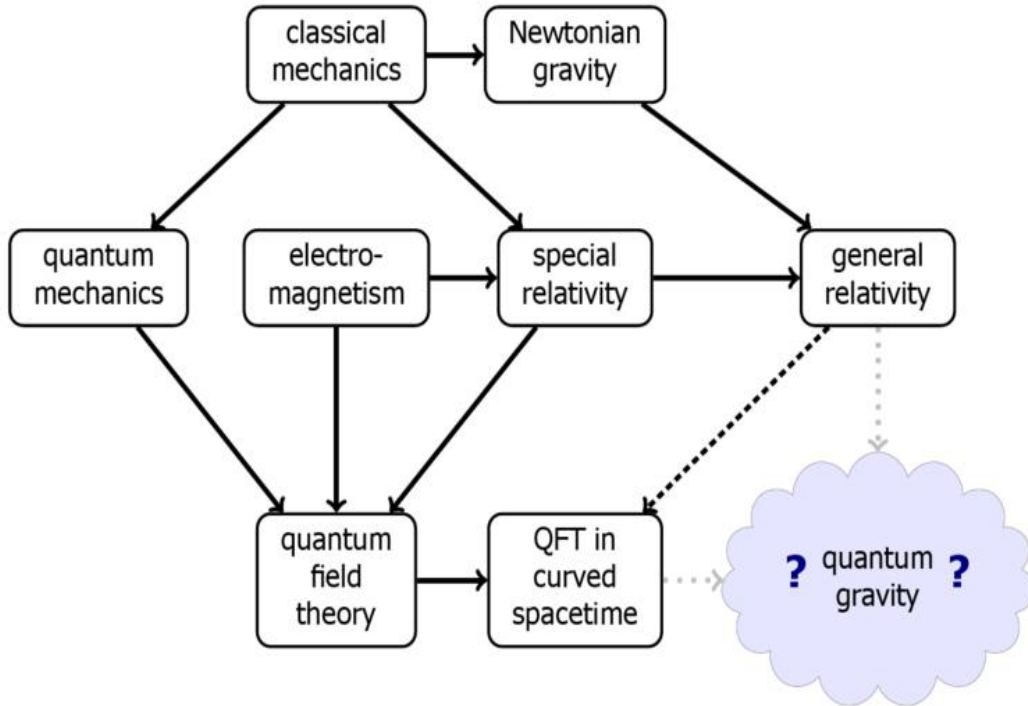
- At low energies, the logic of the renormalization group tells us that, despite the unknown choices of these infinitely many parameters, quantum gravity will reduce to the usual Einstein theory of general relativity.
- On the other hand, if we could probe very high energies where quantum effects take over, then *every one* of the infinitely many unknown parameters would begin to matter, and we could make no predictions at all.

As explained below, there is a way around this problem by treating QG as an effective field theory.

Any meaningful theory of quantum gravity that makes sense and is predictive at all energy scales must have some deep principle that reduces the infinitely many unknown parameters to a finite number that can then be measured.

- One possibility is that normal perturbation theory is not a reliable guide to the renormalizability of the theory, and that there really *is* a UV fixed point for gravity. Since this is a question of non-perturbative quantum field theory, it is difficult to find a reliable answer, but some people still pursue this option.
- Another possibility is that there are new symmetry principles that constrain the parameters and reduce them to a finite set. This is the route taken by string theory, where all of the excitations of the string essentially manifest themselves as new symmetries.

**QUANTUM GRAVITY AS AN EFFECTIVE FIELD THEORY:**



THE precise manner in which quantum-mechanical behaviour at the microscopic level underlies classical behaviour at the macroscopic level remains unclear, despite seventy years of theoretical investigation. Experimentally, the crossover between these regimes can be explored by looking for signatures of quantum-mechanical behaviour—such as tunneling—in macroscopic systems. Magnetic systems (such as small grains, spin glasses and thin films) are often investigated in this way because transitions between different magnetic states can be closely monitored. But transitions between states can be induced by thermal fluctuations, as well as by tunnelling, and definitive identification of macroscopic tunnelling events in these complex systems is therefore difficult. Here we report the results of low-temperature experiments on a single crystal composed of super-paramagnetic manganese clusters (Mn<sub>12</sub>-ac), which clearly demonstrate the existence of quantum-mechanical tunnelling of the bulk magnetization. In an applied magnetic field, the magnetization shows hysteresis loops with a distinct 'staircase' structure: the steps occur at values of the applied field where the energies of different collective spin states of the manganese clusters coincide. At these special values of the field, relaxation from one spin state to another is enhanced above the thermally activated rate by the action of resonant quantum-mechanical tunnelling. These observations corroborate the results of similar experiments performed recently on a system of oriented crystallites made from a powdered sample.

**Young's double slit experiment - Quantum mechanical behaviour**

Young's double slit experiment represents the observation of an interference pattern consistent with a wave nature for objects that traverse the apparatus. This is emphasized in the figures 3 and 4

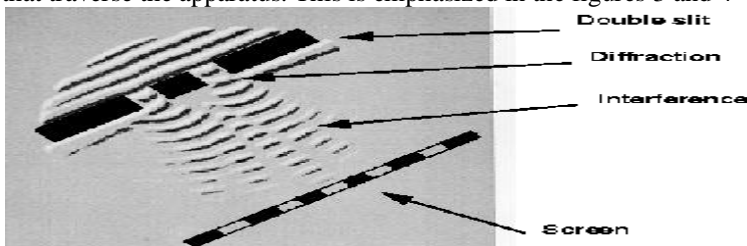


Figure 3: Young's double slit experiment with water waves.

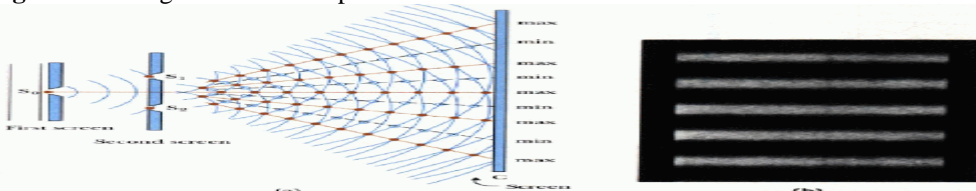
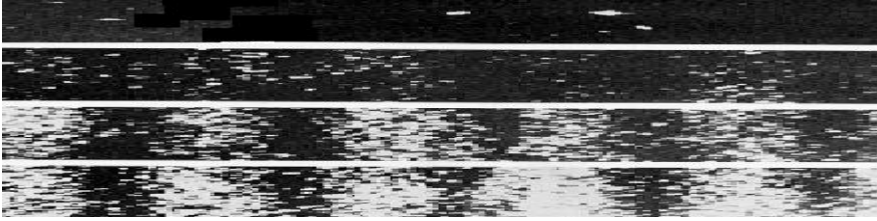


Figure 4: Young's double slit experiment, performed with either light or electrons leads to an interference pattern.

The diffraction and interference effects appear at first sight to be due to the beam of electrons, interfering with each other. However, the interference pattern still results even if only one electron traverses the apparatus at a time. In this case, the pattern is built up gradually from the statistically correlated impacts on many electrons arriving independently at the detection system. This effect is evidenced in figure 5.



**Figure 5:** Young's double slit experiment, performed with electrons in such a way that only one electron is present in the apparatus at any one time.

We see that the electron must in some sense pass through both slits at once and then interfere with itself as it travels towards the detector. Young's double slit experiment has been performed many times in many different ways with electrons (and other particles). The inescapable conclusion is that each electron must be delocalized in both time and space over the apparatus.

Considering the analogies between Young's double slit experiment performed with water waves, electro-magnetic waves and with electrons, and considering the material of the foregoing section, we can now specify some properties for a new theory of mechanics, termed *wave mechanics*.

$$\Psi(\mathbf{r}, t)$$

1. There must be a wave function describing some fundamental property of matter. (We leave further physical interpretation of the wave function open to continuous debate through-out the course.)
2. As with the intensity pattern on the screen for water waves and light waves, the "observable" associated with the wave function indicating the probability of detection of the particle will be the intensity (square of the amplitude) of

$$|\Psi(\mathbf{r}, t)|^2 = \Psi(\mathbf{r}, t)\Psi^*(\mathbf{r}, t)$$

the wave. Mathematically, this is

$$\lambda = h/p$$

3. The wavelength in the wave function will be related to the de Broglie wavelength of the particle .
4. We would like to be able to proceed to develop a differential equation which would specify the time evolution of the wave function, consistent with the conservation of energy and momentum of physical systems.
5. The quantization of energy should arise in a natural way from this formalism, just as it does for other bounded systems that support oscillations.
6. Then we must develop the formalism to enable other observables than simple the probability of detection "position of the particle" to be determined. Examples would be the energy and momentum of the particle.
7. Note the judicious use of the word observable. The actual wave function itself has never yet been observed.

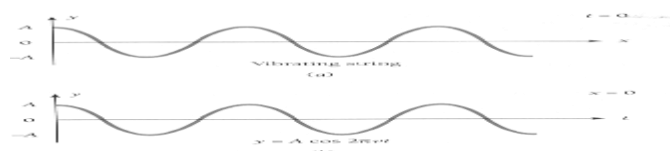
It is clear that an improved understanding of waves in physics is now necessary. To this end, some results from wave motion in physics are reviewed.

A transverse wave train, travelling on a string in the  $+x$ -direction (as in figure 6) may be represented by

$$y(x, t) = A \cos 2\pi\nu \left( t - \frac{x}{v_p} \right) \tag{8}$$

where  $\nu$  is the frequency of the wave and  $v_p$  is its *phase velocity*. The phase velocity is the velocity with which a point on the wave maintaining the same phase appears to be transported.

$$v_p = \lambda\nu \tag{9}$$



+x

**Figure 6:** A transverse wave train, travelling on a string in the +x-direction. It is more common to define the angular frequency (frequency in radians/sec rather than cycles/sec)

$$\omega = 2\pi\nu \quad (10)$$

and the wavenumber

$$k = \frac{2\pi}{\lambda} \quad (\text{by definition})$$

$$= \frac{\omega}{v_p} \quad (\text{substitution with the last two equations})$$

$$= \frac{|p|/\hbar}{\hbar} \quad (\text{using de Broglie's relation}) \quad (11)$$

$$\hbar = h/2\pi$$

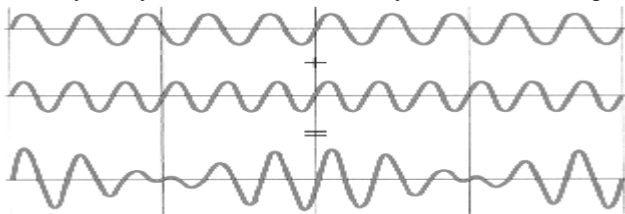
where  $\hbar = h/2\pi$ . The wave equation for the wave moving in the +x-direction can now be written :

$$y(x, t) = A \cos(\omega t - kx) \quad (12)$$

In three dimensions, this equation would be

$$y(\mathbf{r}, t) = A \cos(\omega t - \mathbf{k} \cdot \mathbf{r}) \quad (13)$$

It turns out that in quantum mechanics, a particle will be described as a *wave packet*. By this, we mean a group of (usually infinitely many) waves which mutually interfere, creating a new wave form which exhibits some localization.



**Figure 7:** Two waves of nearly equal wave number combined coherently.

$$k \pm \Delta k$$

This can be illustrated by considering only two waves, of nearly equal wave number ( $k \pm \Delta k$ ), and combining them coherently as in figure 7. Clearly, performing this process with many more waves would achieve a better localization of the wave packet, as illustrated in figure 8.



**Figure 8:** Localisation of a wave packet by combination of many waves.

We find that

$$y = A \cos[(\omega + \Delta\omega/2)t - (k + \Delta k/2)x] + A \cos[(\omega - \Delta\omega/2)t - (k - \Delta k/2)x]$$

... use trigonometric double angle formulae

$$= 2A \cos(\omega t - kx) \cos\left(\frac{\Delta\omega}{2}t - \frac{\Delta k}{2}x\right) \quad (14)$$

The combined wave train exhibits the phenomenon of "beats" as shown in figure 7 where an amplitude modulation envelope is superimposed on the original wave train. The amplitude modulation envelope will clearly have the frequency  $\nu_g = \Delta\omega/\Delta k$ , wave

number  $\Delta k$  and hence the velocity

$$v_g = \Delta\omega/\Delta k$$

The velocity of the localised group of waves (or beat) is known as the group velocity.

$$v_g = v \quad \text{and} \quad v_p = c^2/v \quad *(15)$$

This must be compared to the phase velocity of each wave train making up the wave packet

$$v_p = \frac{\omega}{k} \quad *(16)$$

$$v_g = v \quad \text{and} \quad v_p = c^2/v \quad *(17)$$

Thus the de Broglie wave group associated with a moving particle travels with the same velocity as the particle. The de Broglie waves in the packet have superluminal velocities, however, these do not represent the motion of the particle, and therefore the special relativity is not violated.

Finally, the form of the wave equation, yielding the above expression for a wave train is

$$y(x, t) = -iA \sin(\omega t - kx) \quad *(18)$$

We can verify that  $y(x, t) = Ae^{-i(\omega t - kx)}$  is indeed a solution of the wave equation.

$$y(x, t) = -iA \sin(\omega t - kx)$$

Clearly  $y(x, t) = Ae^{-i(\omega t - kx)}$  is also a solution of the wave equation. It follows that

$$y(x, t) = Ae^{-i(\omega t - kx)} \quad *(19)$$

is also a solution of the wave equation. This can be verified by direct substitution, or by exploiting the fact that any linear combination of solutions of the wave equation is itself a solution of the wave equation

$$e^{-i\theta} = \cos\theta - i \sin\theta$$

(Hint: .)

In fact a second order differential equation should have two constants of integration, which are determined by the boundary conditions of the specific problem. We can show that for the equation above, we could write

$$y(x, t) = A \cos(\omega t - kx) + B \sin(\omega t - kx) \quad *(20)$$

or

$$y(x, t) = C e^{-i(\omega t - kx)} + D e^{+i(\omega t - kx)} \quad *(21)$$

Also find the relationship between the two sets of coefficients. We will use the former set when discussing standing waves (like a guitar string), and the latter set when discussing travelling waves (like a ripple on a large pond)..

The differential system of this model is now \*

$$\frac{dG_{36}}{dt} = (a_{36})^{(7)} G_{37} - [(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}, t)] G_{36} \quad *19$$

$$\frac{dG_{37}}{dt} = (a_{37})^{(7)} G_{36} - [(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37}, t)] G_{37} \quad *20$$

$$\frac{dG_{38}}{dt} = (a_{38})^{(7)} G_{37} - [(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}, t)] G_{38} \quad *21$$

$$\frac{dT_{36}}{dt} = (b_{36})^{(7)} T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}((G_{39}), t)] T_{36} \quad *22$$

$$\frac{dT_{37}}{dt} = (b_{37})^{(7)} T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}((G_{39}), t)] T_{37}$$

\*23

$$\frac{dT_{38}}{dt} = (b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}((G_{39}), t)]T_{38} \quad *24$$

$$+(a''_{36})^{(7)}(T_{37}, t) = \text{First augmentation factor}^*$$

$$-(b'_{36})^{(7)}((G_{39}), t) = \text{First detritions factor}^*$$

Where we suppose\*

$$(A) \quad (a_i)^{(7)}, (a'_i)^{(7)}, (a''_i)^{(7)}, (b_i)^{(7)}, (b'_i)^{(7)}, (b''_i)^{(7)} > 0, \quad i, j = 36, 37, 38$$

(B) The functions  $(a''_i)^{(7)}, (b''_i)^{(7)}$  are positive continuous increasing and bounded.**Definition of**  $(p_i)^{(7)}, (r_i)^{(7)}$ :

$$(a''_i)^{(7)}(T_{37}, t) \leq (p_i)^{(7)} \leq (\hat{A}_{36})^{(7)} \quad *25$$

$$(b''_i)^{(7)}(G, t) \leq (r_i)^{(7)} \leq (b'_i)^{(7)} \leq (\hat{B}_{36})^{(7)} \quad *26$$

$$(C) \quad \lim_{T_2 \rightarrow \infty} (a''_i)^{(7)}(T_{37}, t) = (p_i)^{(7)}$$

$$\lim_{G \rightarrow \infty} (b''_i)^{(7)}((G_{39}), t) = (r_i)^{(7)}$$

**Definition of**  $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}$  : \*27Where  $(\hat{A}_{36})^{(7)}, (\hat{B}_{36})^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}$  are positive constants and  $i = 36, 37, 38$  \*28

They satisfy Lipschitz condition:

$$|(a''_i)^{(7)}(T'_{37}, t) - (a''_i)^{(7)}(T_{37}, t)| \leq (\hat{k}_{36})^{(7)}|T'_{37} - T_{37}|e^{-(\hat{M}_{36})^{(7)}t} \quad *29$$

$$|(b''_i)^{(7)}((G_{39})', t) - (b''_i)^{(7)}((G_{39}), (T_{39}))| < (\hat{k}_{36})^{(7)}\|((G_{39})' - (G_{39}))\|e^{-(\hat{M}_{36})^{(7)}t} \quad *30$$

With the Lipschitz condition, we place a restriction on the behavior of functions  $(a''_i)^{(7)}(T'_{37}, t)$  and  $(a''_i)^{(7)}(T_{37}, t) \cdot (T'_{37}, t)$  and  $(T_{37}, t)$  are points belonging to the interval  $[(\hat{k}_{36})^{(7)}, (\hat{M}_{36})^{(7)}]$ . It is to be noted that  $(a''_i)^{(7)}(T_{37}, t)$  is uniformly continuous. In the eventuality of the fact, that if  $(\hat{M}_{36})^{(7)} = 7$  then the function  $(a''_i)^{(7)}(T_{37}, t)$ , the **first augmentation coefficient would be continuous**.\*

**Definition of**  $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$  :(D)  $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}$ , are positive constants

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 1 \quad *31$$

**Definition of**  $(\hat{P}_{36})^{(7)}, (\hat{Q}_{36})^{(7)}$  :(E) There exists two constants  $(\hat{P}_{36})^{(7)}$  and  $(\hat{Q}_{36})^{(7)}$  which together with  $(\hat{M}_{36})^{(7)}, (\hat{k}_{36})^{(7)}, (\hat{A}_{36})^{(7)}$  and  $(\hat{B}_{36})^{(7)}$  and the constants  $(a_i)^{(7)}, (a'_i)^{(7)}, (b_i)^{(7)}, (b'_i)^{(7)}, (p_i)^{(7)}, (r_i)^{(7)}, i = 36, 37, 38$ , satisfy the inequalities

$$\frac{1}{(\hat{M}_{36})^{(7)}} [(a_i)^{(7)} + (a'_i)^{(7)} + (\hat{A}_{36})^{(7)} + (\hat{P}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1$$

$$\frac{1}{(\hat{M}_{36})^{(7)}} [(b_i)^{(7)} + (b'_i)^{(7)} + (\hat{B}_{36})^{(7)} + (\hat{Q}_{36})^{(7)} (\hat{k}_{36})^{(7)}] < 1 \quad *32 \quad *33$$

**Theorem 7:** if the conditions (A)-(E) above are fulfilled, there exists a solution satisfying the conditions**Definition of**  $G_i(0), T_i(0)$  :

$$G_i(t) \leq (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)}t}, \quad \boxed{G_i(0) = G_i^0 > 0}$$

$$T_i(t) \leq (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)}t}, \quad \boxed{T_i(0) = T_i^0 > 0}$$



**Proof:**

Consider operator  $\mathcal{A}^{(7)}$  defined on the space of sextuples of continuous functions  $G_i, T_i: \mathbb{R}_+ \rightarrow \mathbb{R}_+$  which satisfy

$$G_i(0) = G_i^0, T_i(0) = T_i^0, G_i^0 \leq (\hat{P}_{36})^{(7)}, T_i^0 \leq (\hat{Q}_{36})^{(7)}, \tag{*34}$$

$$0 \leq G_i(t) - G_i^0 \leq (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)}t} \tag{*35}$$

$$0 \leq T_i(t) - T_i^0 \leq (\hat{Q}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)}t} \tag{*36}$$

By

$$\bar{G}_{36}(t) = G_{36}^0 + \int_0^t [(a_{36})^{(7)} G_{37}(s_{(36)}) - ((a'_{36})^{(7)} + a''_{36})^{(7)} (T_{37}(s_{(36)}), s_{(36)})] G_{36}(s_{(36)}) ds_{(36)} \tag{*37}$$

$$\bar{G}_{37}(t) = G_{37}^0 + \int_0^t [(a_{37})^{(7)} G_{36}(s_{(36)}) - ((a'_{37})^{(7)} + (a''_{37})^{(7)} (T_{37}(s_{(36)}), s_{(36)}))] G_{37}(s_{(36)}) ds_{(36)} \tag{*38}$$

$$\bar{G}_{38}(t) = G_{38}^0 + \int_0^t [(a_{38})^{(7)} G_{37}(s_{(36)}) - ((a'_{38})^{(7)} + (a''_{38})^{(7)} (T_{37}(s_{(36)}), s_{(36)}))] G_{38}(s_{(36)}) ds_{(36)} \tag{*39}$$

$$\bar{T}_{36}(t) = T_{36}^0 + \int_0^t [(b_{36})^{(7)} T_{37}(s_{(36)}) - ((b'_{36})^{(7)} - (b''_{36})^{(7)} (G(s_{(36)}), s_{(36)}))] T_{36}(s_{(36)}) ds_{(36)} \tag{*40}$$

$$\bar{T}_{37}(t) = T_{37}^0 + \int_0^t [(b_{37})^{(7)} T_{36}(s_{(36)}) - ((b'_{37})^{(7)} - (b''_{37})^{(7)} (G(s_{(36)}), s_{(36)}))] T_{37}(s_{(36)}) ds_{(36)} \tag{*41}$$

$$\bar{T}_{38}(t) = T_{38}^0 + \int_0^t [(b_{38})^{(7)} T_{37}(s_{(36)}) - ((b'_{38})^{(7)} - (b''_{38})^{(7)} (G(s_{(36)}), s_{(36)}))] T_{38}(s_{(36)}) ds_{(36)}$$

Where  $s_{(36)}$  is the integrand that is integrated over an interval  $(0, t)$  \*42

(a) The operator  $\mathcal{A}^{(7)}$  maps the space of functions satisfying 37,35,36 into itself .Indeed it is obvious that

$$G_{36}(t) \leq G_{36}^0 + \int_0^t [(a_{36})^{(7)} (G_{37}^0 + (\hat{P}_{36})^{(7)} e^{(\hat{M}_{36})^{(7)}s_{(36)}})] ds_{(36)} = (1 + (a_{36})^{(7)}t)G_{37}^0 + \frac{(a_{36})^{(7)}(\hat{P}_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} (e^{(\hat{M}_{36})^{(7)}t} - 1) \tag{*43}$$

From which it follows that

$$(G_{36}(t) - G_{36}^0) e^{-(\hat{M}_{36})^{(7)}t} \leq \frac{(a_{36})^{(7)}}{(\hat{M}_{36})^{(7)}} \left[ ((\hat{P}_{36})^{(7)} + G_{37}^0) e^{-\left(\frac{(\hat{P}_{36})^{(7)} + G_{37}^0}{G_{37}^0}\right)} + (\hat{P}_{36})^{(7)} \right]$$

$(G_i^0)$  is as defined in the statement of theorem 1 \*44

Analogous inequalities hold also for  $G_{37}, G_{38}, T_{36}, T_{37}, T_{38}$

It is now sufficient to take  $\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}}, \frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} < 7$  and to choose

$(\hat{P}_{36})^{(7)}$  and  $(\hat{Q}_{36})^{(7)}$  large to have

$$\frac{(a_i)^{(7)}}{(\hat{M}_{36})^{(7)}} \left[ (\hat{P}_{36})^{(7)} + ((\hat{P}_{36})^{(7)} + G_j^0) e^{-\left(\frac{(\hat{P}_{36})^{(7)} + G_j^0}{G_j^0}\right)} \right] \leq (\hat{P}_{36})^{(7)} \tag{*45}$$

$$\frac{(b_i)^{(7)}}{(\hat{M}_{36})^{(7)}} \left[ ((\hat{Q}_{36})^{(7)} + T_j^0) e^{-\left(\frac{(\hat{Q}_{36})^{(7)} + T_j^0}{T_j^0}\right)} + (\hat{Q}_{36})^{(7)} \right] \leq (\hat{Q}_{36})^{(7)} \tag{*46}$$

In order that the operator  $\mathcal{A}^{(7)}$  transforms the space of sextuples of functions  $G_i, T_i$  satisfying 37,35,36 into itself

The operator  $\mathcal{A}^{(7)}$  is a contraction with respect to the metric

$$d\left( ((G_{39})^{(1)}, (T_{39})^{(1)}), ((G_{39})^{(2)}, (T_{39})^{(2)}) \right) = \sup_i \max_{t \in \mathbb{R}_+} |G_i^{(1)}(t) - G_i^{(2)}(t)| e^{-(\hat{M}_{36})^{(7)}t}, \max_{t \in \mathbb{R}_+} |T_i^{(1)}(t) - T_i^{(2)}(t)| e^{-(\hat{M}_{36})^{(7)}t}$$

Indeed if we denote

**Definition of**  $(\widetilde{G}_{39}), (\widetilde{T}_{39}) :$

$$((\widetilde{G}_{39}), (\widetilde{T}_{39})) = \mathcal{A}^{(7)}((G_{39}), (T_{39}))$$

It results

$$|\widetilde{G}_{36}^{(1)} - \widetilde{G}_i^{(2)}| \leq \int_0^t (a_{36})^{(7)} |G_{37}^{(1)} - G_{37}^{(2)}| e^{-(\widetilde{M}_{36})^{(7)}s_{(36)}} e^{(\widetilde{M}_{36})^{(7)}s_{(36)}} ds_{(36)} + *48$$

$$\int_0^t \{(a_{36}')^{(7)} |G_{36}^{(1)} - G_{36}^{(2)}| e^{-(\widetilde{M}_{36})^{(7)}s_{(36)}} e^{-(\widetilde{M}_{36})^{(7)}s_{(36)}} + (a_{36}'')^{(7)}(T_{37}^{(1)}, s_{(36)}) |G_{36}^{(1)} - G_{36}^{(2)}| e^{-(\widetilde{M}_{36})^{(7)}s_{(36)}} e^{(\widetilde{M}_{36})^{(7)}s_{(36)}} + *49$$

$$G_{36}^{(2)} | (a_{36}'')^{(7)}(T_{37}^{(1)}, s_{(36)}) - (a_{36}'')^{(7)}(T_{37}^{(2)}, s_{(36)}) | e^{-(\widetilde{M}_{36})^{(7)}s_{(36)}} e^{(\widetilde{M}_{36})^{(7)}s_{(36)}}\} ds_{(36)} *47$$

Where  $s_{(36)}$  represents integrand that is integrated over the interval  $[0, t]$

From the hypotheses it follows that:

$$|(G_{39})^{(1)} - (G_{39})^{(2)}| e^{-(\widetilde{M}_{36})^{(7)}t} \leq \frac{1}{(\widetilde{M}_{36})^{(7)}} ((a_{36})^{(7)} + (a_{36}')^{(7)} + (\widehat{A}_{36})^{(7)} + (\widehat{P}_{36})^{(7)} (\widehat{k}_{36})^{(7)}) d(((G_{39})^{(1)}, (T_{39})^{(1)}); (G_{39})^{(2)}, (T_{39})^{(2)}) *50$$

And analogous inequalities for  $G_i$  and  $T_i$ . Taking into account the hypothesis (37,35,36) the result follows

**Remark 7:** The fact that we supposed  $(a_{36}'')^{(7)}$  and  $(b_{36}'')^{(7)}$  depending also on  $t$  can be considered as not conformal with the reality, however we have put this hypothesis in order that we can postulate condition necessary to prove the uniqueness of the solution bounded by  $(\widehat{P}_{36})^{(7)} e^{(\widetilde{M}_{36})^{(7)}t}$  and  $(\widehat{Q}_{36})^{(7)} e^{(\widetilde{M}_{36})^{(7)}t}$  respectively of  $\mathbb{R}_+$ .

If instead of proving the existence of the solution on  $\mathbb{R}_+$ , we have to prove it only on a compact then it suffices to consider that  $(a_i'')^{(7)}$  and  $(b_i'')^{(7)}, i = 36,37,38$  depend only on  $T_{37}$  and respectively on  $(G_{39})$  (and not on  $t$ ) and hypothesis can be replaced by a usual Lipschitz condition.

\*51

**Remark 2:** There does not exist any  $t$  where  $G_i(t) = 0$  and  $T_i(t) = 0$

$$G_i(t) \geq G_i^0 e^{-\int_0^t \{(a_i')^{(7)} - (a_i'')^{(7)}(T_{37}(s_{(36)}), s_{(36)})\} ds_{(36)}} \geq 0$$

$$T_i(t) \geq T_i^0 e^{-(b_i')^{(7)}t} > 0 \text{ for } t > 0 *52$$

**Definition of**  $((\widetilde{M}_{36})^{(7)})_1, ((\widetilde{M}_{36})^{(7)})_2$  and  $((\widetilde{M}_{36})^{(7)})_3 :$

**Remark 3:** if  $G_{36}$  is bounded, the same property have also  $G_{37}$  and  $G_{38}$ . indeed if

$$G_{36} < (\widetilde{M}_{36})^{(7)} \text{ it follows } \frac{dG_{37}}{dt} \leq ((\widetilde{M}_{36})^{(7)})_1 - (a_{37}')^{(7)} G_{37} \text{ and by integrating}$$

$$G_{37} \leq ((\widetilde{M}_{36})^{(7)})_2 = G_{37}^0 + 2(a_{37})^{(7)} ((\widetilde{M}_{36})^{(7)})_1 / (a_{37}')^{(7)}$$

In the same way, one can obtain

$$G_{38} \leq ((\widetilde{M}_{36})^{(7)})_3 = G_{38}^0 + 2(a_{38})^{(7)} ((\widetilde{M}_{36})^{(7)})_2 / (a_{38}')^{(7)}$$

If  $G_{37}$  or  $G_{38}$  is bounded, the same property follows for  $G_{36}, G_{38}$  and  $G_{36}, G_{37}$  respectively.

\*53

**Remark 7:** If  $G_{36}$  is bounded, from below, the same property holds for  $G_{37}$  and  $G_{38}$ . The proof is analogous with the preceding one. An analogous property is true if  $G_{37}$  is bounded from below.

\*54

**Remark 5:** If  $T_{36}$  is bounded from below and  $\lim_{t \rightarrow \infty} ((b_i'')^{(7)}((G_{39})(t), t)) = (b_{37}')^{(7)}$  then  $T_{37} \rightarrow \infty$ .

**Definition of**  $(m)^{(7)}$  and  $\varepsilon_7 :$

Indeed let  $t_7$  be so that for  $t > t_7$

$$(b_{37})^{(7)} - (b'_i)^{(7)}((G_{39})(t), t) < \varepsilon_7, T_{36}(t) > (m)^{(7)}$$

\*55

Then  $\frac{dT_{37}}{dt} \geq (a_{37})^{(7)}(m)^{(7)} - \varepsilon_7 T_{37}$  which leads to

$$T_{37} \geq \left( \frac{(a_{37})^{(7)}(m)^{(7)}}{\varepsilon_7} \right) (1 - e^{-\varepsilon_7 t}) + T_{37}^0 e^{-\varepsilon_7 t} \text{ If we take } t \text{ such that } e^{-\varepsilon_7 t} = \frac{1}{2} \text{ it results}$$

$T_{37} \geq \left( \frac{(a_{37})^{(7)}(m)^{(7)}}{2} \right)$ ,  $t = \log \frac{2}{\varepsilon_7}$  By taking now  $\varepsilon_7$  sufficiently small one sees that  $T_{37}$  is unbounded. The same property holds for  $T_{38}$  if  $\lim_{t \rightarrow \infty} (b''_{38})^{(7)}((G_{39})(t), t) = (b'_{38})^{(7)}$

We now state a more precise theorem about the behaviors at infinity of the solutions of equations 37 to 72

\*

**Behavior of the solutions of the system Quantum Information and Quantum mechanical Behaviour:**

**Theorem 2:** If we denote and define

**Definition of**  $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$  :

(a)  $(\sigma_1)^{(7)}, (\sigma_2)^{(7)}, (\tau_1)^{(7)}, (\tau_2)^{(7)}$  four constants satisfying

$$-(\sigma_2)^{(7)} \leq -(a'_{36})^{(7)} + (a'_{37})^{(7)} - (a''_{36})^{(7)}(T_{37}, t) + (a''_{37})^{(7)}(T_{37}, t) \leq -(\sigma_1)^{(7)}$$

$$-(\tau_2)^{(7)} \leq -(b'_{36})^{(7)} + (b'_{37})^{(7)} - (b''_{36})^{(7)}((G_{39}), t) - (b''_{37})^{(7)}((G_{39}), t) \leq -(\tau_1)^{(7)}$$

\*56

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**Definition of**  $(v_1)^{(7)}, (v_2)^{(7)}, (u_1)^{(7)}, (u_2)^{(7)}, v, u$  :

(b) By  $(v_1)^{(7)} > 0, (v_2)^{(7)} < 0$  and respectively  $(u_1)^{(7)} > 0, (u_2)^{(7)} < 0$  the roots of the equations  $(a_{37})^{(7)}(v^{(7)})^2 + (\sigma_1)^{(7)}v^{(7)} - (a_{36})^{(7)} = 0$  and  $(b_{37})^{(7)}(u^{(7)})^2 + (\tau_1)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$  and

\*59

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**Definition of**  $(\bar{v}_1)^{(7)}, (\bar{v}_2)^{(7)}, (\bar{u}_1)^{(7)}, (\bar{u}_2)^{(7)}$  :

By  $(\bar{v}_1)^{(7)} > 0, (\bar{v}_2)^{(7)} < 0$  and respectively  $(\bar{u}_1)^{(7)} > 0, (\bar{u}_2)^{(7)} < 0$  the roots of the equations  $(a_{37})^{(7)}(v^{(7)})^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)} = 0$

and  $(b_{37})^{(7)}(u^{(7)})^2 + (\tau_2)^{(7)}u^{(7)} - (b_{36})^{(7)} = 0$

**Definition of**  $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}, (v_0)^{(7)}$  :-

(c) If we define  $(m_1)^{(7)}, (m_2)^{(7)}, (\mu_1)^{(7)}, (\mu_2)^{(7)}$  by

$$(m_2)^{(7)} = (v_0)^{(7)}, (m_1)^{(7)} = (v_1)^{(7)}, \text{ if } (v_0)^{(7)} < (v_1)^{(7)}$$

$$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (\bar{v}_1)^{(7)}, \text{ if } (v_1)^{(7)} < (v_0)^{(7)} < (\bar{v}_1)^{(7)},$$

$$\text{and } (v_0)^{(7)} = \frac{a_{36}^0}{a_{37}^0}$$

$$(m_2)^{(7)} = (v_1)^{(7)}, (m_1)^{(7)} = (v_0)^{(7)}, \text{ if } (\bar{v}_1)^{(7)} < (v_0)^{(7)}$$

\*62

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and analogously

$$(\mu_2)^{(7)} = (u_0)^{(7)}, (\mu_1)^{(7)} = (u_1)^{(7)}, \text{ if } (u_0)^{(7)} < (u_1)^{(7)}$$

$$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (\bar{u}_1)^{(7)}, \text{ if } (u_1)^{(7)} < (u_0)^{(7)} < (\bar{u}_1)^{(7)},$$

$$\text{and } (u_0)^{(7)} = \frac{T_{36}^0}{T_{37}^0}$$

$(\mu_2)^{(7)} = (u_1)^{(7)}, (\mu_1)^{(7)} = (u_0)^{(7)}$ , if  $(\bar{u}_1)^{(7)} < (u_0)^{(7)}$  where  $(u_1)^{(7)}, (\bar{u}_1)^{(7)}$  are defined by 59 and 67 respectively

\*68

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Then the solution of the system Quantum Information(machine-Computer) and Quantum Mechanical Behaviour satisfies the inequalities

$$G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \leq G_{36}(t) \leq G_{36}^0 e^{(S_1)^{(7)}t}$$

where  $(p_i)^{(7)}$  is defined in the foregoing

\*71

$$\frac{1}{(m_7)^{(7)}} G_{36}^0 e^{((S_1)^{(7)} - (p_{36})^{(7)})t} \leq G_{37}(t) \leq \frac{1}{(m_2)^{(7)}} G_{36}^0 e^{(S_1)^{(7)}t}$$

\*72

$$\left( \frac{(a_{38})^{(7)} G_{36}^0}{(m_1)^{(7)}((S_1)^{(7)} - (p_{36})^{(7)} - (S_2)^{(7)})} \left[ e^{((S_1)^{(7)} - (p_{36})^{(7)})t} - e^{-(S_2)^{(7)}t} \right] + G_{38}^0 e^{-(S_2)^{(7)}t} \leq G_{38}(t) \leq \frac{(a_{38})^{(7)} G_{36}^0}{(m_2)^{(7)}((S_1)^{(7)} - (a'_{38})^{(7)})} \left[ e^{(S_1)^{(7)}t} - e^{-(a'_{38})^{(7)}t} \right] + G_{38}^0 e^{-(a'_{38})^{(7)}t} \right)$$

\*73

$$T_{36}^0 e^{(R_1)^{(7)}t} \leq T_{36}(t) \leq T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}$$

\*74

$$\frac{1}{(\mu_1)^{(7)}} T_{36}^0 e^{(R_1)^{(7)}t} \leq T_{36}(t) \leq \frac{1}{(\mu_2)^{(7)}} T_{36}^0 e^{((R_1)^{(7)} + (r_{36})^{(7)})t}$$

\*75

$$\frac{(b_{38})^{(7)} T_{36}^0}{(\mu_1)^{(7)}((R_1)^{(7)} - (b'_{38})^{(7)})} \left[ e^{(R_1)^{(7)}t} - e^{-(b'_{38})^{(7)}t} \right] + T_{38}^0 e^{-(b'_{38})^{(7)}t} \leq T_{38}(t) \leq$$

$$\frac{(a_{38})^{(7)} T_{36}^0}{(\mu_2)^{(7)}((R_1)^{(7)} + (r_{36})^{(7)} + (R_2)^{(7)})} \left[ e^{((R_1)^{(7)} + (r_{36})^{(7)})t} - e^{-(R_2)^{(7)}t} \right] + T_{38}^0 e^{-(R_2)^{(7)}t}$$

\*76

**Definition of**  $(S_1)^{(7)}, (S_2)^{(7)}, (R_1)^{(7)}, (R_2)^{(7)}$ :-

$$\text{Where } (S_1)^{(7)} = (a_{36})^{(7)}(m_2)^{(7)} - (a'_{36})^{(7)}$$

$$(S_2)^{(7)} = (a_{38})^{(7)} - (p_{38})^{(7)}$$

$$(R_1)^{(7)} = (b_{36})^{(7)}(\mu_2)^{(7)} - (b'_{36})^{(7)}$$

$$(R_2)^{(7)} = (b'_{38})^{(7)} - (r_{38})^{(7)} \quad *77$$

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**Proof** From the sytemal equations of Quantum Information and Quantum Mechanical Behaviour, and the resultant equations we get:

$$\frac{dv^{(7)}}{dt} = (a_{36})^{(7)} - \left( (a'_{36})^{(7)} - (a'_{37})^{(7)} + (a''_{36})^{(7)}(T_{37}, t) \right) - (a''_{37})^{(7)}(T_{37}, t)v^{(7)} - (a_{37})^{(7)}v^{(7)}$$

**Definition of**  $v^{(7)}$  :-

$$v^{(7)} = \frac{G_{36}}{G_{37}}$$

It follows

$$- \left( (a_{37})^{(7)}(v^{(7)})^2 + (\sigma_2)^{(7)}v^{(7)} - (a_{36})^{(7)} \right) \leq \frac{dv^{(7)}}{dt} \leq - \left( (a_{37})^{(7)}(v^{(7)})^2 + (\sigma_1)^{(7)}v^{(7)} - (a_{36})^{(7)} \right)$$

From which one obtains

**Definition of**  $(\bar{v}_1)^{(7)}, (v_0)^{(7)}$  :-

I. For  $0 < \boxed{(v_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}} < (v_1)^{(7)} < (\bar{v}_1)^{(7)}$

$$v^{(7)}(t) \geq \frac{(v_1)^{(7)} + (C)^{(7)}(v_2)^{(7)} e^{[-(a_{37})^{(7)}(v_1)^{(7)} - (v_0)^{(7)}]t}}{1 + (C)^{(7)} e^{[-(a_{37})^{(7)}(v_1)^{(7)} - (v_0)^{(7)}]t}}, \quad \boxed{(C)^{(7)} = \frac{(v_1)^{(7)} - (v_0)^{(7)}}{(v_0)^{(7)} - (v_2)^{(7)}}$$

it follows  $(v_0)^{(7)} \leq v^{(7)}(t) \leq (v_1)^{(7)}$  \*80

81

In the same manner , we get

$$v^{(7)}(t) \leq \frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)} e^{[-(a_{37})^{(7)}(\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}]t}}{1 + (\bar{C})^{(7)} e^{[-(a_{37})^{(7)}(\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}]t}}, \quad \boxed{(\bar{C})^{(7)} = \frac{(\bar{v}_1)^{(7)} - (v_0)^{(7)}}{(v_0)^{(7)} - (\bar{v}_2)^{(7)}}$$

From which we deduce  $(v_0)^{(7)} \leq v^{(7)}(t) \leq (\bar{v}_1)^{(7)}$  \*82

83

II. If  $0 < (v_1)^{(7)} < (v_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0} < (\bar{v}_1)^{(7)}$  we find like in the previous case,

$$(v_1)^{(7)} \leq \frac{(v_1)^{(7)} + (C)^{(7)}(v_2)^{(7)} e^{[-(a_{37})^{(7)}(v_1)^{(7)} - (v_2)^{(7)}]t}}{1 + (C)^{(7)} e^{[-(a_{37})^{(7)}(v_1)^{(7)} - (v_2)^{(7)}]t}} \leq v^{(7)}(t) \leq$$

$$\frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)} e^{[-(a_{37})^{(7)}(\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}]t}}{1 + (\bar{C})^{(7)} e^{[-(a_{37})^{(7)}(\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}]t}} \leq (\bar{v}_1)^{(7)} \quad *84$$

III. If  $0 < (v_1)^{(7)} \leq (\bar{v}_1)^{(7)} \leq \boxed{(v_0)^{(7)} = \frac{G_{36}^0}{G_{37}^0}}$ , we obtain

$$(v_1)^{(7)} \leq v^{(7)}(t) \leq \frac{(\bar{v}_1)^{(7)} + (\bar{C})^{(7)}(\bar{v}_2)^{(7)} e^{[-(a_{37})^{(7)}(\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}]t}}{1 + (\bar{C})^{(7)} e^{[-(a_{37})^{(7)}(\bar{v}_1)^{(7)} - (\bar{v}_2)^{(7)}]t}} \leq (v_0)^{(7)}$$

And so with the notation of the first part of condition (c) , we have

**Definition of**  $v^{(7)}(t)$  :-

$$(m_2)^{(7)} \leq v^{(7)}(t) \leq (m_1)^{(7)}, \quad \boxed{v^{(7)}(t) = \frac{G_{36}(t)}{G_{37}(t)}}$$

In a completely analogous way, we obtain

**Definition of**  $u^{(7)}(t)$  :-

$$(\mu_2)^{(7)} \leq u^{(7)}(t) \leq (\mu_1)^{(7)}, \quad \boxed{u^{(7)}(t) = \frac{T_{36}(t)}{T_{37}(t)}}$$

Now, using this result and replacing it in system equations for the system Quantum Information and Quantum Mechanical Behaviour we obtain:

**Particular case :**

If  $(a_{36})^{(7)} = (a_{37})^{(7)}$ , then  $(\sigma_1)^{(7)} = (\sigma_2)^{(7)}$  and in this case  $(v_1)^{(7)} = (\bar{v}_1)^{(7)}$  if in addition  $(v_0)^{(7)} = (v_1)^{(7)}$  then  $v^{(7)}(t) = (v_0)^{(7)}$  and as a consequence  $G_{36}(t) = (v_0)^{(7)} G_{37}(t)$  **this also defines  $(v_0)^{(7)}$  for the special case.**

Analogously if  $(b_{36})^{(7)} = (b_{37})^{(7)}$ , then  $(\tau_1)^{(7)} = (\tau_2)^{(7)}$  and then

$(u_1)^{(7)} = (\bar{u}_1)^{(7)}$  if in addition  $(u_0)^{(7)} = (u_1)^{(7)}$  then  $T_{36}(t) = (u_0)^{(7)} T_{37}(t)$  This is an important consequence of the relation between  $(v_1)^{(7)}$  and  $(\bar{v}_1)^{(7)}$ , **and definition of  $(u_0)^{(7)}$ .**

\*85, 86, 87

#### IV. STATIONARY SOLUTIONS AND STABILITY

Stationary solutions and stability curve representative of the variation of the system Quantum Information and Quantum Mechanical Behaviour variation curve lies below the tangent at  $(G_{39}) = G_0$  for  $(G_{39}) < G_0$  and above the tangent for  $(G_{39}) > G_0$ . Wherever such a situation occurs the point  $G_0$  is called the “**point of inflexion**”. In this case, the tangent has a positive slope that simply means the rate of change of Quantum Mechanical Behaviour vis a vis Quantum Information is

greater than zero. Above factor shows that it is possible, to draw a curve that has a point of inflexion at a point where the tangent (slope of the curve) is horizontal.

### Stationary value :

In all the cases  $(G_{39}) = G_0$ ,  $(G_{39}) < G_0$ ,  $(G_{39}) > G_0$  the condition that the rate of change of oxygen consumption is maximum or minimum holds. When this condition holds we have stationary value. We now infer that :

1. A necessary and sufficient condition for there to be stationary value of  $(G_{39})$  is that the rate of change of oxygen consumption function at  $G_0$  is zero.
2. A sufficient condition for the stationary value at  $G_0$ , to be maximum is that the acceleration of the oxygen consumption is less than zero.
3. A sufficient condition for the stationary value at  $G_0$ , to be minimum is that acceleration of oxygen consumption is greater than zero.
4. With the rate of change of  $(G_{39})$  namely oxygen consumption defined as the accentuation term and the dissipation term, we are sure that the rate of change of Quantum Mechanical Behaviour-Quantum Information system is always positive.
5. Concept of stationary state is mere methodology although there might be closed system exhibiting symptoms of stationeries.

We can prove the following

**Theorem 3:** If  $(a_i'')^{(7)}$  and  $(b_i'')^{(7)}$  are independent on  $t$ , and the conditions :

$$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} < 0$$

$$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a_{36})^{(7)}(p_{36})^{(7)} + (a'_{37})^{(7)}(p_{37})^{(7)} + (p_{36})^{(7)}(p_{37})^{(7)} > 0$$

$$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} > 0,$$

$$(b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - (b'_{36})^{(7)}(r_{37})^{(7)} - (b'_{37})^{(7)}(r_{37})^{(7)} + (r_{36})^{(7)}(r_{37})^{(7)} < 0$$

with  $(p_{36})^{(7)}, (r_{37})^{(7)}$  as defined by equation 37 are satisfied, then the system

\*88

$$(a_{36})^{(7)}G_{37} - [(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37})]G_{36} = 0 \quad *89$$

$$(a_{37})^{(7)}G_{36} - [(a'_{37})^{(7)} + (a''_{37})^{(7)}(T_{37})]G_{37} = 0 \quad *90$$

$$(a_{38})^{(7)}G_{37} - [(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37})]G_{38} = 0 \quad *91$$

$$(b_{36})^{(7)}T_{37} - [(b'_{36})^{(7)} - (b''_{36})^{(7)}(G_{39})]T_{36} = 0 \quad *92$$

$$(b_{37})^{(7)}T_{36} - [(b'_{37})^{(7)} - (b''_{37})^{(7)}(G_{39})]T_{37} = 0 \quad *93$$

$$(b_{38})^{(7)}T_{37} - [(b'_{38})^{(7)} - (b''_{38})^{(7)}(G_{39})]T_{38} = 0$$

\*94

has a unique positive solution, which is an equilibrium solution for the system Quantum Mechanical Behaviour(Quantum Entanglement, Coherence, Decoherence) and Quantum Information(Computer).

### Proof:

(a) Indeed the first two equations have a nontrivial solution  $G_{36}, G_{37}$  if

$$F(T_{39}) =$$

$$(a'_{36})^{(7)}(a'_{37})^{(7)} - (a_{36})^{(7)}(a_{37})^{(7)} + (a'_{36})^{(7)}(a''_{37})^{(7)}(T_{37}) + (a'_{37})^{(7)}(a''_{36})^{(7)}(T_{37}) + (a''_{36})^{(7)}(T_{37})(a''_{37})^{(7)}(T_{37}) = 0$$

\*95

**Definition and uniqueness of  $T_{37}^*$  :-**

After hypothesis  $f(0) < 0, f(\infty) > 0$  and the functions  $(a_i'')^{(7)}(T_{37})$  being increasing, it follows that there exists a unique  $T_{37}^*$  for which  $f(T_{37}^*) = 0$ . With this value, we obtain from the three first equations

$$G_{36} = \frac{(a_{36})^{(7)}G_{37}}{[(a'_{36})^{(7)} + (a''_{36})^{(7)}(T_{37}^*)]}, \quad G_{38} = \frac{(a_{38})^{(7)}G_{37}}{[(a'_{38})^{(7)} + (a''_{38})^{(7)}(T_{37}^*)]}$$

(b) By the same argument, the equations 92,93 admit solutions  $G_{36}, G_{37}$  if

$$\varphi(G_{39}) = (b'_{36})^{(7)}(b'_{37})^{(7)} - (b_{36})^{(7)}(b_{37})^{(7)} - [(b'_{36})^{(7)}(b''_{37})^{(7)}(G_{39}) + (b'_{37})^{(7)}(b''_{36})^{(7)}(G_{39})] + (b''_{36})^{(7)}(G_{39})(b''_{37})^{(7)}(G_{39}) = 0 \quad *96$$

97

Where in  $(G_{39})(G_{36}, G_{37}, G_{38}), G_{36}, G_{38}$  must be replaced by their values from 96. It is easy to see that  $\varphi$  is a decreasing function in  $G_{37}$  taking into account the hypothesis  $\varphi(0) > 0, \varphi(\infty) < 0$  it follows that there exists a unique  $G_{37}^*$  such that  $\varphi(G^*) = 0$

Finally we obtain the unique solution of the defined system Quantum Mechanical Behaviour and Quantum Information:  $G_{37}^*$  given by  $\varphi((G_{39})^*) = 0, T_{37}^*$  given by  $f(T_{37}^*) = 0$  and

$$G_{36}^* = \frac{(a_{36})^{(7)} G_{37}^*}{[(a_{36})^{(7)} + (a_{36})^{(7)}(T_{37}^*)]} \quad , \quad G_{38}^* = \frac{(a_{38})^{(7)} G_{37}^*}{[(a_{38})^{(7)} + (a_{38})^{(7)}(T_{37}^*)]} \quad *98$$

$$T_{36}^* = \frac{(b_{36})^{(7)} T_{37}^*}{[(b_{36})^{(7)} - (b_{36})^{(7)}((G_{39})^*)]} \quad , \quad T_{38}^* = \frac{(b_{38})^{(7)} T_{37}^*}{[(b_{38})^{(7)} - (b_{38})^{(7)}((G_{39})^*)]} \quad *99$$

Obviously, these values represent an equilibrium solution of the system Quantum Mechanical Behaviour and Quantum mechanical System(Quantum Information)

### ASYMPTOTIC STABILITY ANALYSIS

One of the studies that come to mind when one thinks of the Asymptotic Stability Analysis, is the study by Komech On asymptotic stability of solitary waves for Schrödinger equation coupled to nonlinear oscillator, The long-time asymptotics is analyzed for finite energy solutions of the 1D Schrödinger equation coupled to a nonlinear oscillator; mathematically the system under study is a Nonlinear Schrödinger equation, whose nonlinear term includes a Dirac delta. The coupled system is invariant with respect to the phase rotation group  $U(1)$ . The article, which extends the results of a previous one, provides a proof of asymptotic stability of solitary wave solutions in the case that the linearization contains a single discrete oscillatory mode satisfying a non-degeneracy assumption of the type known as the Fermi Golden Rule. In this article we continue the study of large time asymptotics for a model  $U(1)$ -invariant nonlinear Schrödinger equation Main focus is on the role that certain solitary waves (also referred to as nonlinear bound states, or solitons) play in the description of the solution for large times.

### Equilibrium: Stable or Unstable?

**Equilibrium** is a state of a system which does not change.

If the dynamics of a system is described by a differential equation (or a system of differential equations), then equilibria can be estimated by setting a derivative (all derivatives) to zero.

### Example: Logistic model

$$\frac{dN}{dt} = r_0 N \left(1 - \frac{N}{K}\right)$$

To find equilibrium we have to solve the equation:  $dN/dt = 0$ :

$$r_0 N \left(1 - \frac{N}{K}\right) = 0$$

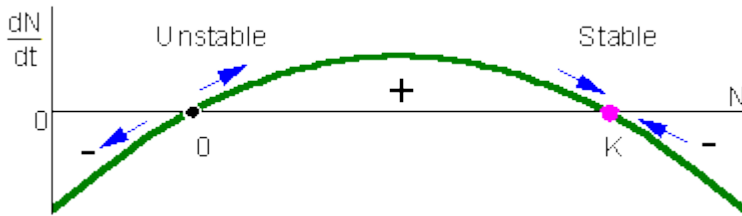
This equation has two roots:  $N=0$  and  $N=K$ . An equilibrium may be stable or unstable. For example, the equilibrium of a pencil standing on its tip is unstable; The equilibrium of a picture on the wall is (usually) stable.



An equilibrium is considered **stable** (for simplicity we will consider asymptotic stability only) if the system always returns to it after small disturbances. If the system moves away from the equilibrium after small disturbances, then the equilibrium is **unstable**.

The notion of stability can be applied to other types of attractors (limit cycle, chaos), however, the general definition is more complex than for equilibria. Stability is probably the most important notion in science because it refers to what we call "reality". Everything should be stable to be observable. For example, in quantum mechanics, energy levels are those that are stable because unstable levels cannot be observed.

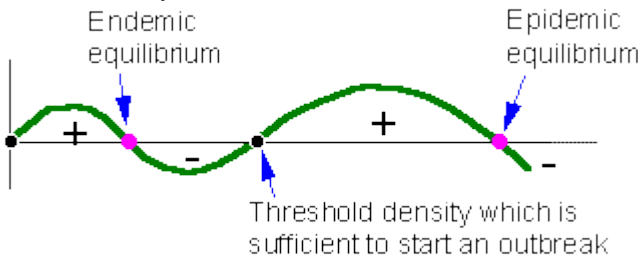
Now, let's examine stability of 2 equilibria points in the logistic model.



In this figure, population growth rate,  $dN/dt$ , is plotted versus population density,  $N$ . This is often called a **phase-plot** of population dynamics. If  $0 < N < K$ , then  $dN/dt > 0$  and thus, population grows (the point in the graph moves to the right). If  $N < 0$  or  $N > K$  (of course,  $N < 0$  has no biological sense), then population declines (the point in the graph moves to the left). The arrows show that the equilibrium  $N=0$  is unstable, whereas the equilibrium  $N=K$  is stable. From the biological point of view, this means that after small deviation of population numbers from  $N=0$  (e.g., immigration of a small number of organisms), the population never returns back to this equilibrium. Instead, population numbers increase until they reach the stable equilibrium  $N=K$ . After any deviation from  $N=K$  the population returns back to this stable equilibrium.

The difference between stable and unstable equilibria is in the slope of the line on the phase plot near the equilibrium point. Stable equilibria are characterized by a negative slope (negative feedback) whereas unstable equilibria are characterized by a positive slope (positive feedback).

The second example is the bark beetle model with two stable and two unstable equilibria. Stable equilibria correspond to endemic and epidemic populations. Endemic populations are regulated by the amount of susceptible trees in the forest. Epidemic populations are limited by the total number of trees because mass attack of beetle females may overcome the resistance of any tree.



**Stability of models with several variables**

Detection of stability in these models is not that simple as in one-variable models. Let's consider a predator-prey model with two variables: (1) density of prey and (2) density of predators. Dynamics of the model is described by the system of 2 differential equations:

$$\begin{cases} \frac{dH}{dt} = f(H, P) \\ \frac{dP}{dt} = g(H, P) \end{cases}$$

This is the 2-variable model in a general form. Here,  $H$  is the density of prey, and  $P$  is the density of predators. The first step is to find equilibrium densities of prey ( $H^*$ ) and predator ( $P^*$ ). We need to solve a system of equations:

$$\begin{cases} f(H^*, P^*) = 0 \\ g(H^*, P^*) = 0 \end{cases}$$

The second step is to linearize the model at the equilibrium point ( $H = H^*, P = P^*$ ) by estimating the Jacobian matrix:

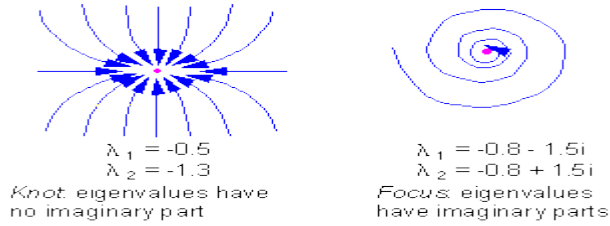
$$A = \begin{vmatrix} \frac{\partial f}{\partial H} & \frac{\partial f}{\partial P} \\ \frac{\partial g}{\partial H} & \frac{\partial g}{\partial P} \end{vmatrix}$$

Third, eigenvalues of matrix  $A$  should be estimated. The number of eigenvalues is equal to the number of state variables. In our case there will be 2 eigenvalues. Eigenvalues are generally complex numbers. If real parts of all eigenvalues are negative, then the equilibrium is stable. If at least one eigenvalue has a positive real part, then the equilibrium is unstable.

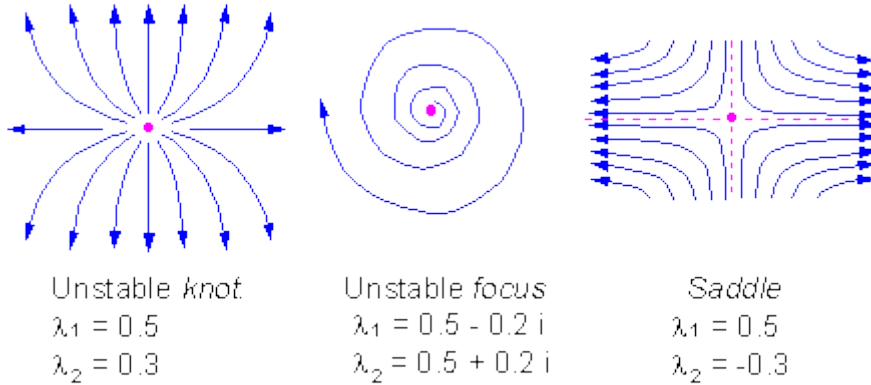


Eigenvalues are used here to reduce a 2-dimensional problem to a couple of 1-dimensional problem problems. Eigenvalues have the same meaning as the slope of a line in phase plots. Negative real parts of eigenvalues indicate a negative feedback. It is important that ALL eigenvalues have negative real parts. If one eigenvalue has a positive real part then there is a direction in a 2-dimensional space in which the system will not tend to return back to the equilibrium point.

There are 2 types of stable equilibrium in a two-dimensional space: **knot** and **focus**



There are 3 types of unstable equilibrium in a two-dimensional space: **knot**, **focus**, and **saddle**



**Stability in discrete-time models**

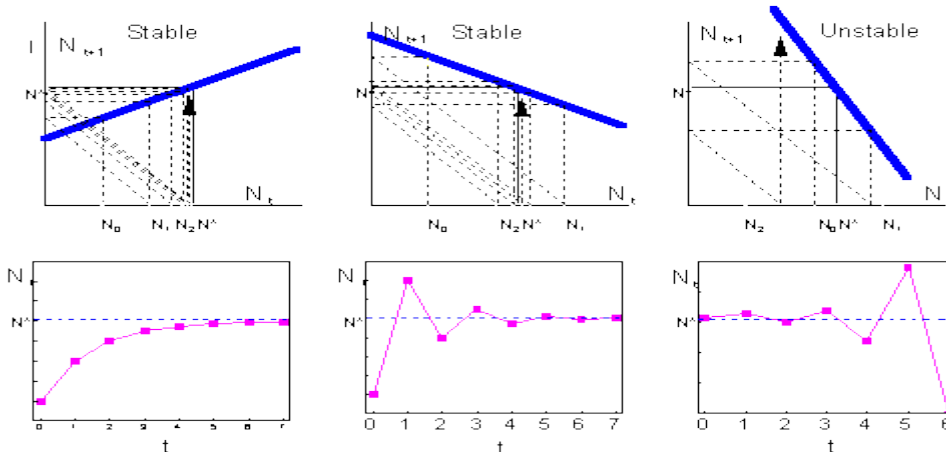
Consider a discrete-time model (a difference equation) with one state variable:

$$N_{t+1} = f(N_t)$$

This model is stable if and only if :

$$-1 < \frac{df}{dN_t} < 1$$

where  $\frac{df}{dN_t}$  is the slope of a thick line in graphs below:



You can check this yourself using the following Excel spreadsheet:



Excel spreadsheet "ricker.xls"

If the slope is positive but less than 1, then the system approaches the equilibrium monotonically (left). If the slope is negative and greater than -1, then the system exhibits oscillations because of the "overcompensation" (center). Overcompensation means that the system jumps over the equilibrium point because the negative feedback is too strong. Then it returns back and again jumps over the equilibrium.

Continuous-time models with 1 variable never exhibit oscillations. In discrete-time models, oscillations are possible even with 1 variable. What causes oscillations is the delay between time steps. Overcompensation is a result of large time steps. If time steps were smaller, then the system would not jump over the equilibrium but will approach to it gradually.

Now we will analyze stability in the Ricker's model. This model is a discrete-time analog of the logistic model:

$$N_{t+1} = N_t \cdot \exp\left[r \left(1 - \frac{N_t}{K}\right)\right]$$

First, we need to find the equilibrium population density  $N^*$  by solving the equation:

$$N^* = N^* \cdot \exp\left[r \left(1 - \frac{N^*}{K}\right)\right]$$

This equation is obtained by substituting  $N_{t+1}$  and  $N_t$  with the equilibrium population density  $N^*$  in the initial equation. The roots are:  $N^* = 0$  and  $N^* = K$ . We are not interested in the first equilibrium ( $N^* = 0$ ) because there is no population. Let's estimate the slope  $df/dN$  at the second equilibrium point:

$$\frac{df}{dN_t} = \left(1 - \frac{r \cdot N_t}{K}\right) \cdot \exp\left[r \left(1 - \frac{N_t}{K}\right)\right] = 1 - r$$

$$N_t = N^* = K$$

Now we can apply the condition of stability:

$$-1 < 1 - r < 1$$

$$0 < r < 2$$

Thus, the Ricker's model has a stable equilibrium  $N^* = K$  if  $0 < r < 2$ .

If a discrete time model has more than one state variable, then the analysis is similar to that in continuous-time models. The first step is to find equilibria. The second step is to linearize the model at the equilibrium state, i.e., to estimate the Jacobian matrix. The third step is to estimate eigenvalues of this matrix. The only difference from continuous models is the condition of stability. Discrete-time models are stable (asymptotically stable) if and only if all eigenvalues lie in the circle with the radius = 1 in the complex plain.

**Theorem 7:** If the conditions of the previous theorem are satisfied and if the functions  $(a_i'')^{(7)}$  and  $(b_i'')^{(7)}$  belong to  $C^{(7)}(\mathbb{R}_+)$  then the above equilibrium point is asymptotically stable.

**Proof:** Denote

\*

**Definition of**  $G_i, T_i$  :-

$$G_i = G_i^* + G_i \quad , \quad T_i = T_i^* + T_i$$

$$\frac{\partial (a_{37}'')^{(7)}}{\partial T_{37}}(T_{37}^*) = (q_{37})^{(7)} \quad , \quad \frac{\partial (b_i'')^{(7)}}{\partial G_j}((G_{39})^{**}) = s_{ij} \quad *100, 101$$

Then taking into account equations of the system Quantum Information and Quantum Mechanical Behaviour and neglecting the terms of power 2, we obtain\*

$$\frac{dG_{36}}{dt} = -((a'_{36})^{(7)} + (p_{36})^{(7)})G_{36} + (a_{36})^{(7)}G_{37} - (q_{36})^{(7)}G_{36}^*T_{37} \quad *102$$

$$\frac{dG_{37}}{dt} = -((a'_{37})^{(7)} + (p_{37})^{(7)})G_{37} + (a_{37})^{(7)}G_{36} - (q_{37})^{(7)}G_{37}^*T_{37} \quad *103$$

$$\frac{dG_{38}}{dt} = -((a'_{38})^{(7)} + (p_{38})^{(7)})G_{38} + (a_{38})^{(7)}G_{37} - (q_{38})^{(7)}G_{38}^*T_{37} \quad *104$$

$$\frac{dT_{36}}{dt} = -((b'_{36})^{(7)} - (r_{36})^{(7)})T_{36} + (b_{36})^{(7)}T_{37} + \sum_{j=36}^{38} (s_{(36)(j)})T_{36}^*G_j \quad *105$$

$$\frac{dT_{37}}{dt} = -((b'_{37})^{(7)} - (r_{37})^{(7)})T_{37} + (b_{37})^{(7)}T_{36} + \sum_{j=36}^{38} (s_{(37)(j)})T_{37}^*G_j \quad *106$$

$$\frac{dT_{38}}{dt} = -((b'_{38})^{(7)} - (r_{38})^{(7)})T_{38} + (b_{38})^{(7)}T_{37} + \sum_{j=36}^{38} (s_{(38)(j)})T_{38}^*G_j \quad *107$$

The characteristic equation of this system is

$$\begin{aligned}
& ((\lambda)^{(7)} + (b'_{38})^{(7)} - (r_{38})^{(7)}) \{ ((\lambda)^{(7)} + (a'_{38})^{(7)} + (p_{38})^{(7)}) \\
& \left[ ((\lambda)^{(7)} + (a'_{36})^{(7)} + (p_{36})^{(7)}) (q_{37})^{(7)} G_{37}^* + (a_{37})^{(7)} (q_{36})^{(7)} G_{36}^* \right] \\
& \left( ((\lambda)^{(7)} + (b'_{36})^{(7)} - (r_{36})^{(7)}) s_{(37),(37)} T_{37}^* + (b_{37})^{(7)} s_{(36),(37)} T_{37}^* \right) \\
& + \left( ((\lambda)^{(7)} + (a'_{37})^{(7)} + (p_{37})^{(7)}) (q_{36})^{(7)} G_{36}^* + (a_{36})^{(7)} (q_{37})^{(7)} G_{37}^* \right) \\
& \left( ((\lambda)^{(7)} + (b'_{36})^{(7)} - (r_{36})^{(7)}) s_{(37),(36)} T_{37}^* + (b_{37})^{(7)} s_{(36),(36)} T_{36}^* \right) \\
& \left( ((\lambda)^{(7)})^2 + (a'_{36})^{(7)} + (a'_{37})^{(7)} + (p_{36})^{(7)} + (p_{37})^{(7)} \right) (\lambda)^{(7)} \\
& \left( ((\lambda)^{(7)})^2 + (b'_{36})^{(7)} + (b'_{37})^{(7)} - (r_{36})^{(7)} + (r_{37})^{(7)} \right) (\lambda)^{(7)} \\
& + \left( ((\lambda)^{(7)})^2 + (a'_{36})^{(7)} + (a'_{37})^{(7)} + (p_{36})^{(7)} + (p_{37})^{(7)} \right) (\lambda)^{(7)} (q_{38})^{(7)} G_{38} \\
& + ((\lambda)^{(7)} + (a'_{36})^{(7)} + (p_{36})^{(7)}) \left( (a_{38})^{(7)} (q_{37})^{(7)} G_{37}^* + (a_{37})^{(7)} (a_{38})^{(7)} (q_{36})^{(7)} G_{36}^* \right) \\
& \left( ((\lambda)^{(7)} + (b'_{36})^{(7)} - (r_{36})^{(7)}) s_{(37),(38)} T_{37}^* + (b_{37})^{(7)} s_{(36),(38)} T_{36}^* \right) \} = 0
\end{aligned}$$

And as one sees, all the coefficients are positive. It follows that all the roots have negative real part, and this proves the theorem.

### DISSIPATION OF QUANTUM MECHANICAL EFFECTS:

The need for a quantum-mechanical formalism for systems with dissipation which is applicable to the radiation field of a cavity is discussed. Two methods that have been used in this connection are described. The first, which starts with the classical Newtonian equation of motion for a damped oscillator and applies the conventional formal quantization techniques, leads to an exact solution; but subsequent discussion shows that this method is invalid, the results being unacceptable from a quantum-mechanical viewpoint. The second method, which considers the interaction of two systems, the lossless oscillator and the loss mechanism, is adopted in the present article. No special model is used for the loss mechanism, but this mechanism is assumed to have a large number of densely-spaced energy states.

The approximations with respect to the loss mechanism that underlie the concept of dissipation are discussed. These approximations are then applied to the analysis, and a differential equation for a coordinate operator of the harmonic oscillator is obtained which has the formal appearance of the Newtonian equation of motion for a driven damped harmonic oscillator, the driving term being an operator referring to the loss mechanism. The presence of the driving term is responsible for the difference between the present theory and that of the first method mentioned above. A solution of the differential equation for the coordinate operator is given explicitly. An examination of the physical significance of the solution shows that the driving term is responsible not only for the thermal fluctuations which are due to the loss mechanism, but also for the proper commutation relationship of the conjugate coordinates of the oscillator and for its zero-point fluctuations.

A generalization of the solution to provide for a classical driving force and coupled atomic systems is given. The results are then restated in a form that refers to the loss mechanism only through the two parameters by which it is usually described—the dissipation constant and the temperature.

### Some selected Examples:

#### A. O. Caldeira and A. J. Leggett studies the following system:

quantum system which can tunnel, at  $T=0$ , out of a metastable state and whose interaction with its environment is adequately described in the classically accessible region by a phenomenological friction coefficient  $\eta$ , is considered. By only assuming that the environment response is linear, it is found that dissipation multiplies the tunneling probability by the factor  $\exp[-A\eta(\Delta q)^2/\hbar]$ , where  $\Delta q$  is the "distance under the barrier" and  $A$  is a numerical factor which is generally of order unity

Measurements of the tunneling rate  $\Gamma$  out of the zero-voltage state for several Nb edge junctions with differing shunt capacitances are described. At zero temperature, increasing the shunt capacitance lowers  $\Gamma$  in agreement with dissipative calculations of the macroscopic-quantum-tunneling rate. As temperature increases,  $\ln[\Gamma(T)/\Gamma(0)] \propto T^2$  as recently predicted.

#### Quantum dissipation

From Wikipedia, the free encyclopedia

**Quantum Dissipation** is the branch of physics that studies the quantum analogues of the process of irreversible loss of energy observed at the classical level. Its main purpose is to derive the laws of classical dissipation from the framework of quantum mechanics. It shares many features with the subjects of quantum decoherence and quantum theory of measurement.

#### **Models of Quantum Dissipation**

The main problem to address to study dissipation at the quantum level is the way to envisage the mechanism of irreversible loss of energy. Quantum mechanics usually deal with the Hamiltonian formalism, where the total energy of the system is a conserved quantity. So in principle it would not be possible to describe dissipation in this framework.

The idea to overcome this issue consists on splitting the total system in two parts: the quantum system where dissipation occurs, and a so-called environment or bath where the energy of the former will flow towards. The way both systems are coupled depends on the details of the microscopic model, and hence, the description of the bath. To include an irreversible flow of energy (i.e., to avoid Poincaré recurrences in which the energy eventually flows back to the system), requires that the

bath contain an infinite number of degrees of freedom. Notice that by virtue of the principle of universality, it is expected that the particular description of the bath will not affect the essential features of the dissipative process, as far as the model contains the minimal ingredients to provide the effect.

The simplest way to model the bath was proposed by Feynman and Vernon in a seminal paper from 1963<sup>[1]</sup>. In this description the bath is a sum of an infinite number of harmonic oscillators, that in quantum mechanics represents a set of free bosonic particles.

#### [edit]The Caldeira-Leggett model

In 1981 Amir Caldeira and Anthony J. Leggett proposed a simple model to study in detail the way dissipation arises from a quantum point of view<sup>[2]</sup>. It describes a quantum particle in one-dimension coupled to a bath. The Hamiltonian reads:

$$H = \frac{P^2}{2M} + V(X) + \sum_i \left( \frac{p_i^2}{2m_i} + \frac{1}{2} m_i \omega_i^2 q_i^2 \right) + X \sum_i C_i q_i + X^2 \sum_i \frac{C_i^2}{2m_i \omega_i^2},$$

The first two terms correspond to the Hamiltonian of a quantum particle of mass  $M$  and momentum  $P$ , in a potential  $V$  at position  $X$ . The third term describes the bath as a sum of infinite harmonic oscillators with masses  $m_i$  and momentum  $p_i$ , at positions  $q_i$ .  $\omega_i$  are the frequencies of the harmonic oscillators. The next term describes the way system and bath are coupled. In the Caldeira - Leggett model the bath is coupled to the position of the particle.  $C_i$  are coefficients which depend on the details of the coupling. The last term is a counter-term which must be included to ensure that dissipation is homogeneous in all space. As the bath couples to the position, if this term is not included the model is not translationally invariant, in the sense that the coupling is different wherever the quantum particle is located. This gives rise to an unphysical renormalization of the potential, which can be shown to be suppressed by including the counter-term.

To provide a good description of the dissipation mechanism, a relevant quantity is the bath spectral function, defined as follows:

$$J(\omega) = \frac{\pi}{2} \sum_i \frac{C_i^2}{m_i \omega_i} \delta(\omega - \omega_i)$$

The bath spectral function provides a constraint in the choice of the coefficients  $C_i$ . When this function has the form  $J(\omega) = \eta \omega$ , the corresponding classical kind of dissipation can be shown to be Ohmic. A more generic form is  $J(\omega) \propto \omega^s$ . In this case, if  $s > 1$  the dissipation is called "super-ohmic", while if  $s < 1$  is sub-ohmic. An example of a super-ohmic bath is the electro-magnetic field under certain circumstances.

As mentioned, the main idea in the field of quantum dissipation is to explain the way classical dissipation can be described from a quantum mechanics point of view. To get the classical limit of the Caldeira - Leggett model, the bath must be integrated out (or traced out), which can be understood as taking the average over all the possible realizations of the bath and studying the effective dynamics of the quantum system. As a second step, the limit  $\hbar \rightarrow 0$  must be taken to recover classical mechanics. To proceed with those technical steps mathematically, the path integral description of quantum mechanics is usually employed. The resulting classical equations of motion are:

$$M \frac{d^2}{dt^2} X(t) = -\frac{\partial V(X)}{\partial X} - \int_0^T dt' \alpha(t-t')(X(t) - X(t'))$$

where:

$$\alpha(t-t') = \frac{1}{2\pi} \int_0^\infty J(\omega) e^{-\omega|t-t'|} d\omega$$

is a kernel which characterizes the effective force that affects the motion of the particle in the presence of dissipation. For so-called Markovian baths, which do not keep memory of the interaction with the system, and for Ohmic dissipation, the equations of motion simplify to the classical equations of motion of a particle with friction:

$$M \frac{d^2}{dt^2} X(t) = -\frac{\partial V(X)}{\partial X} - \eta \frac{dX(t)}{dt}$$

Hence, one can see how Caldeira-Leggett model fulfills the goal of getting classical dissipation from the quantum mechanics framework. The Caldeira-Leggett model has been used to study **quantum dissipation** problems since its introduction in 1981, being extensively used as well in the field of quantum decoherence.

#### [edit]The dissipative two-level system

This particular realization of the Caldeira - Leggett model deserves special attention due to its interest in the field of Quantum Computation. The aim of the model is to study the effects of dissipation in the dynamics of a particle that can hop between two different positions. This reduced Hilbert space allows the problem to be described in terms of  $1/2$  spin operators. The resulting Hamiltonian is also referred in the literature as the Spin-Boson model, reading:

$$H = \Delta S_x + \sum_i \left( \frac{p_i^2}{2m_i} + \frac{1}{2} m_i \omega_i^2 q_i^2 \right) + S_z \sum_i C_i q_i,$$

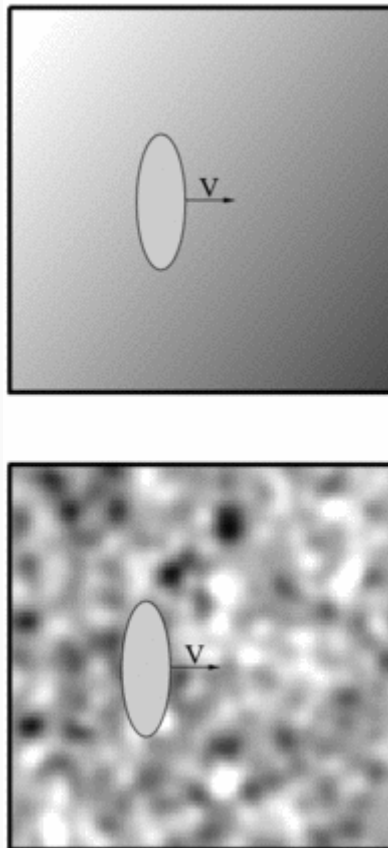
where  $S_i = \frac{\sigma_i}{2}$ ,  $i = x, y, z$  are proportional to the Pauli matrices  $\sigma_i$ , and  $\Delta$  is the probability of hopping between the two possible positions. Notice that in this model the counter-term is no longer needed, as the coupling to  $S_z$  gives already homogeneous dissipation.

The model has many applications. In **quantum dissipation** it is used as a simple model to study the dynamics of a dissipative particle confined in a double-well potential. In the context of Quantum Computation it represents a qubit coupled to an environment, which can produce decoherence. In the study of amorphous solids it provides the basis of the standard theory to describe their thermodynamic properties.

The dissipative two-level systems represents also a paradigm in the study of quantum phase transitions. For a critical value of the coupling to the bath it shows a phase transition from a regime in which the particle is delocalized among the two positions to another in which it is localized in only one of them. The transition is of Kosterlitz-Thouless kind, as can be seen by deriving the Renormalization group flow equations for the hopping term.

Dissipation model for extended environment

From Wikipedia, the free encyclopedia



□

(a) The Brownian particle in the Caldeira-Leggett model experiences a fluctuating homogeneous field of force. (b) In case of the DLD model the fluctuating field is farther characterized by a finite correlation distance. The background image is a “snapshot” of the fluctuating environment. Namely, the gray levels correspond to the “height” of an instantaneous potential which is experienced by the Brownian particle.

A unified model for *Diffusion Localization and Dissipation* (DLD), optionally termed *Diffusion with Local Dissipation*, has been introduced for the study of *Quantal Brownian Motion* (QBM) in dynamical disorder<sup>[1], [2]</sup>. It can be regarded as a generalization of the familiar Caldeira-Leggett model.

$$\mathcal{H} = \frac{p^2}{2m} + V(x) + \mathcal{H}_{int} + \mathcal{H}_{bath}$$

$$\mathcal{H}_{bath} = \sum_{\alpha} \left( \frac{P_{\alpha}^2}{2m_{\alpha}} + \frac{1}{2} m_{\alpha} \omega_{\alpha}^2 Q_{\alpha}^2 \right)$$

$$\mathcal{H}_{int} = - \sum_{\alpha} c_{\alpha} Q_{\alpha} u(x - x_{\alpha})$$

where  $Q_{\alpha}$  denotes the dynamical coordinate of the  $\alpha$  scatterer or bath mode.  $u(x - x_{\alpha})$  is the interaction potential, and  $c_{\alpha}$  are coupling constants. The spectral characterization of the bath is analogous to that of the Caldeira-Leggett model:

$$\frac{\pi}{2} \sum_{\alpha} \frac{c_{\alpha}^2}{m_{\alpha} \omega_{\alpha}} \delta(\omega - \omega_{\alpha}) \delta(x - x_{\alpha}) = J(\omega)$$

i.e. the oscillators that appear in the Hamiltonian are distributed uniformly over space, and in each location have the same spectral distribution  $J(\omega)$ . Optionally the environment is characterized by the power spectrum of the fluctuations  $S(q, \omega)$ , which is determined by  $J(\omega)$  and by the assumed interaction  $u(r)$ . See examples.

The model can be used to describes the dynamics of a Brownian particle in an Ohmic environment whose fluctuations are uncorrelated in space This should be contrasted with the Zwanzig-Caldeira-Leggett model, where the induced fluctuating force is assumed to be uniform in space (see figure).

At high temperatures the propagator possesses a Markovian property and one can write down an equivalent Master equation. Unlike the case of the Zwanzig-Caldeira-Leggett model, genuine quantum mechanical effects manifest themselves due to the disordered nature of the environment.

Using Wigner picture of the dynamics one can distinguish between two different mechanisms for destruction of coherence: scattering mechanism and smearing mechanism. The analysis of dephasing can be extended to the low temperature regime by using a semiclassical strategy. In this context the dephasing rate SP formula can be derived. Various results can be derived for ballistic, chaotic, diffusive, both ergodic and non-ergodic motion.

**In the following we give example of a recent study done in brief measure, not wholly or in full measure but very substantially as a review percussion:**

#### **Quantum Dissipation versus Classical Dissipation for Generalized Brownian motion (For details See Doron Cohen)**

Authors' try to clarify what are the genuine quantal effects that **are associated with** generalized Brownian Motion (BM). All the quantal effects that are **associated with** the Zwanzig-Feynman-Vernon-Caldeira-Leggett model are (formally) a solution of the classical Langevin equation. Non-stochastic, genuine quantum mechanical effects are found for a model that takes into account either the disordered or the chaotic nature of some environment.

The role of **dissipation in** curve crossing phenomena in condensed phases is discussed. Adiabaticity criteria are found in two regimes via path integral arguments. In the high temperature regime a stochastic quantum Langevin approach is developed. In the low temperature region an instanton method is used. In both cases the curve crossing time scale competes with the time scale of molecular motion. Dissipation causes the process to be more likely to be adiabatic.

Quantum optics is one of the liveliest fields in physics at present. While it has been a dominant research field for at least two decades, with much graduated but incremental activity, in the past few years it has started **to impact the** study of Quantum Computation. There are plenty of good research monographs in this field, but it was felt that there was a genuine need for a straightforward account .This is a field which attracts the brightest students at present, in part because of the **extraordinary progress** in the field (e.g. **the implementation of teleportation, quantum cryptography, Schrödinger cat states, Bell violations of local realism and the like**).

The presentation is almost entirely concerned with the quantized electromagnetic field and its effects on atoms, and how nonclassical light behaves. One aim of presently is to connect **quantum optics** with the newly developing subject of **quantum information processing**.

Topics covered are: single-mode field quantization in a cavity, quantization of multimode fields, the issue of the quantum phase, coherent states, quasi-probability distributions in phase space, atom-field interactions, the Jaynes-Cummings model, quantum coherence theory, beam splitters and interferometers, nonclassical field states with squeezing, etc., test of local realism with entangled photons from down-conversion, experimental realizations of cavity quantum electrodynamics, trapped ions, etc., issues regarding decoherence, and some applications to quantum information processing, particularly quantum cryptography. Recent studies include involvement of computational work, some more extensively than others.

#### **History**

The ancient world already was wrestling with the nature of light as rays. By the seventeenth century the two rival concepts of waves and corpuscles were well established. Maxwell, in the second half of the nineteenth century, laid the foundations of

modern field theory, with a detailed account of light as electromagnetic waves and at that point classical physics seemed triumphant, with “minor” worries about the nature of black-body radiation and of the photoelectric effect. These of course were the seeds of the quantum revolution. Planck, an inherently conservative theorist, was led rather reluctantly, it seems, to propose that thermal radiation **was emitted and absorbed** in discrete quanta in order to explain the spectra of thermal bodies. It was Einstein who generalized this idea so that these new quanta represented the light itself rather than the processes of **absorption and emission**, and was able to describe how matter and radiation could come into **equilibrium** (introducing on the way the idea of stimulated emission), and how the photoelectric effect could be explained. By 1913, Bohr applied the basic idea of quantization to atomic dynamics and was able to predict the positions of atomic spectral lines. Gilbert Lewis, a chemist, coined the word photon well after the light quanta idea itself was introduced. In 1926 Lewis said:

It would seem appropriate to speak of one of these hypothetical entities as a particle of light, a corpuscle of light, a light quantum, or light quant, if we are to assume that it spends only a minute fraction of its existence as a carrier of radiant energy, while the rest of the time it remains as an important structural element within the atom . . . I therefore take the liberty of proposing for this hypothetical new atom, which is not light but plays an important part in every process of radiation, the name photon

Clearly Lewis’s idea and ours are rather distantly connected!

De Broglie in a remarkable leap of imagination generalized what we knew about light quanta, **exhibiting** wave and particle properties to matter itself. Heisenberg, Schrödinger and Dirac laid the foundations of quantum mechanics in an amazingly short period from 1925 to 1926. They gave us the whole machinery we still use: representations, quantum-state evolution, unitary transformations, perturbation theory and more. The intrinsic probabilistic nature of quantum mechanics was uncovered by Max Born, who proposed the idea of probability amplitudes which allowed a fully quantum treatment of interference.

Fermi and Dirac, pioneers of quantum mechanics, were also among the first to address the question of how quantized light interacts with atomic sources and propagates. Fermi’s *Reviews of Modern Physics* article in the 1930s, based on lectures he gave in Ann Arbor, summarize what was known at that time within the context of nonrelativistic quantum electrodynamics in the Coulomb gauge. His treatment of interference (especially Lipmann fringes) still repays reading today. It is useful to quote Willis Lamb in this context:

Begin by deciding how much of the universe needs to be brought into the discussion. Decide what normal modes are needed for an adequate treatment. Decide how to model the light sources and work out how they drive the system [4].

Weisskopf and Wigner applied the newly developed ideas of non-relativistic quantum mechanics to the dynamics of spontaneous emission and resonance fluorescence, predicting the exponential law for excited-state decay. This work already exhibited the self-energy problems, which were to plague quantum electrodynamics for the next 20 years until the development of the renormalization programme by Schwinger, Feynman, Tomonaga, and Dyson. The observation of the **anomalous magnetic moment** of the electron by Kusch, and of radiative level shifts of atoms by Lamb and Retherford, were the highlights of this era. The interested reader will find the history of this period very ably described by Schweber in his magisterial account of QED. This period of research demonstrated the importance of considering the vacuum as a field which had observable consequences. In a remarkable development in the late 1940s, triggered by the observation that colloids were more stable than expected from considerations of van der Waals interactions, Casimir showed that long-range intermolecular forces were intrinsically quantum electrodynamic. He **linked them to the idea** of zero-point motion of the field and showed that metal plates in vacuum attract as a consequence of such zero-point motion.

Einstein had continued his study of the basic nature of quantum mechanics and in 1935 in a remarkable paper with Podolsky and Rosen was able to show how peculiar quantum correlations were. The ideas in this paper were to explode into one of the most active parts of modern physics with the development by Bohm and Bell of concrete predictions of the nature of **these correlations**; this laid the foundations of what was to become the new subject of quantum information processing.

Optical coherence had been investigated for many years using amplitude interference: a first-order correlation. Hanbury Brown and Twiss in the 1950s worked on intensity correlations as a tool in stellar interferometry, and showed how thermal photon detection events were “bunched.” This **led to the** development of the theory of photon statistics and photon counting and to the beginnings of quantum optics as a separate subject. At the same time as ideas of photon statistics were being developed, researchers had begun to investigate coherence in **light-matter interactions**. Radio-frequency spectroscopy had already been initiated with atomic beams with the work of Rabi, Ramsey and others. Sensitive optical pumping probes of light interaction with atoms were developed in the 1950s and 1960s by Kastler, Brossel, Series, Dodd and others.

By the early 1950s, Townes and his group, and Basov and Prokhorov, had developed molecular microwave sources of radiation: the new masers, based on precise initial state preparation, population inversion and stimulated emission. Ed Jaynes in the 1950s played a major role in studies of whether quantization played a role in maser operation (and this set the stage for much later work on fully quantized atom-field coupling in what became known as the Jaynes-Cummings model). Extending the maser idea to the optical regime and the development of lasers of course revolutionized modern physics and technology.

Glauber, Wolf, Sudarshan, Mandel, Klauder and many others developed a quantum theory of coherence based on coherent states and photodetection. Coherent states allowed us to describe the behaviour of light in phase space, using the quasi-probabilities developed much earlier by Wigner and others.

For several years after the development of the laser there were no tuneable sources: researchers interested in the details of atom-light or molecule-light interactions had to rely on molecular chance resonances. Nevertheless, this led to the beginning of the study of coherent interactions and coherent transients such as photon echoes, self-induced transparency, optical notation and so on (well described in the standard monograph by Allen and Eberly). Tuneable lasers became available in the

early 1970s, and the dye laser in particular transformed precision studies in quantum optics and laser spectroscopy. Resonant interactions, coherent transients and the like became much more straightforward to study and led to the beginnings of quantum optics proper as we now understand it: for the first time we were able to study the dynamics of single atoms interacting with light in a non-perturbative manner. Stroud and his group initiated studies of resonance fluorescence with the observation of the splitting of resonance fluorescence spectral lines into component parts by the coherent driving predicted earlier by Mollow. Mandel, Kimble and others demonstrated how the resonance fluorescence light was antibunched, a feature studied by a number of theorists including Walls, Carmichael, Cohen-Tannoudji, Mandel and Kimble. The observation of antibunching and the associated (but in equivalent) sub-Poissonian photon statistics laid the foundation of the study of “non-classical light”. During the 1970s, several experiments explored the nature of photons: their indivisibility and the buildup of interference at the single photon level. Laser cooling rapidly developed in the 1980s and 1990s and allowed the preparation of states of matter under precise control. Indeed, this has become a major subject in its own right and we have taken the decision here to exclude laser cooling from this text.

Following the development of high-intensity pulses of light from lasers, a whole set of nonlinear optical phenomena were investigated, starting with the pioneering work in Ann Arbor by Franken and co-workers. Harmonic generation, parametric down-conversion and other phenomena were demonstrated. For the most part, none of this early work on nonlinear optics required field quantization and quantum optics proper for its description. But there were early signs that some could well do so: quantum nonlinear optics was really initiated by the study by Burnham and Weinberg of unusual nonclassical correlations in down-conversion. In the hands of Mandel and many others, these correlations in down-conversion became the fundamental tool used to uncover fundamental insights into quantum optics.

Until the 1980s, essentially all light fields investigated had phase-independent noise; this changed with the production of squeezed light sources with phase-sensitive noise. These squeezed light sources enabled us to investigate Heisenberg uncertainty relations for light fields. Again, parametric down-conversion proved to be the most effective tool to generate such unusual light fields.

Quantum opticians realized quite early that were atoms to be confined in resonators, then atomic radiative transition dynamics could be dramatically changed. Purcell, in a remarkable paper in 1946 within the context of magnetic resonance, had already predicted that spontaneous emission rates previously thought of as pretty immutable were in fact modified by enclosing the source atom within a cavity whose mode structure and densities are significantly different from those of free space. Putting atoms within resonators or close to mirrors became possible at the end of the 1960s. By the 1980s the theorists' dream of studying single atoms interacting with single modes of the electromagnetic field became possible. At this point the transition dynamics becomes wholly reversible, as the atom coherently exchanges excitation with the field, until coherence is eventually lost through a dissipative “decoherence” process. This dream is called the Jaynes–Cummings model after its proposers and forms a basic building block of quantum optics .

New fundamental concepts in information processing, leading to quantum cryptography and quantum computation, have been developed in recent years by Feynman, Benioff, Deutsch, Jozsa, Bennett, Ekert and others. Instead of using classical bits that can represent either the values 0 or 1, the basic unit of a quantum computer is a quantum mechanical two-level system (qubit) that can exist in coherent superpositions of the logical values 0 and 1. A set of  $n$  qubits can then be in a superposition of up to  $2^n$  different states, each representing a binary number. Were we able to control and manipulate say 1500 qubits, we could access more states than there are particles in the visible universe. Computations are implemented by unitary transformations, which act on all states of a superposition simultaneously. Quantum gates form the basic units from which these unitary transformations are built up. In related developments, absolutely secure encryption can be guaranteed by using quantum sources of light.

The use of the quantum mechanical superpositions and entanglement results in a high degree of parallelism, which can increase the speed of computation exponentially. A number of problems which cannot feasibly be tackled on a classical computer can be solved efficiently on a quantum computer. In 1994 a quantum algorithm was discovered by Peter Shor that allows the solution of a practically important problem, namely factorization, with such an exponential increase of speed. Subsequently, possible experimental realizations of a quantum computer have been proposed, for example in linear ion traps and nuclear magnetic resonance schemes. Presently we are at a stage where quantum gates have been demonstrated in these two implementations. Quantum computation is closely related to quantum cryptography and quantum communication. Basic experiments demonstrating the in-principle possibility of these ideas have been carried out in various laboratories.

The linear ion trap is one of the most promising systems for quantum computation and is one we study in this book in detail. The quantum state preparation (laser cooling and optical pumping) in this system is a well-established technique, as is the state measurement by electron shelving and fluorescence. Singly charged ions of an atom such as calcium or beryllium are trapped and laser cooled to micro-Kelvin temperatures, where they form a string lying along the axis of a linear radio-frequency (r.f.) Paul trap. The internal state of any one ion can be exchanged with the quantum state of motion of the whole string. This can be achieved by illuminating the ion with a pulse of laser radiation at a frequency tuned below the ion's internal resonance by the vibrational frequency of one of the normal modes of oscillation of the string. This ouples single phonons into and out of the vibrational mode. The motional state can then be coupled to the internal state of another ion by directing the laser onto the second ion and applying a similar laser pulse. In this way general transformations of the quantum state of all the ions can be generated. The ion trap has several features to recommend it. It can achieve processing on quantum bits without the need for any new technological breakthroughs, such as micro-fabrication techniques or new cooling methods. The state of any ion can be measured and re-prepared many times without problem, which is an important feature for implementing quantum error correction protocols.



Trapped atoms or ions can be **strongly coupled to an** electromagnetic field mode in a cavity, which permits the powerful **combination of** quantum processing and long-distance quantum communication. This suggests ways in which we may construct quantum memories. These systems can in principle realize a quantum processor larger than any which could be thoroughly simulated by classical computing but the decoherence generated by dephasing and **spontaneous emission is a formidable obstacle.**

Entangled states are **the key ingredient** for certain forms of quantum cryptography and for quantum teleportation. Entanglement is also **responsible for** the power of quantum computing, which, under ideal conditions, **can accomplish** certain tasks exponentially faster than any classical computer. A deeper understanding of **the role of quantum** entanglement in quantum information theory **will allow us** to improve existing applications and to develop new methods of **quantum information manipulation.**

What then is the future of quantum optics? It underpins a great deal of laser science and novel atomic physics. It may even be the vehicle by which we can realize a whole new technology whereby quantum mechanics permits the processing and transmission of information in wholly novel ways. But of course, whatever we may predict now to emerge will be confounded by the unexpected: the field remains an adventure repeatedly throwing up the unexpected.

**ESSENTIAL PREDICATIONS, PREDICATIONAL ANTERIORITIES, PRIMORDIAL EXACTITUDE, ONTOLOGICAL CONSONANCE OF VARIOUS VARIABLES OF QUANTUM COMPUTING SUCH AS ELECTROMAGNETIC FIELD, HARMONIC OSCILLATORS, PHASE INFORMATION, AND SPONTANEOUS DECAY; WITH SOME CONCOMITANT AND CORRESPONDING STUDIES DONE IN ABSTRACT AND SENTENTIOUS MANNER:**

THE electromagnetic field can be quantized in terms of harmonic oscillators representing modes of the electromagnetic field, with states describing how many excitations (photons) are present in each normal mode.

1. Coherent states, superposition states carrying phase information are closely interrelated to each other
2. Light and matter interact.
3. Quantification of notions of coherence in terms of optical field correlation functions.
4. Optical elements such as beam splitters and interferometers, **manipulate the** states of light.
5. Nonclassical states have properties which are dictated by their fundamental quantum nature.
6. Spontaneous emission and decay in an open environment
7. How quantum optical sources of radiation can be used to provide tests of fundamental quantum mechanics, including tests of nonlocality and Bell inequalities.
8. Atoms confined in cavities and trapped laser-cooled ions **can be used to study** basic interaction phenomena.
9. Applications to the newly emerging problems of quantum information processing.

For a given 2-dimensional dissipative discrete **map generating** chaotic dynamics author presents the phenomenological construction of a quantum mechanical master equation **which reduces to the** given map in the classical limit. **Global dissipation**, caused by the non-invertibility of the map, and local dissipation, **caused by the** local contraction of the map, are both incorporated in the description. The behavior in the two opposite limits of vanishing **local dissipation and of strong local dissipation** is analyzed exactly. Using the representation of the statistical operator by the Wigner distribution, the classical and semi-classical limit has been studied. An estimate of the critical time is obtained, which determines the **crossover** between classical and quantum mechanical behavior in the chaotic state. This critical time diverges logarithmically for  $\hbar \rightarrow 0$ . In the main body of the text Emphasis is given to physical meaning **of dissipation** of background **fields due to** particle creation and statistical effects in **interacting** quantum field theories and in semiclassical gravitational theories. We indicate the possible existence of a **fluctuation-dissipation relation** for non-equilibrium quantum fields as occurring in cosmological particle **creation and back reaction processes**. Thus in the total, the sum and substance of the write is that it can be conjectured that all effective theories, including quantum gravity, **could manifest** dissipative behavior.

**Effect of** dissipation on the quantum-mechanical behavior of a macroscopic variable, in particular in situations **involving tunnelling** through a classically impenetrable barrier. Question of dissipation can be placed in the context of quantum measurement theory, and emphasize the importance of the distinction between the "adiabatic" and "dissipative" aspects **of the coupling of a macroscopic** variable to its environment. Next, there is a possible theoretical framework for the problem, with particular attention to how far the necessary input **parameters can be deduced** from the purely classical behavior of the system in question. Theoretical results obtained within this framework for the effects **of dissipation both on the decay of a metastable** state ("macroscopic quantum tunnelling") and on the coherent oscillations of a macroscopic two-state system ("macroscopic quantum coherence").

**SOME RECENT DEVELOPMENTS:**

The aim of recent research has been is the description of the electrical transport properties of Nanoscale systems. These properties **are influenced by** fundamental physical properties such as quantum mechanical coherence, dissipation, and **interaction among the charge carriers**. They are used to develop methods to describe the large variety of Nanoscale systems, but also work on the description of new measurable observables like current noise and **counting statistics** that provide a deeper understanding of the relevant transport processes. The following examples give an overview of the research:

**Electron transport through quantum dot systems and molecules**

In Nanoscale quantum dots electronic interactions play a **dominant role**, leading, e.g. to the Coulomb blockade. Electron transport and current noise using various methods like diagrammatic perturbation theory, full counting statistics, or path integral techniques are also taken in to consideration. Similar questions are relevant for charge transport through molecules. However, in long molecules (e.g. DNA) also inelastic transport processes and dissipation of energy to the molecular environment play an important role.

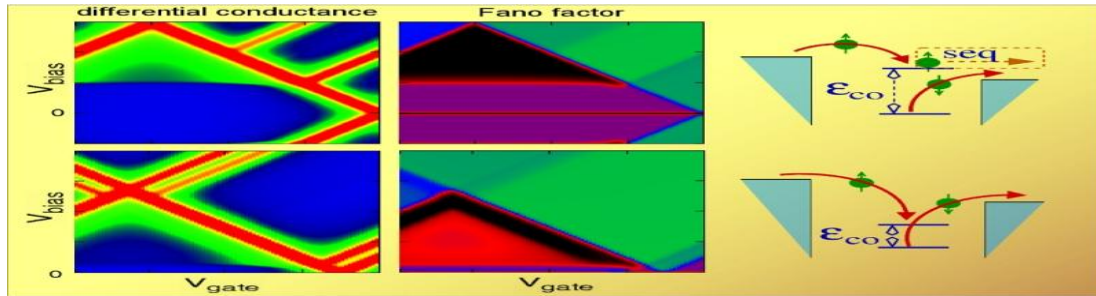


Figure 1: Differential conductance and shot **noise (Fano+ factor) of** a quantum dot with two energy levels. Within the so-called "Coulomb diamond" (area at low transport bias  $V_{bias}$  **limited by** the red lines of high differential conductance) second order processes ("**Co-tunneling**") are dominant. The sketches (right) show the dominant processes at small transport bias  $V_{bias}$ .

**Electronic properties of carbon nanotubes and graphene**

Carbon nanotubes and graphene are often considered as ideal examples of coherent quantum transport. In experiments, however, often effects due to disorder, in homogeneity of the environment, or phonon scattering are of importance. Some work studies observables like current noise in various experimental situations.

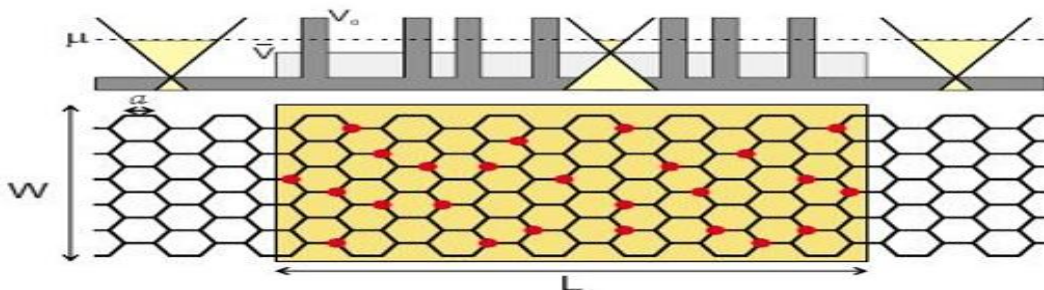


Figure 2: Sketch of the structure of a graphene ribbon with binary disorder. The disorder causes local scattering potentials  $V_s$  and a shift of the chemical potential  $\mu$  away from the "Dirac points". The **conductivity depends on** the length of the graphene ribbon.

**Transport through hybrid structures**

In hybrid structures, materials of strongly differing (or even complementary) properties are brought together. An example is the combination of superconducting and ferromagnetic metals. Transport through such structures show phenomena that are so far poorly understood, e.g. Non-local Andreev reflections, that we describe by Green function methods. Hybrid structures are particularly interesting for applications, as their integration with established (semiconductor) technologies is feasible.

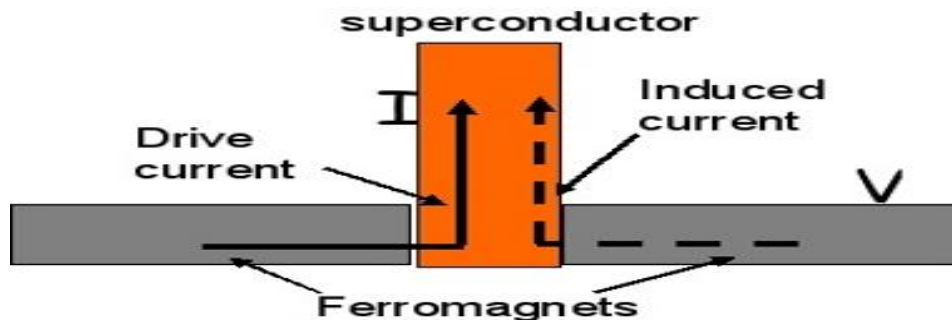


Figure 3: Sketch of a superconducting-ferromagnetic hybrid structure. The behavior of the non-local transport properties is under investigation.

**Atomic contacts**

The smallest possible transport system consists of single atoms. Here, the difficulty lies in the manufacturing of stable structure, with the possibility to control them via external "knobs". Our research applies a combination of Monte Carlo simulations and tight binding transport calculations, to make the bridge from a purely statistical description to possible applications of such systems with predictable transport properties.

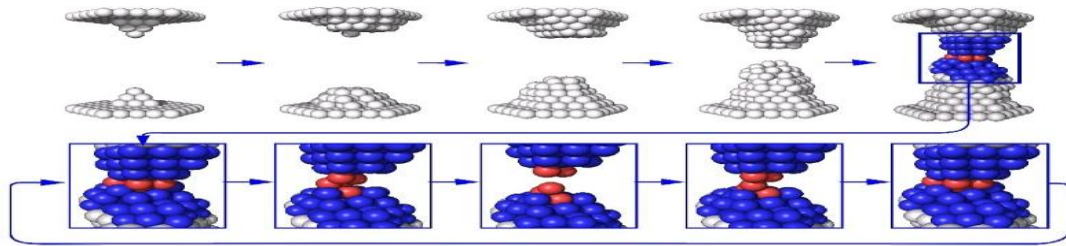


Figure 4: Monte Carlo simulation of the growth of an atomic contact of silver. In specific configurations the controlled opening and closing of the contact is possible.

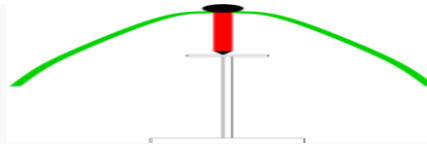
**T Symmetry** is the symmetry of physical laws under a **time reversal** transformation:

$$T : t \mapsto -t.$$

Although in restricted contexts one may find this symmetry, the observable universe itself does not show symmetry under time reversal, primarily due to the second law of thermodynamics. Time *asymmetries* are generally distinguished as between those intrinsic to the dynamic laws of nature, and those due to the initial conditions of our universe. The T-*asymmetry* of the weak force is of the first kind, while the T-*asymmetry* of the second law of thermodynamics is of the second kind.

### Invariance

Physicists also discuss the time-reversal invariance of local and/or macroscopic descriptions of physical systems, independent of the invariance of the underlying microscopic physical laws. For example, Maxwell's equations with material absorption or Newtonian mechanics with friction are not time-reversal invariant at the macroscopic level where they are normally applied, even if they are invariant at the microscopic level when one includes the atomic motions the "lost" energy is translated into.



A toy called the teeter-totter illustrates the two aspects **of time reversal invariance**. When set into motion atop a pedestal, the figure oscillates for a very long time. The toy is engineered to minimize friction and illustrate the reversibility of Newton's laws of motion. However, the mechanically stable state of the toy is when the figure falls down from the pedestal into one of arbitrarily many positions. This is an illustration of the law of increase of entropy through Boltzmann's identification of the logarithm of the number of states with the entropy.

### Macroscopic phenomena: the second law of thermodynamics

Our daily experience shows that T-symmetry does not hold for the behavior of bulk materials. Of these macroscopic laws, most notable is the second law of thermodynamics. Many other phenomena, such as the relative motion of bodies with friction, or viscous motion of fluids, **reduce to this, because the underlying mechanism is the dissipation of usable energy (for example, kinetic energy) into heat.**

Is this time-asymmetric **dissipation** really inevitable? This question has been considered by many physicists, often in the context of Maxwell's. The name comes from a thought experiment described by James Clerk Maxwell in which a microscopic demon guards a gate between two halves of a room. It only lets slow molecules into one half, only fast ones into the other. By eventually making one side of the room cooler than before and the other hotter, it seems **to reduce** the entropy of the room, **and reverse** the arrow of time. Many analysts have been made of this; all show that when the entropy of room and demon are taken together, this total entropy **does increase**. Modern analyses of this problem have taken into account Claude E. Shannon's relation between entropy and information. Many interesting results in modern computing are **closely related** to this problem — reversible computing, quantum computing and physical limits to computing, are examples. These seemingly metaphysical questions are today, in these ways, slowly being converted to the stuff of the physical sciences.

The current consensus hinges upon the Boltzmann-Shannon identification of the logarithm of phase space volume with the negative of Shannon, and hence to entropy. In this notion, a fixed initial state of a macroscopic system corresponds to relatively low **entropy because the** coordinates of the molecules of the body **are constrained**. As the system **evolves in** the presence of dissipation, the molecular coordinates can move into larger volumes of phase space, **becoming more** uncertain, and thus **leading to** increase in entropy.

One can, however equally well imagine a state of the universe in which the motions of all of the particles at one instant were the reverse (strictly, the CPT reverse). Such a state would then evolve in reverse, so presumably entropy would decrease (Loschmidt's paradox). Why is 'our' state preferred over the other?

One position is to say that the constant increase of entropy we observe **happens only because** of the initial state of our universe. Other possible states of the universe (for example, a universe at heat death equilibrium) would actually **result in** no increase of entropy. In this view, the apparent T-asymmetry of our universe is a problem in cosmology: why did the universe start with low entropy? This view, if it remains viable in the light of future cosmological observation, would connect this problem to one of the big open questions beyond the reach of today's physics — the question of initial conditions of the universe.

### **QUANTUM DISSIPATION AT MACROSCOPIC LEVEL- BLACK HOLES AND COSMOLOGY:**

An object can cross through the event horizon of a black hole from the outside, and then fall rapidly to the central region where our understanding of physics breaks down. Since within a black hole the forward light-cone is directed towards the center and the backward light-cone is directed outward, it is not even possible to define time-reversal in the usual manner. The only way anything can escape from a black hole is as Hawking radiation.

The time reversal of a black hole would be a hypothetical object known as a white hole. From the outside they appear similar. While a black hole has a beginning and is inescapable, a white hole has an ending and cannot be entered. The forward light-cones of a white hole are directed outward; and its backward light-cones are directed towards the center.

The event horizon of a black hole may be thought of as a surface moving outward at the local speed of light and is just on the edge between escaping and falling back. The event horizon of a white hole is a surface moving inward at the local speed of light and is just on the edge between being swept outward and succeeding in reaching the center. They are two different kinds of horizons—the horizon of a white hole is like the horizon of a black hole turned inside-out.

The modern view of black hole irreversibility is to **relate it to** the second law of thermodynamics, since black holes are viewed as thermodynamic objects. Indeed, according to the Gauge-gravity duality conjecture, all microscopic processes in a black hole are **reversible**, and only the collective **behavior is irreversible**, as in any other macroscopic, thermal system.

Kinetic consequences: detailed balance and Onsager reciprocal relations

In physical and chemical kinetics, T-symmetry of the mechanical microscopic **equations implies two important** laws: the principle of detailed **balance and** the Onsager reciprocal relations. T-symmetry of the microscopic description together with its kinetic **consequences are** called microscopic reversibility.

Effect of time reversal on some variables of classical physics

Classical variables that do not change upon time reversal include:

$\vec{x}$ , Position of a particle in three-space

$\vec{a}$ , Acceleration of the particle

$\vec{F}$ , Force on the particle

$E$ , Energy of the particle

$\phi$ , Electric potential (voltage)

$\vec{E}$ , Electric field

$\vec{D}$ , Electric displacement

$\rho$ , Density of electric charge

$\vec{P}$ , Electric polarization

Energy density of the electromagnetic field

Maxwell stress tensor

All masses, charges, coupling constants, and other physical constants, except those associated with the weak force.

**Odd**

**Classical variables that time reversal negates include:**

$t$ , The time when an event occurs

$\vec{v}$ , Velocity of a particle

$\vec{P}$ , Linear momentum of a particle

$\vec{l}$ , Angular momentum of a particle (both orbital and spin)

$\vec{A}$ , Electromagnetic vector potential

$\vec{B}$ , Magnetic induction

$\vec{H}$ , Magnetic field

$\vec{j}$ , Density of electric current

$\vec{M}$ , Magnetization

$\vec{S}$ , Poynting vector

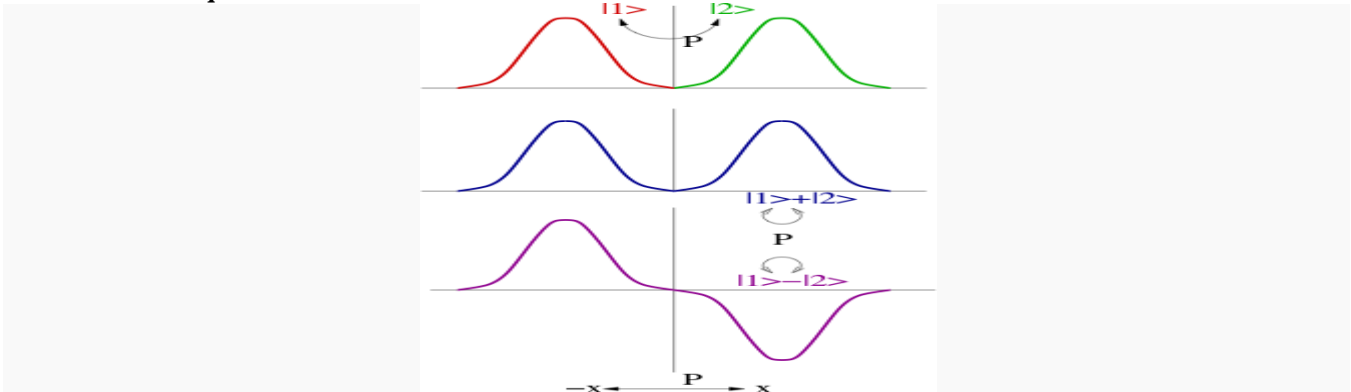
Power (rate of work done).

**Microscopic phenomena: time reversal invariance**

Since most systems are asymmetric under time reversal, it is interesting to ask whether there are phenomena **that do have** this symmetry. In classical mechanics, a **velocity reverses** under the operation of  $T$ , but **acceleration does not**. Therefore, one models dissipative phenomena through terms that are odd in  $v$ . However, delicate experiments in which **known sources of dissipation are removed** reveal that the laws of mechanics are time reversal invariant. **Dissipation itself is originated in the second law of thermodynamics.**

The motion of a charged body in a magnetic field,  $B$  **involves the** velocity through the Lorentz force term  $v \times B$ , and might seem at first to be asymmetric under  $T$ . A closer look assures us that  $B$  also changes sign under time reversal. This happens because a magnetic field **is produced** by an electric current,  $J$ , **which reverses sign** under  $T$ . Thus, the motion of classical charged particles in electromagnetic fields is also time reversal invariant. (Despite this, it is still useful to consider the time-reversal non-invariance in a *local* sense when the external field is held fixed, as when the magneto-optic effect is analyzed. This allows one to analyze the conditions under **which optical phenomena** that locally break time-reversal, such as Faraday isolators, can occur.) **The laws of gravity also** seem to be time reversal invariant in classical mechanics.

In physics one separates the laws of motion, called kinematics, from the laws of force, called dynamics. Following the classical kinematics of Newton's laws of motion, the kinematics of quantum mechanics is built in such a way that it presupposes nothing about the time reversal symmetry of the dynamics. In other words, if the **dynamics are invariant**, then the kinematics will allow it to remain invariant; if the dynamics is not, and then the kinematics will also show this. The structure of the quantum laws of motion is richer.

**Time reversal in quantum mechanics**

Two-dimensional representations of parity are given by a pair of quantum states that go into each other under parity. However, this representation can always be **reduced to** linear combinations of states, each of which is either even or odd under parity. One says that all irreducible representations of parity are one-dimensional. **Kramers' theorem** states that time reversal need not have this property **because it** is represented by an anti-unitary operator.

Fundamental properties of time reversal in quantum mechanics are:

1. that it must be **represented as** an anti-unitary operator,
2. that **it protects** non-degenerate quantum states from having an electric dipole moment,
3. that it has two-dimensional representations with the property  $T^2 = -1$ .

The strangeness of this result is clear if one compares it with parity. If parity **transforms a** pair of quantum states into each other, then the sum and difference of these two basis states are states of good parity. **Time reversal does not behave like this.** It seems **to violate the** theorem that all abelian groups be represented by one dimensional irreducible representation. **The reason it does this is that it is represented by an anti-unitary operator.** It thus opens the way to spinors in quantum mechanics.

**Anti-unitary representation of time reversal**

Eugene Wigner showed that a symmetry operation  $S$  of a Hamiltonian is represented, in quantum mechanics either by a **unitary operator,  $S = U$ , or an antiunitary one,  $S = UK$  where**  $U$  is unitary, and  $K$  denotes complex conjugation. These are the only operations that act on Hilbert space so as to preserve the *length* of the projection of any one state-vector onto another state-vector.

Consider the **parity operator**. Acting on the position, it **reverses the** directions of space, so that  $P^{-1}xP = -x$ . Similarly, it **reverses the** direction of *momentum*, so that  $PpP^{-1} = -p$ , where  $x$  and  $p$  are the position and momentum operators. This preserves the canonical  $[x, p] = i\hbar$ , where  $\hbar$  is the reduced Planck constant, only if  $P$  is chosen to be unitary,  $PiP^{-1} = i$ .

On the other hand, for time reversal, the time-component of the momentum is the energy. If time reversal were implemented as a unitary operator, it **would reverse the sign of the energy** just as space-reversal **reverses the sign of the momentum**. This is not possible, because, unlike momentum, energy is always positive. Since energy in quantum mechanics is defined as **the phase factor**  $\exp(-iEt)$  that one gets when one moves forward in time, the way to reverse time while preserving the sign of the energy is to reverse the sense of "i", so that the sense of phases is reversed.

Similarly, any operation that reverses the sense of phase, which changes the sign of  $i$ , will turn positive energies into negative energies unless it also changes the direction of time. So every antiunitary symmetry in a theory with positive energy must reverse the direction of time. The only antiunitary symmetry is time reversal, together with a unitary symmetry that does not reverse time.

Given the *time reversal* operator  $T$ , it does nothing to the  $x$ -operator,  $TxT^{-1} = x$ , but it reverses the direction of  $p$ , so that  $TpT^{-1} = -p$ . The canonical commutator is invariant only if  $T$  is chosen to be anti-unitary, i.e.,  $TiT^{-1} = -i$ . For a particle with spin  $J$ , one can use the representation

$$T = e^{-i\pi J_y/\hbar} K,$$

where  $J_y$  is the  $y$ -component of the spin, and use of  $TJT^{-1} = -J$  has been made.

### Electric dipole moments

This has an interesting consequence on the electric dipole moment (EDM) of any particle. The EDM is defined through the shift in the energy of a state when it is put in an external electric field:  $\Delta e = d \cdot E + E \cdot \delta \cdot E$ , where  $d$  is called the EDM and  $\delta$ , the induced dipole moment. One important property of an EDM is that the energy shift due to it changes sign under a parity transformation. However, since  $\mathbf{d}$  is a vector, its expectation value in a state  $|\psi\rangle$  must be proportional to  $\langle \psi | J | \psi \rangle$ . Thus, under time reversal, an invariant state must have vanishing EDM. In other words, a non-vanishing EDM signals both  $P$  and  $T$  symmetry-breaking.

It is interesting to examine this argument further, since one feels that some molecules, such as water, must have EDM irrespective of whether  $\mathbf{T}$  is symmetry. This is correct: if a quantum system has degenerate ground states that transform into each other under parity, then time reversal need not be broken to give EDM.

Experimentally observed bounds on the electric dipole moment of the nucleon currently set stringent limits on the violation of time reversal symmetry in the strong interactions, and their modern theory: quantum chromodynamics. Then, using the CPT invariance of a relativistic quantum field theory, this puts strong bounds on strong CP violation. Experimental bounds on the electron electric dipole moment also place limits on theories of particle physics and their parameters.

### Kramers' theorem

For  $T$ , which is an anti-unitary  $Z_2$  symmetry generator

$$T^2 = UKUK = U U^* = U (U^T)^{-1} = \Phi,$$

where  $\Phi$  is a diagonal matrix of phases. As a result,  $U = \Phi U^T$  and  $U^T = U \Phi$ , showing that

$$U = \Phi U \Phi.$$

This means that the entries in  $\Phi$  are  $\pm 1$ , as a result of which one may have either  $T^2 = \pm 1$ . This is specific to the anti-unitarity of  $T$ . For a unitary operator, such as the parity, any phase is allowed. Next, take a Hamiltonian invariant under  $T$ . Let  $|a\rangle$  and  $T|a\rangle$  be two quantum states of the same energy. Now, if  $T^2 = -1$ , then one finds that the states are orthogonal: a result called **Kramers' theorem**. This implies that if  $T^2 = -1$ , then there is a twofold degeneracy in the state. This result in non-relativistic quantum mechanics presages the spin statistics theorem of quantum field theory.

Quantum states that give unitary representations of time reversal, i.e., have  $\mathbf{T}^2=1$ , are characterized by a multiplicative quantum number, sometimes called the **T-parity**.

Time reversal transformation for fermions in quantum field theories can be represented by an 8-component spinor in which the above mentioned **T-parity** can be a complex number with unit radius. The CPT invariance is not a theorem but a **better to have** property in these classes of theories.

### Time reversal of the known dynamical laws

Particle physics codified the basic laws of dynamics into the standard model. This is formulated as a quantum field theory that has CPT symmetry, i.e., the laws are invariant under simultaneous operation of time reversal, parity and charge conjugation. However, time reversal itself is seen not to be symmetry (this is usually called CP violation). There are two possible origins of this asymmetry, one through the mixing of different flavours of quarks in their weak decays, the second through a direct CP violation in strong interactions. The first is seen in experiments, the second is strongly constrained by the non-observation of the EDM of a neutron.

It is important to stress that this time reversal violation is unrelated to the second law of thermodynamics, because due to the conservation of the CPT symmetry, the effect of time reversal is to rename particles as antiparticles and *vice versa*. Thus the second law of thermodynamics is thought to originate in the initial conditions in the universe.

### SOME RECENT DEVELOPMENTS IN THE CONTEXT OF QUANTUM INFORMATION OR QUANTUM MECHANICAL BEHAVIOUR DISSIPATION: A REVIEW STUDY THAT SUBSTANTIATES THE DISSIPATION COEFFICIENT USAGE IN THE MODEL:

**Influence of dissipation on phase tunneling in Josephson-junctions (For details see. Zwerger)**

The coupling between the phase and the electromagnetic field in the case of a tunnel junction is treated by Feynman's path integral method. It is shown that the elimination of the field leads to a frequency dependent mass for the motion of the

phase  $\psi$ , which is simply related to the effective dielectric constant of the junction. Considering tunneling as a motion in

imaginary time **one obtains** a polaron like mass enhancement **connected** to the dielectric function at positive imaginary frequencies, which **essentially leads to** the Caldeira-Leggett **reduction** of the elastic tunneling probability. In the weak damping limit it is shown that the emission of real excitations during tunneling is a higher order effect. At low temperatures the damping finally **is determined** by the line width of electromagnetic radiation at the Josephson plasma frequency

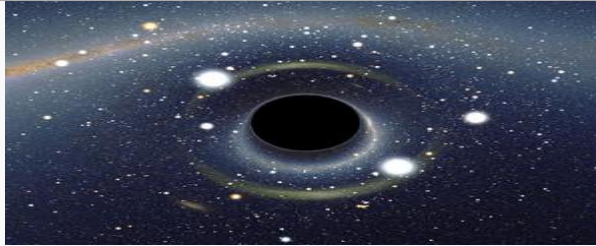
### **The quantum-mechanical two-level system with dissipation and feedback: multimode applied electric-field case**

A two-level dissipative quantum-mechanical system subjected to a small applied multimode electric field plus a feedback term proportional to the induced dipole moment is discussed. The long-term behaviour of the induced dipole moment and of the upper level occupation probability is represented by a series expansion in terms of the applied electric-field amplitudes. First-order and third-order internal, external, and intrinsic polarizabilities **are introduced** and calculated for the model. Certain ratios of third-order to first-order polarizabilities are shown to have a common value for the internal, external, and intrinsic cases. Clausius-Mossotti relations for the linear and third-order susceptibilities **are derived**.

### **The quantum mechanical fluctuation-dissipation theorem:**

**In this paper, Kubo's spectral theorem is re-derived and developed into a fluctuation-dissipation theorem for systems which fluctuate about their equilibrium positions. This is accomplished by introducing a general link between the operator corresponding to the observable and the operator which joins the external force to the Hamiltonian. For certain systems this link takes on simple forms and the theorem then reduces to previously derived results. It is also shown how different forms of the link are related to different Langevin equations which are the basis of the corresponding classical theory.**

Black hole: Quantum Information Loss:



Simulated view of a black hole (center) in front of the Large. Note the gravitational lensing effect, which **produces** two enlarged but highly distorted views of the Cloud. Across the top, the Milky Way disk appears distorted into an arc.

A **black hole** is a region of spacetime whose gravitational field is so strong that nothing which enters it, not even light, can escape. The theory of general relativity predicts that a sufficiently compact mass **will deform** spacetime to form a black hole. Around a black hole there is a mathematically defined surface called an **event horizon** that marks the point of no return. It is called "black" because it absorbs all the light that hits the horizon, reflecting nothing, just like a perfect **black body in thermodynamics**. Quantum mechanics predicts that black holes emit radiation like a black body with a finite temperature. This temperature is **inversely proportional** to the mass of the black hole, making it difficult to observe this radiation for **black holes of stellar mass** or greater.

Objects whose **gravity field** is too strong for light to escape were first considered in the 18th century by John Michell and Pierre-Simon Laplace. The first modern solution of general relativity that would characterize a black hole was found by Karl Schwarzschild in 1916, although its interpretation as a region of space from which nothing can escape was not fully appreciated for another four decades. Long considered a mathematical curiosity, it was during the 1960s that theoretical work showed black holes were a generic prediction of general relativity. The discovery of **neutron stars** sparked interest in **gravitationally compact** objects as a possible astrophysical reality.

Black holes of stellar mass are expected to form when a star of more than 5 solar masses runs out of energy fuel. **This results** in the outer layers of gas being thrown out in a supernova explosion. The core of the star **collapses** and **becomes** super dense where even the atomic nuclei are squeezed together. The energy density at the core goes to infinity. After a black hole has **formed** it can continue to grow **by absorbing mass** from its surroundings. By absorbing other stars and **merging with** other black holes, supermassive black holes of millions of solar masses may form. There is general consensus that supermassive black holes exist in the centers of most galaxies. In particular, there is strong evidence of a black hole of more than 4 million solar masses at the center of our galaxy, the Milky Way.

Despite its invisible interior, the presence of a black hole can be inferred through **its interaction** with other **matter and with light** and other electromagnetic radiation. From stellar movement, the mass and location of an invisible companion object can be calculated. A half-dozen or so binary star systems have been discovered by Astronomers where one of the stars is invisible, yet must surely exist since it pulls with enough gravitational force on the other visible star to make it orbit around their common center of gravity. Therefore these invisible stars are thought to be good **candidate black holes**. **Astronomers have identified numerous stellar black hole candidates in binary systems by studying the movement of their companion stars in this way.**

**GRAVITATIONAL LENSING-PRODUCTION OF DISTORTED PICTURE:**



Simulation of gravitational lensing by a black hole, **which distorts** the image of a galaxy in the background (larger animation)

The idea of a body so massive that even light could not escape was first put forward by geologist John Michell in a letter written to Henry Cavendish in 1783 of the Royal Society:

If the semi-diameter of a sphere of the same density as the Sun were to exceed that of the Sun in the proportion of 500 to 1, a body falling from an infinite height towards it would have acquired at its surface greater velocity than that of light, and consequently supposing light to be attracted by the same force in proportion to its vis inertiae, with other bodies, all light emitted from such a body would be made to return towards it by its own proper gravity.

—John Michell

In 1796, mathematician Pierre-Simon Laplace promoted the same idea in the first and second editions of his book *Exposition du système du Monde* (it was removed from later editions). Such "dark stars" were largely ignored in the nineteenth century, since it was not understood how a massless wave such as light could be influenced by gravity.

#### **STABILITY ANALYSIS OR UNSTABILITY AT MACROSCOPIC LEVEL: A POINTER TO IMPORTANCE TO STABILITY ANALYSIS CARRIED OUT IN THE MIODEL:**

In 1915, Albert Einstein developed his theory of general relativity, having earlier shown that gravity does influence light's motion. Only a few months later, Karl Schwarzschild found a solution to Einstein field equations, which describes the gravitational field of a point mass and a spherical mass. A few months after Schwarzschild, Johannes Droste, a student of Hendrik Lorentz, independently gave the same solution for the point mass and wrote more extensively about its properties. This solution had a peculiar behaviour at what is now called the Schwarzschild radius, where it became singular, meaning that some of the terms in the Einstein equations became infinite. The nature of this surface was not quite understood at the time. In 1924, Arthur Eddington showed that the singularity disappeared after a change of coordinates (see Eddington–Finkelstein coordinates), although it took until 1933 for Georges Lemaître to realize that this meant the singularity at the Schwarzschild radius was an unphysical coordinate singularity.<sup>3</sup>

In 1931, Subrahmanyan Chandrasekhar calculated, using special relativity, that a non-rotating body of electron-degenerate matter above a certain limiting mass (now called the Chandrasekhar at 1.4 solar masses) has no stable solutions. His arguments were opposed by many of his contemporaries like Eddington and Lev Landau, who argued that some yet unknown mechanism would stop the collapse. They were partly correct: a white dwarf slightly more massive than the Chandrasekhar limit will collapse into a neutron star, which is itself stable because of the Pauli exclusion principle. But in 1939, Robert Oppenheimer and others predicted that neutron stars above approximately three solar masses (the Tolman–Oppenheimer–Volkoff limit) would collapse into black holes for the reasons presented by Chandrasekhar, and concluded that no law of physics was likely to intervene and stop at least some stars from collapsing to black holes.

Oppenheimer and his co-authors interpreted the singularity at the boundary of the Schwarzschild radius as indicating that this was the boundary of a bubble in which time stopped. This is a valid point of view for external observers, but not for infalling observers. Because of this property, the collapsed stars were called "frozen stars, because an outside observer would see the surface of the star frozen in time at the instant where its collapse takes it inside the Schwarzschild radius.

#### **BLACK HOLES: THE MACROSCOPIC LEVEL IMPECCABLE TESTIMONY FOR THE QUNTUM DISSIPATION EFFECTS (HAWKING RADIATION):**

In 1958, David Finkelstein identified the Schwarzschild surface as an event horizon, "a perfect unidirectional membrane: causal influences can cross it in only one direction". This did not strictly contradict Oppenheimer's results, but extended them to include the point of view of infalling observers. Finkelstein's solution extended the Schwarzschild solution for the future of observers falling into a black hole. A complete extension had already been found by Martin Kruskal,

These results came at the beginning of the golden age of general relativity, which was marked by general relativity and black holes becoming mainstream subjects of research. This process was helped by the discovery of pulsars in 1967, which, by 1969, were shown to be rapidly rotating neutron stars. Until that time, neutron stars, like black holes, were regarded as just theoretical curiosities; but the discovery of pulsars showed their physical relevance and spurred a further interest in all types of compact objects that might be formed by gravitational collapse.



In this period more general black hole solutions were found. In 1963, Roy Kerr found the exact solution for a rotating black hole. Two years later, Ezra Newman found the axisymmetric solution for a black hole that is both rotating and electrically charged. Through the work of Werner Israel, Brandon Carter, and David Robinson, the no-hair theorem emerged, stating that a stationary black hole solution is completely described by the three parameters of the Kerr–Newman metric; mass, angular momentum, and electric charge.

For a long time, it was suspected that the strange features of the black hole solutions were pathological artifacts from the symmetry conditions imposed, and that the singularities would not appear in generic situations. This view was held in particular by Vladimir Belinsky, Isaak Khalatnikov, and Evgeny Lifshitz, who tried to prove that no singularities appear in generic solutions. However, in the late sixties Roger Penrose and Stephen Hawking used global techniques to prove those singularities are generic.

Work by James Bardeen, Jacob Bekenstein, Carter, and Hawking in the early 1970s led to the formulation of black hole thermodynamics. These laws describe the behaviour of a black hole in close analogy to the laws of thermodynamics by relating mass to energy, area to entropy, and surface gravity to temperature. The analogy was completed when Hawking, in 1974, showed that quantum field theory predicts that black holes should radiate like a black body with a temperature proportional to the surface gravity of the black hole.

The term "black hole" was first publicly used by John Wheeler during a lecture in 1967. Although he is usually credited with coining the phrase, he always insisted that it was suggested to him by somebody else..

Properties and structure

The no-hair theorem states that, once it achieves a stable condition after formation, a black hole has only three independent physical properties: mass, charge, and angular momentum. Any two black holes that share the same values for these properties, or parameters, are indistinguishable according to classical (i.e. non-quantum) mechanics. These properties are special because they are visible from outside a black hole. For example, a charged black hole repels other like charges just like any other charged object. Similarly, the total mass inside a sphere containing a black hole can be found by using the gravitational analog of Gauss's law, the ADM mass, far away from the black hole. Likewise, the angular momentum can be measured from far away using frame dragging by the gravitomagnetic field.

### QUANTUM INFORMATION LOSS PARADOX IN BLACK HOLE:

When an object falls into a black hole, any information about the shape of the object or distribution of charge on it is evenly distributed along the horizon of the black hole, and is lost to outside observers. The behavior of the horizon in this situation is a dissipative system that is closely analogous to that of a conductive stretchy membrane with friction and electrical—the membrane paradigm. This is different from other field theories like electromagnetism, which do not have any friction or resistivity at the microscopic level, because they are time-reversible. Because a black hole eventually achieves a stable state with only three parameters, there is no way to avoid losing information about the initial conditions: the gravitational and electric fields of a black hole give very little information about what went in. The information that is lost includes every quantity that cannot be measured far away from the black hole horizon, including the total baryon number, lepton number, and all the other nearly conserved pseudo-charges of particle physics. This behavior is so puzzling that it has been called the black hole information loss paradox.

#### **Physical properties**

The simplest black holes have mass but neither electric charge nor angular momentum. These black holes are often referred to as Schwarzschild black holes after Karl Schwarzschild who discovered this solution in 1916.<sup>1</sup> According to Birkhoff's theorem, it is the only vacuum solution that is spherically symmetric. This means that there is no observable difference between the gravitational field of such a black hole and that of any other spherical object of the same mass. The popular notion of a black hole "sucking in everything" in its surroundings is therefore only correct near a black hole's horizon; far away, the external gravitational field is identical to that of any other body of the same mass.

Solutions describing more general black holes also exist. Charged black holes are described by the Reissner–Nordström metric, while the Kerr metric describes a rotating black hole. The most general stationary black hole solution known is the Kerr–Newman metric, which describes a black hole with both charge and angular momentum.

While the mass of a black hole can take any positive value, the charge and angular momentum are constrained by the mass. In Planck units, the total electric charge  $Q$  and the total angular momentum  $J$  are expected to satisfy

$$Q^2 + \left(\frac{J}{M}\right)^2 \leq M^2$$

for a black hole of mass  $M$ . Black holes saturating this inequality are called extremal. Solutions of Einstein's equations that violate this inequality exist, but they do not possess an event horizon. These solutions have so-called naked singularities that can be observed from the outside, and hence are deemed unphysical. The hypothesis rules out the formation of such singularities, when they are created through the gravitational collapse of realistic matter. This is supported by numerical simulations.

Due to the relatively large strength of the electromagnetic force, black holes forming from the collapse of stars are expected to retain the nearly neutral charge of the star. Rotation, however, is expected to be a common feature of compact

objects. The black-hole candidate binary X-ray source **GRS 1915+105<sup>[41]</sup>** appears to have an angular momentum near the maximum allowed value.

### Black hole classifications

#### Class\*Mass\*Size

Supermassive black hole\* $\sim 10^5$ – $10^9 M_{\text{Sun}}$ \* $\sim 0.001$ – $10$  AU

Intermediate-mass black hole\* $\sim 10^3 M_{\text{Sun}}$ \* $\sim 10^3$  km =  $R_{\text{Earth}}$

Stellar black hole\* $\sim 10 M_{\text{Sun}}$ \* $\sim 30$  km

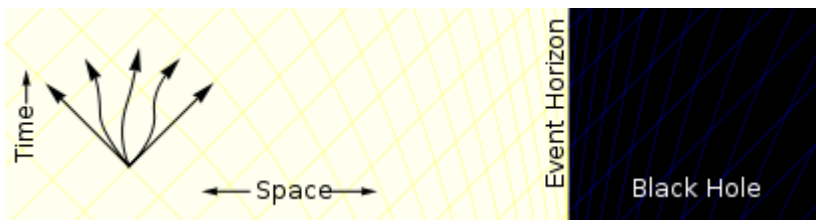
Micro black hole\*up to  $\sim M_{\text{Moon}}$ \*up to  $\sim 0.1$  mm

Black holes are commonly classified according to their mass, independent of angular momentum  $J$  or electric charge  $Q$ . The size of a black hole, as determined by the radius of the event horizon, or **Schwarzschild radius**, is roughly proportional to the mass  $M$  through

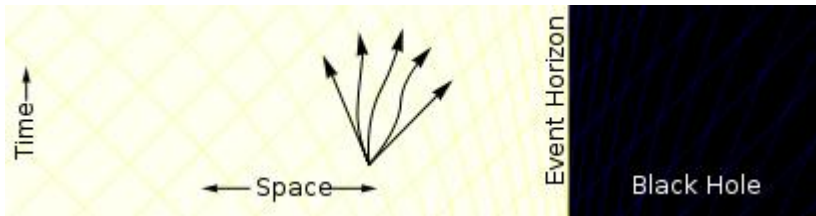
$$r_{\text{sh}} = \frac{2GM}{c^2} \approx 2.95 \frac{M}{M_{\text{Sun}}} \text{ km},$$

where  $r_{\text{sh}}$  is the Schwarzschild radius and  $M_{\text{Sun}}$  is the **mass of the Sun**. This relation is exact only for black holes with zero charge and angular momentum; for more general black holes it can differ up to a factor of 2.

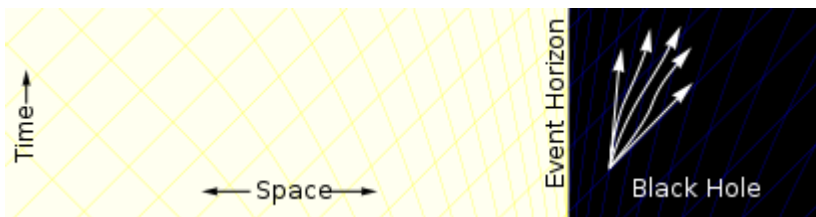
### Event horizon



Far away from the black hole a particle can move in any direction, as illustrated by the set of arrows. It is only restricted by the speed of light.



Closer to the black hole spacetime *starts to deform*. There are more *paths going towards* the black hole than paths moving away



Inside of the event horizon all paths bring the particle *closer* to the center of the black hole. It is no longer possible for the particle to escape. Particle is captured

The defining feature of a black hole is the appearance of an event horizon—a boundary in spacetime through which matter and light can only pass **inward towards** the mass of the black hole. Nothing, not even light, can escape from inside the event horizon. The event horizon is referred to as such because if an event **occurs within** the boundary, **information from that event cannot reach an outside observer, making it impossible to determine if such an event occurred**

As predicted by general relativity, the presence of a mass **deforms spacetime** in such a way that the paths taken by particles bend towards the mass. At the event horizon of a black hole, this deformation becomes so strong that there are no paths that lead away from the black hole.

To a distant observer, clocks near a black hole appear to tick more slowly than those further away from the black hole. **Due to this effect**, known as **gravitational time dilation**, an object falling into a black hole appears **to slow down** as it approaches the event horizon, taking an infinite time to reach it. At the same time, all processes on this **object slow down** **causing** emitted light to appear redder and dimmer, an effect known as **gravitational redshift**. Eventually, at a point just before it reaches the event horizon, the falling object becomes so dim that it can no longer be seen.

On the other hand, an observer falling into a black hole does not notice any of these effects as he crosses the event horizon. According to his own clock, he crosses the event horizon after a finite time, although he is **unable to determine** exactly when he crosses it, as it is impossible to determine the location of the event horizon from local observations

The shape of the event horizon of a black hole is always approximately spherical. For non-rotating (static) black holes the geometry is precisely spherical, while for rotating black holes the sphere is somewhat oblate.

### **GRAVITATIONAL SINGULARITY-ANOTHER CARDINAL WAY OF DISSIPATION AND ENTANGLEMENT WITH QUANTUM INFORMATION:**

At the center of a black hole as described by general relativity lies a **gravitational singularity**, a region where the spacetime curvature becomes infinite. For a non-rotating black hole, this region takes the shape of a single point and for a **rotating black hole**, it is **smeared out to** form a **ring singularity** lying in the plane of rotation. In both cases, the singular region has zero volume. It can also be shown that the singular region contains all the mass of the black hole solution. The singular region can thus be thought of as having **infinite density**.

Observers falling into a Schwarzschild black hole (i.e. non-rotating and no charges) cannot avoid being carried into the singularity, once they cross the event horizon. They can prolong the experience by accelerating away to slow their descent, but only up to a point; after attaining a certain ideal velocity, it is best **to free fall** the rest of the way. When they reach the singularity, they are **crushed to** infinite density and their mass is added to the **total of the** black hole. Before that happens, they will have been torn apart by the growing **tidal forces** in a process sometimes referred to as **spaghettification** or the "noodle effect".

### **THE ULTIMATE ELDORADO OF DESTRUCTION/OBLITERATION OF INFORMATION QUANTUM INFORMATION LOSS SYNDROME AND ARMAGEDDON OR APOCHRYPHAL ANEURISM-THE BLACK HOLE SPAGATEFICATION**

In the case of a charged (Reissner–Nordström) or rotating (Kerr) black hole, it is possible to avoid the singularity. Extending these solutions as far as possible reveals the hypothetical possibility of exiting the black hole into a different spacetime with the **black hole acting as a wormhole**. The possibility of traveling to another universe is however only theoretical, since any perturbation will destroy this possibility. It also appears to be possible to follow **closed timelike curves** (going back to one's own past) around **the Kerr singularity**, which lead to problems with **causality** like the **grandfather paradox**. It is expected that none of these peculiar effects would survive in a proper quantum mechanical treatment of rotating and charged black holes

The appearance of singularities in general relativity is commonly perceived as signaling the breakdown of the theory.<sup>[62]</sup> This breakdown, however, is expected; it occurs in a situation where **quantum mechanical effects** should describe these actions due to the extremely high density and therefore particle interactions. To date, it has not been possible to combine quantum and gravitational effects into a single theory. It is generally expected that a theory of **quantum gravity** will feature black holes without singularities.

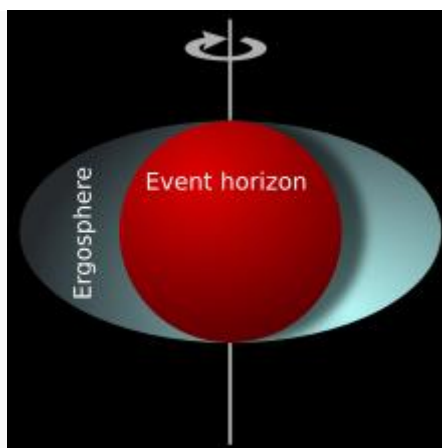
#### **Photon sphere**

The photon sphere is a spherical boundary of zero thickness such **that photons** moving **along tangents to** the sphere will be trapped in a circular orbit. For non-rotating black holes, the photon sphere has a radius 1.5 times the Schwarzschild radius. The orbits are **dynamically unstable**, hence any small perturbation (such as a particle of infalling matter) **will grow over time**, either setting it on an outward trajectory escaping the black hole or on an inward spiral eventually crossing the event horizon.

While light can still escape from inside the photon sphere, any light that crosses the photon sphere on an inbound trajectory will be captured by the black hole. Hence any light reaching an outside observer from inside the photon sphere must have been emitted by objects inside the photon sphere but still outside of the event horizon.

**Other compact objects**, such as **neutron stars**, can also have photon spheres. This follows from the fact that the gravitational field of an object **does not depend** on its actual size; hence any object that is smaller than 1.5 times the Schwarzschild radius corresponding to its mass will indeed have a photon sphere.

#### **Ergo sphere**



The ergo sphere is an oblate spheroid region outside of the event horizon, where objects **cannot remain** stationary.

Rotating black holes are surrounded by a region of spacetime in which it is impossible to stand still, called the ergo sphere. This is **the result of** a process known as **frame-dragging**; general relativity predicts that any rotating mass will tend to slightly "drag" along the spacetime immediately surrounding it. Any object near the rotating mass will tend to start moving in the direction of rotation. For a rotating black hole, this effect becomes so strong near the event horizon that an object would have to move faster than the speed of light in the opposite direction to just stand still.

The ergo sphere of a black hole is bounded by the (outer) event horizon on the inside and an **oblate spheroid**, which coincides with the event horizon at the poles and is noticeably wider around the equator. The outer boundary is sometimes called the **ergo surface**. Objects and radiation can escape normally from the ergo sphere. Through the **Penrose process**, objects can emerge from the ergo sphere with more energy than they entered. This energy is taken from the rotational energy of the black hole causing it to slow down.

Formation and evolution

Considering the exotic nature of black holes, it may be natural to question if such bizarre objects could exist in nature or to suggest that they are merely pathological solutions to Einstein's equations. Einstein himself wrongly thought that **black holes would not form**, because he held that the angular momentum of collapsing particles would stabilize their motion at some radius. This led the general relativity community to dismiss all results to the contrary for many years. However, a minority of relativists continued to contend that black holes were physical objects, and by the end of the 1960s, they had persuaded the majority of researchers in the field that there is no obstacle to forming an event horizon.

Once an event horizon forms, Penrose proved that a singularity will form somewhere inside it. Shortly afterwards, Hawking showed that many cosmological solutions describing **the Big Bang** have singularities without scalar fields or other exotic matter (see **Penrose-Hawking singularity theorems**). The **Kerr solution, the no-hair theorem** and the laws of **black hole thermodynamics** showed that the physical properties of black holes were simple and comprehensible, making them respectable subjects for research. The primary formation process for black holes is expected to be **the gravitational collapse** of heavy objects such as stars, but there are also more exotic processes that can lead to the production of black holes.

### Gravitational collapse

Gravitational collapse **occurs when** an object's internal **pressure** is insufficient **to resist(E) the** object's own gravity. For stars this **usually occurs** either because a star has too little "fuel" left to maintain its temperature through **stellar nucleosynthesis**, or because a star that would have been stable **receives extra** matter in a way that does not raise its core temperature. In either case the star's temperature is no longer high enough to **prevent it from collapsing** under its own weight. The ideal gas law explains the **connection** between pressure, temperature, and volume.

The collapse may be stopped by the **degeneracy pressure** of the star's constituents, condensing the matter in an **exotic denser state**. The **result is** one of the various types of compact STARS. The type of compact star formed **depends on the** mass of the remnant—the matter left over after the outer layers have been blown away, such from a supernova explosion or by pulsations leading to **a planetary nebula**. Note that this mass can be substantially less than the original star—remnants exceeding 5 solar masses **are produced** by stars that were over 20 solar masses before the collapse.

If the mass of the remnant exceeds about 3–4 solar masses (the **Tolman–Oppenheimer–Volkoff limit**<sup>1</sup>)—either because the original star was very heavy or because the remnant collected additional mass through **accretion** of matter—even the degeneracy pressure **of neutrons** is insufficient to stop the collapse. No known mechanism (except possibly quark degeneracy pressure, see **quark star**) is powerful enough to stop the implosion and the object will inevitably **collapse to form a black hole**.

The gravitational collapse of heavy stars is assumed to be responsible for the formation of **stellar mass black holes**. **Star formation** in the early universe may have resulted in very massive stars, which upon their collapse would **have produced** black holes of up to  $10^3$  solar masses. These black holes could be the seeds of the supermassive black holes found in the centers of most galaxies.

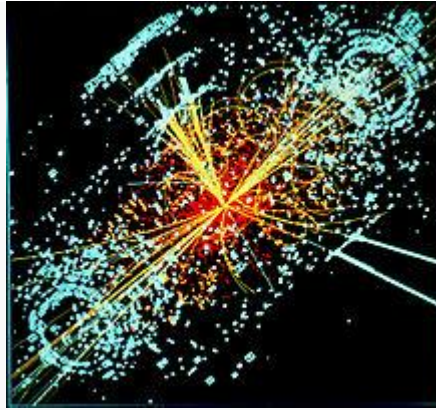
While most of the energy released during gravitational collapse is emitted very quickly, an outside observer does not actually see the end of this process. Even though the collapse takes a finite amount of time from the **reference frame** of infalling matter, a distant observer sees the infalling material slow and halts just above the event horizon, due to **gravitational time dilation**. Light from the collapsing material takes longer and longer to reach the observer, with the light emitted just before the event horizon forms is delayed an infinite amount of time. Thus the external observer never sees the formation of the event horizon; instead, the collapsing material seems to become dimmer and increasingly red-shifted, eventually fading away.

### Primordial black holes in the Big Bang

Gravitational collapse requires great density. In the current epoch of the universe these high densities are only found in stars, but in the early universe shortly after the bang densities were much greater, possibly allowing for the creation of black holes. The high density alone is not enough to allow the formation of black holes since a uniform mass distribution will not allow the mass to bunch up. In order for **primordial black holes** to form in such a dense medium, there must be initial density perturbations that can then grow under their own gravity. Different models for the early universe vary widely in their predictions of the size of these perturbations. Various models predict the creation of black holes, ranging from a **Planck**

mass to hundreds of thousands of solar masses.<sup>[25]</sup> Primordial black holes could thus account for the creation of any type of black hole.

### High-energy collisions



A simulated event in the CMS detector, a collision in which a micro black hole may be created.

Gravitational collapse is not the only process that could create black holes. In principle, black holes could be formed in **high-energy collisions** that achieve sufficient density. As of 2002, no such events have been detected, either directly or indirectly as a deficiency of the mass balance in **particle accelerator** experiments. This suggests that there must be a lower limit for the mass of black holes. Theoretically, this boundary is expected to lie around the **Planck mass** ( $m_P = \sqrt{\hbar c/G} \approx 1.2 \times 10^{19} \text{ GeV}/c^2 \approx 2.2 \times 10^{-8} \text{ kg}$ ), where quantum effects are expected to invalidate the predictions of general relativity. This would put the creation of black holes firmly out of reach of any high energy process occurring on or near the Earth. However, certain developments in quantum gravity suggest that the Planck mass could be much lower: some **braneworld** scenarios for example put the boundary as low as **1 TeV/c<sup>2</sup>**. This would make it conceivable for **micro black holes** to be created in the high energy collisions occurring when **cosmic rays** hit the Earth's atmosphere, or possibly in the new **Large Hadron Collider at CERN**. Yet these theories are very speculative, and the creation of black holes in these processes is deemed unlikely by many specialists. Even if micro black holes should be formed in these collisions, it is expected that they would **evaporate** in about  $10^{-25}$  seconds, posing no threat to the Earth.

### Growth

Once a black hole has formed, it can continue to grow by absorbing additional matter. Any black hole will continually absorb gas and interstellar from its direct surroundings and omnipresent **cosmic background radiation**. This is the primary process through which supermassive black holes seem to have grown. A similar process has been suggested for the formation of **intermediate-mass black holes in globular clusters**.

Another possibility is for a black hole to merge with other objects such as stars or even other black holes. This is thought to have been important especially for the early development of supermassive black holes, which could have formed from the coagulation of many smaller objects. The process has also been proposed as the origin of **some intermediate-mass black holes**.

### Evaporation-Another Dissipatory Mode:

In 1974, Hawking showed that black holes are not entirely black but emit small amounts of thermal radiation;<sup>[30]</sup> an effect that has become known as **Hawking radiation**. By **applying quantum** to a static black hole background, he determined that a black hole should emit particles in a perfect **black body spectrum**. Since **Hawking's publication**, many others have verified the result through various approaches.<sup>[84]</sup> If **Hawking's theory of black hole radiation** is correct, then black holes are expected to shrink and evaporate over time because they lose mass by the emission of photons and other particles. The temperature of this thermal spectrum (**Hawking temperature**) is proportional to the **surface gravity** of the black hole, which, for a Schwarzschild black hole, is inversely proportional to the mass. Hence, large black holes emit less radiation than small black holes.

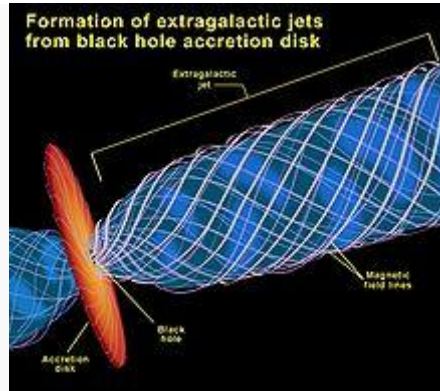
A stellar black hole of one solar mass has a Hawking temperature of about **100 nanokelvins**. This is far less than the 2.7 K temperature of the **cosmic microwave background radiation**. Stellar mass or larger black holes receive more mass from the cosmic microwave background than they emit through Hawking radiation and thus will grow instead of shrink. To have a Hawking temperature larger than 2.7 K (and be able to evaporate), a black hole needs to have less mass than **the Moon**. Such a black hole would have a diameter of less than a tenth of a millimeter.

If a black hole is very small the radiation effects are expected to become very strong. Even a black hole that is heavy compared to a human would evaporate in an instant. A black hole the weight of a car would have a diameter of about  $10^{-24} \text{ m}$  and take a nanosecond to evaporate, during which time it would briefly have luminosity more than 200 times that of the sun. Lower mass black holes are expected to evaporate even faster; for example, a black hole of mass  $1 \text{ TeV}/c^2$  would take less than  $10^{-88}$  seconds to evaporate completely. For such a small black hole, **quantum gravitation** effects are expected to play an important role and could even—although current developments in quantum gravity do not indicate so<sup>[87]</sup>—hypothetically make such a small black hole stable.

By their very nature, black holes do not directly emit any signals other than the hypothetical Hawking radiation; since the Hawking radiation for an astrophysical black hole is predicted to be very weak, this makes it impossible to directly detect astrophysical black holes from the Earth. A possible exception to the Hawking radiation being weak is the last stage of the evaporation of light (primordial) black holes; searches for such flashes in the past has proven unsuccessful and provides stringent limits on the possibility of existence of light primordial black holes NASA's **Fermi Gamma-ray Space Telescope** launched in 2008 will continue the search for these flashes

Astrophysicists searching for black holes thus have to rely on indirect observations. A black hole's existence can sometimes be inferred by observing its gravitational interactions with its surroundings. A project run by **MIT's Haystack Observatory** is attempting to observe the event horizon of a black hole directly. Initial results are encouraging

**Accretion of matter-The Accentuation Coefficient in the Model:**



Formation of extragalactic jets from a black hole's accretion disk

Due to conservation of angular momentum, gas falling into the gravitational well created by a massive object will typically form a disc-like structure around the object. Friction within the disc causes angular momentum to be transported outward, allowing matter to fall further inward, releasing potential energy and increasing the temperature of the gas. In the case of compact objects such as white dwarfs, neutron, and black holes, the gas in the inner regions becomes so hot that it will emit vast amounts of radiation (mainly X-rays), which may be detected by telescopes. This process of accretion is one of the most efficient energy-producing processes known; up to 40% of the rest mass of the accreted material can be emitted in radiation (In nuclear fusion only about 0.7% of the rest mass will be emitted as energy.) In many cases, accretion discs are accompanied by relativistic jets emitted along the poles, which carry away much of the energy. The mechanism for the creation of these jets is currently not well understood.

As such many of the universe's more energetic phenomena have been attributed to the accretion of matter on black holes. In particular, active and quasars are believed to be the accretion discs of supermassive black holes.<sup>[93]</sup> Similarly, X-ray binaries are generally accepted to be binary star systems in which one of the two stars is a compact object accreting matter from its companion.<sup>[93]</sup> It has also been suggested that some ultra luminous X-ray sources may be the accretion discs of intermediate-mass black holes.<sup>[94]</sup>

#### X-ray binaries

X-ray binaries are binary star systems that are luminous in the X-ray part of the spectrum. These X-ray emissions are generally thought to be caused by one of the component stars being a compact object accreting matter from the other (regular) star. The presence of an ordinary star in such a system provides a unique opportunity for studying the central object and determining if it might be a black hole.



Artist impression of a binary system with an accretion disk around a black hole being fed by material from the companion star.

If such a system emits signals that can be directly traced back to the compact object, it cannot be a black hole. The absence of such a signal does, however, not exclude the possibility that the compact object is a neutron star. By studying the

companion star it is often possible to obtain the orbital parameters of the system and obtain an estimate for the mass of the compact object. If this is much larger than the Tolman–Oppenheimer–Volkoff limit (that is, the maximum mass a neutron star can have before collapsing) then the object cannot be a neutron star and is generally expected to be a black hole.

The first strong candidate for a black hole, **Cygnus X-1, was discovered in this way by Charles Thomas Bolton**, Louise Webster and Paul Murdin in 1972. Some doubt, however, remained due to the uncertainties resultant from the companion star being much heavier than the candidate black hole. Currently, better candidates for black holes are found in a class of X-ray binaries called soft X-ray transients.<sup>[93]</sup> In this class of system the companion star is relatively low mass allowing for more accurate estimates in the black hole mass. Moreover, these systems are only active in X-ray for several months once every 10–50 years. During the period of low X-ray emission (called quiescence), the accretion disc is extremely faint allowing for detailed observation of the companion star during this period. One of the best such candidates is **V404 Cyg**.

#### ***Quiescence and advection-dominated accretion flow***

The faintness of the accretion disc during quiescence is suspected to be caused by the flow entering a mode called **an advection-dominated accretion flow** (ADAF). In this mode, almost all the energy generated by friction in the disc is swept along with the flow instead of radiated away. If this model is correct, then it forms strong qualitative evidence for the presence of an event horizon. Because, if the object at the center of the disc had a solid surface, it would emit large amounts of radiation as the highly energetic gas hits the surface, an effect that is observed for neutron stars in a similar state

#### ***Quasi-periodic oscillations***

The X-ray emission from accretion disks sometimes flickers at certain frequencies. These signals are called quasi-periodic oscillations and are thought to be caused by material moving along the inner edge of the accretion disk (the innermost stable circular orbit). As such their frequency is linked to the mass of the compact object. They can thus be used as an alternative way to determine the mass of potential black holes.<sup>[100]</sup>

#### **Galactic nuclei**

" to describe galaxies with unusual characteristics, such as **unusual spectral line emission** and very strong radio emission. Theoretical and observational studies have shown that the activity in these active galactic nuclei (AGN) may be explained by the presence of supermassive black holes. The models of these AGN consist of a central black hole that may be millions or billions of times more massive than the Sun; a disk of **gas and dust called an accretion disk**; and **two jets** that are perpendicular to the accretion disk.<sup>1</sup>

Although supermassive black holes are expected to be found in most AGN, only some galaxies' nuclei have been more carefully studied in attempts to both identify and measure the actual masses of the central supermassive black hole candidates. Some of the most notable galaxies with supermassive black hole candidates include **the Andromeda Galaxy, M32, M87, NGC 3115, NGC 3377, NGC 4258, and the Sombrero Galaxy.**

It is now widely accepted that the center of (nearly) every galaxy (not just active ones) contains a supermassive black hole. The close observational correlation between the mass of this hole and the velocity dispersion of the host **galaxy's bulge, known as the M-sigma relation**, strongly suggests a connection between the formation of the black hole and the galaxy itself.

Currently, the best evidence for a supermassive black hole comes from studying **the proper motion of stars near the center of our own Milky Way. Since 1995 astronomers have tracked the motion of 90 stars in a region called Sagittarius A\*. By fitting their motion to Keplerian orbits they were able to infer in 1998 that 2.6 million solar masses must be contained in a volume with a radius of 0.02 lightyears** Since then one of the stars—called S2—has completed a full orbit. From the orbital data they were able to place better constraints on the mass and size of the object causing the orbital motion of stars in the Sagittarius A\* region, finding that there is a spherical mass of 4.3 million solar masses contained within a radius of less than 0.002 lightyears. While this is more than 3000 times the Schwarzschild radius corresponding to that mass, it is at least consistent with the central object being a supermassive black hole, and no "realistic cluster [of stars] is physically tenable.

#### **Gravitational lensing**

The deformation of spacetime around a massive object causes light rays to be deflected much like light passing through an **optic lens. This phenomenon is known as gravitational lensing. Observations have been made of weak gravitational lensing, in which photons are deflected by only a few arcseconds. However, it has never been directly observed for a black hole. One possibility for observing gravitational lensing by a black hole would be to observe stars in orbit around the black hole. There are several candidates for such an observation in orbit around Sagittarius A\***

#### **Alternatives**

The evidence for stellar black holes strongly relies on the existence of an upper limit for the mass of a neutron star. The size of this limit heavily depends on the assumptions made about the properties of dense matter. New **exotic phases of matter could push up this bound. A phase of free quarks at high density might allow the existence of dense quark stars, and some supersymmetric models predict the existence of Q stars** Some extensions of the standard model posit **the existence of preons as fundamental building blocks of quarks and leptons, which could hypothetically form preon stars. These hypothetical models could potentially explain a number of observations** of stellar black hole candidates. However, it can be shown from general arguments in general relativity that any such object will have a maximum mass.

Since the average density of a black hole inside its Schwarzschild radius is inversely proportional to the square of its mass, supermassive black holes are much less dense than stellar black holes (the average density of a  $10^8$  solar mass black hole is comparable to that of water) Consequently, the physics of matter forming a supermassive black hole is much better understood and the possible alternative explanations for supermassive black hole observations are much more mundane. For example, a supermassive black hole could be modeled by a large cluster of very dark objects. However, typically such alternatives are not stable enough to explain the supermassive black hole candidates.

The evidence for stellar and supermassive black holes implies that in order for black holes not to form, general relativity must fail as a theory of gravity, perhaps due to the **onset of quantum corrections**. A much anticipated feature of a theory of quantum gravity is that it will not feature singularities or event horizons (and thus no black holes) In recent years, much attention has been drawn by the fuzz ball model in string theory. Based on calculations in specific situations in string theory, the proposal suggest that generically the individual states of a black hole solution do not have an event horizon or singularity, but that for a classical/semi-classical observer the statistical average of such states does appear just like an ordinary black hole in general relativity.

Open questions

### **Entropy and thermodynamics**

The formula for the Bekenstein–Hawking entropy (S) of a black hole, which depends on the area of the black hole (A). The constants are the speed of light (c), the Boltzmann (k), Newton's constant (G), and the reduced Planck constant (ħ).

In 1971, Hawking showed under general conditions that the total area of the event horizons of any collection of classical black holes can never decrease, even if they collide and merge This result, now known as the second law of black hole mechanics, is remarkably similar to the second law of thermodynamics, which states that the total entropy of a system can never decrease. As with classical objects at absolute zero temperature, it was assumed that black holes had zero entropy. If this were the case, the second law of thermodynamics would be violated by entropy-laden matter entering a black hole, resulting in a decrease of the total entropy of the universe. Therefore, Bekenstein proposed that a black hole should have an entropy, and that it should be proportional to its horizon area

The link with the laws of thermodynamics was further strengthened by Hawking's discovery that quantum field theory predicts that a black hole radiates blackbody radiation at a constant temperature. This seemingly causes a violation of the second law of black hole mechanics, since the radiation will carry away energy from the black hole causing it to shrink. The radiation, however also carries away entropy, and it can be proven under general assumptions that the sum of the entropy of the matter surrounding a black hole and one quarter of the area of the horizon as measured in Planck units is in fact always increasing. This allows the formulation of the first law of black hole mechanics as an analogue of the first law of thermodynamics, with the mass acting as energy, the surface gravity as temperature and the area as entropy

One puzzling feature is that the entropy of a black hole scales with its area rather than with its volume, since entropy is normally an extensive quantity that scales linearly with the volume of the system. This odd property led Gerard 't Hooft and Leonard Susskind to propose the holographic principle, which suggests that anything that happens in a volume of spacetime can be described by data on the boundary of that volume.

Although general relativity can be used to perform a semi-classical calculation of black hole entropy, this situation is theoretically unsatisfying. In statistical mechanics, entropy is understood as counting the number of microscopic configurations of a system that have the same macroscopic qualities (such as mass, charge, pressure, etc.). Without a satisfactory theory of quantum gravity, one cannot perform such a computation for black holes. Some progress has been made in various approaches to quantum gravity. In 1995, Andrew Strominger and Cumrun Vafa showed that counting the microstates of a specific supersymmetric black hole in string theory reproduced the Bekenstein–Hawking entropy Since then, similar results have been reported for different black holes both in string theory and in other approaches to quantum gravity like loop quantum gravity

### **Black hole unitarity<.>**

An open question in fundamental physics is the so-called information loss paradox, or black hole unitarity paradox. Classically, the laws of physics are the same run forward or in reverse (T-symmetry). Liouville's theorem dictates conservation of phase space volume, which can be thought of as "conservation of information", so there is some problem even in classical physics. In quantum mechanics, this corresponds to a vital property called unitarity, which has to do with the conservation of probability (it can also be thought of as a conservation of quantum phase space volume as expressed by the density matrix).

Open questions

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The link with the laws of thermodynamics was further strengthened by Hawking's discovery that quantum field theory predicts that a black hole radiates blackbody radiation at a constant temperature. This seemingly causes a violation of the second law of black hole mechanics, since the radiation will carry away energy from the black hole(-) causing it to shrink. The radiation, however also carries away (E)entropy, and it can be proven under general assumptions that the sum of the entropy of the matter surrounding a black hole and one quarter of the area of the horizon as measured in Planck units is in fact always increasing. This allows the formulation of the first law of black hole mechanics as an analogue of the first law of thermodynamics, with the mass acting as energy, the surface gravity as temperature and the area as entropy

One puzzling feature is that the entropy of a black hole scales with its area rather than with its volume, since entropy is normally an extensive quantity that scales linearly with the volume of the system. This odd property led Gerard 't Hooft and Leonard Susskind to propose the holographic principle, which suggests that anything that happens in a volume of spacetime can be described by data on the boundary of that volume.

Although general relativity can be used to perform a semi-classical calculation of black hole entropy, this situation is theoretically unsatisfying. In statistical mechanics, entropy is understood as counting the number of microscopic configurations of a system that have the same macroscopic qualities (such as mass, charge, pressure, etc.). Without a satisfactory theory of quantum gravity, one cannot perform such a computation for black holes. Some progress has been made in various approaches to quantum gravity. In 1995, Andrew Strominger and Cumrun Vafa showed that counting the microstates of a specific supersymmetric black hole in string theory reproduced the Bekenstein–Hawking entropy Since then, similar results have been reported for different black holes both in string theory and in other approaches to quantum gravity like loop quantum gravity.

**Black hole unitarity Black hole information paradox**

An open question in fundamental physics is the so-called information loss paradox, or black hole unitarity paradox. Classically, the laws of physics are the same run forward or in reverse (T-symmetry). Liouville's theorem dictates conservation of phase space volume, which can be thought of as "conservation of information", so there is some problem even in classical physics. In quantum mechanics, this corresponds to a vital property called unitarity, which has to do with the conservation of probability (it can also be thought of as a conservation of quantum phase space volume as expressed by the density matrix)

**Some Salient Notes Relating to the relation between the Quantum Information and Quantum Mechanical Behaviour**

^ The set of possible paths, or more accurately the future light cone containing all possible world line is tilted in this way in Eddington–Finkelstein coordinates, but in other coordinates the light cones are not tilted in this way, for example in Schwarzschild coordinates they simply narrow without tilting as one approaches the event horizon, and in Kruskal–Szekeres coordinates the light cones don't change shape or orientation at all.^ This is true only for 4-dimensional spacetimes. In higher dimensions more complicated horizon topologies like a black ring are possible^ In particular, he assumed that all matter satisfies the weak energy condition.

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