On Semi*-Connected and Semi*-Compact Spaces

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Abstract: *The purpose of this paper is to introduce the concepts of semi*-connected spaces, semi*-compact spaces and semi*-Lindelof spaces. We investigate their basic properties. We also discuss their relationship with already existing concepts.*
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I. Introduction

In 1974, Das defined the concept of semi-connectedness in topology and investigated its properties. Compactness is one of the most important, useful and fundamental concepts in topology. In 1981, Dorsett introduced and studied the concept of semi-compact spaces. Since then, Hanna and Dorsett, Ganster and Mohammad S. Sarsak investigated the properties of semi-compact spaces. In 1990, Ganster defined and investigated semi-Lindelöf spaces.

In this paper, we introduce the concepts of semi*-connected spaces, semi*-compact spaces and semi*-Lindelöf spaces. We investigate their basic properties. We also discuss their relationship with already existing concepts.

II. Preliminaries

Throughout this paper (X, τ) will always denote a topological space on which no separation axioms are assumed, unless explicitly stated. If A is a subset of the space (X, τ), $Cl(A)$ and $Int(A)$ denote the closure and the interior of A respectively.

Definition 2.1: A subset A of a topological space (X, τ) is called
(i) ***generalized closed*** (briefly g-closed)[11] if $Cl(A)\subseteq U$ whenever $A\subseteq U$ and U is open in X.
(ii) ***generalized open*** (briefly g-open)[11] if $X\backslash A$ is g-closed in X.

Definition 2.2: Let A be a subset of X. The ***generalized closure*** [6] of A is defined as the intersection of all g-closed sets containing A and is denoted by $Cl^*(A)$.

Definition 2.3: A subset A of a topological space (X, τ) is called
(i) ***semi-open*** [10] (resp. ***semi*-open***[14]) if $A\subseteq Cl(Int(A))$ (resp. $A\subseteq Cl^*(Int(A))$).
(ii) ***semi-closed*** [1] (resp. ***semi*-closed***[15]) if $X\backslash A$ is semi-open (resp. semi*-open) or equivalently if $Int(Cl(A))\subseteq A$ (resp. $Int^*(Cl(A))\subseteq A$).
(iii) ***semi-regular*** [2] (resp. ***semi*-regular*** [15]) if it is both semi-open and semi-closed (resp. both semi*-open and semi*-closed).

The class of all semi-open (resp. semi-closed, semi*-open, semi*-closed) sets is denoted by $SO(X, \tau)$(resp. $SC(X, \tau)$, $S^*O(X, \tau)$, $S^*C(X,\tau)$).

Definition 2.4: Let A be a subset of X. Then the ***semi*-closure*** [15] of A is defined as the intersection of all semi*-closed sets containing A and is denoted by $s^*Cl(A)$.

Theorem 2.5[14]**:** (i) Every open set is semi*-open.
(ii) Every semi*-open set is semi-open.

Definition 2.6: If A is a subset of X, the semi*-frontier [13] of A is defined by $s^*Fr(A)=s^*Cl(A)\backslash s^*Int(A)$.

Theorem 2.7[13]**:** Let A be a subset of a space X. Then A is semi*-regular if and only if $s^*Fr(A)=\phi$.

Theorem 2.8[15]**:** If A is a subset of X, then
(i) $s^*Cl(X\backslash A)=X\backslash s^*Int(A)$.
(ii) $s^*Int(X\backslash A)=X\backslash s^*Cl(A)$.

Definition 2.9: A topological space X is said to be connected [18] (resp. semi-connected [3]) if X cannot be expressed as the union of two disjoint nonempty open (resp. semi-open) sets in X.

Theorem 2.10 [18]**:** A topological space X is connected if and only if the only clopen subsets of X are ϕ and X.

Definition 2.11: A collection B of open (resp. semi-open) sets in X is called an open (resp. semi-open) cover of $A\subseteq X$ if $A\subseteq\cup\{U_\alpha : U_\alpha\in B\}$ holds.

Definition 2.12: A space X is said to be compact [18] (resp. semi-compact [4]) if every open (resp. semi-open) cover of X has a finite subcover.

Definition 2.13: A space X is said to be Lindel**ö**f [18] (resp. semi-Lindel**ö**f [8]) if every cover of X by open (resp. semi-open) sets contains a countable sub cover.

Definition 2.14: A function $f:X\longrightarrow Y$ is said to be
(i) semi*-continuous [16] if $f^{-1}(V)$ is semi*-open in X for every open set V in Y.
(ii) semi*-irresolute [17] if $f^{-1}(V)$ is semi*-open in X for every semi*-open set V in Y.
(iii) semi*-open [16] if $f(V)$ is semi*-open in Y for every open set V in X.

(iv) semi*-closed [16] if $f(V)$ is semi*-closed in Y for every closed set V in X.
(v) pre-semi*-open [16] if $f(V)$ is semi*-open in Y for every semi*-open set V in X.
(vi) pre-semi*-closed [16] if $f(V)$ is semi*-closed in Y for every semi*-closed set V in X.
(vii) totally semi*-continuous [17] if $f^{-1}(V)$ is semi*-regular in X for every open set V in Y.
(vii) strongly semi*-continuous [17] if $f^{-1}(V)$ is semi*-regular in X for every subset V in Y.
(viii) contra-semi*-continuous [16] if $f^{-1}(V)$ is semi*-closed in X for every open set V in Y.
(ix) contra-semi*-irresolute [17] if $f^{-1}(V)$ is semi*-closed in X for every semi*-open set V in Y.

Theorem 2.15: Let $f:X\longrightarrow Y$ be a function. Then
(i) f is semi*-continuous if and only if $f^{-1}(F)$ is semi*-closed in X for every closed set F in Y.[16]
(ii) f is semi*-irresolute if and only if $f^{-1}(F)$ is semi*-closed in X for every semi*-closed set F in Y.[17]
(iii) f is contra-semi*-continuous if and only if $f^{-1}(F)$ is semi*-open in X for every closed set F in Y.[16]
(iv) f is contra-semi*-irresolute if and only if $f^{-1}(F)$ is semi*-open in X for every semi*-closed set F in Y.[17]

Remark 2.16:[14] If (X, τ) is a locally indiscrete space, then $\tau = S^*O(X, \tau) = SO(X, \tau)$.

Theorem 2.17:[14] A subset A of X is semi*-open if and only if A contains a semi*-open set about each of its points.

III. Semi*-connected spaces

In this section we introduce semi*-connected spaces and investigate their basic properties.

Definition 3.1: A topological space X is said to be ***semi*-connected*** if X cannot be expressed as the union of two disjoint nonempty semi*-open sets in X.

Theorem 3.2: (i) If a space X is semi*-connected, then it is connected.
(ii) If a space X is semi-connected, then it is semi*-connected.

Proof: (i) Let X be semi*-connected. Suppose X is not connected. Then there exist disjoint non-empty open sets A and B such that $X=A\cup B$. By Theorem 2.5(i), A and B are semi*-open sets. This is a contradiction to X is semi*-connected. This proves (i).
(ii) Let X be semi-connected. Suppose X is not semi*-connected. Then there exist disjoint non-empty semi*-open sets A and B such that $X=A\cup B$. By Theorem 2.5(ii), A and B are semi-open sets. This is a contradiction to X is semi-connected. This proves (ii).

Remark 3.3: The converse of the above theorem is not true as shown in the following example.

Example 3.4: Consider the space (X, τ) where $X=\{a, b, c, d\}$ and $\tau=\{\phi, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}, X\}$. Clearly, (X, τ) is connected but not semi*-connected.

Example 3.5: It can be verified that the space (X, τ) where $X=\{a, b, c, d\}$ and $\tau=\{\phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, X\}$ is semi*-connected but not semi-connected.

Theorem 3.6: A topological space X is semi*-connected if and only if the only semi*- regular subsets of X are ϕ and X itself.

Proof: Necessity: Suppose X is a semi*-connected space. Let A be non-empty proper subset of X that is semi*-regular. Then A and X\A are non-empty semi*-open sets and $X=A\cup(X\setminus A)$. This is a contradiction to the assumption that X is semi*-connected.

Sufficiency: Suppose $X=A\cup B$ where A and B are disjoint non-empty semi*-open sets. Then $A=X\setminus B$ is semi*-closed. Thus A is a non-empty proper subset that is semi*-regular. This is a contradiction to our assumption.

Theorem 3.7: A topological space X is semi*-connected if and only if every semi*-continuous function of X into a discrete space Y with at least two points is a constant function.

Proof: Necessity: Let f be a semi*-continuous function of the semi*-connected space into the discrete space Y. Then for each $y\in Y$, $f^{-1}(\{y\})$ is a semi*-regular set of X. Since X is semi*-connected, $f^{-1}(\{y\})=\phi$ or X. If $f^{-1}(\{y\})=\phi$ for all $y\in Y$, then f ceases to be a function. Therefore $f^{-1}(\{y_0\})=X$ for a unique $y_0\in Y$. This implies $f(X)=\{y_0\}$ and hence f is a constant function. **Sufficiency:** Let U be a semi*- regular set in X. Suppose $U\neq\phi$. We claim that U=X. Otherwise, choose two fixed points y_1 and y_2 in Y. Define $f:X\longrightarrow Y$ by $f(x)=\begin{cases} y_1 \text{ if } x\in U \\ y_2 \text{ otherwise}\end{cases}$

Then for any open set V in Y, $f^{-1}(V)=\begin{cases} U & \text{if V contains } y_1 \text{ only} \\ X\setminus U & \text{if V contains } y_2 \text{ only} \\ X & \text{if V contains both } y_1 \text{ and } y_2 \\ \Phi & \text{otherwise}\end{cases}$

In all the cases $f^{-1}(V)$ is semi*-open in X. Hence f is a non-constant semi*-continuous function of X into Y. This is a contradiction to our assumption. This proves that the only semi*- regular subsets of X are ϕ and X and hence X is semi*-connected.

Theorem 3.8: A topological space X is semi*-connected if and only if every nonempty proper subset of X has non-empty semi*-frontier.

Proof: Suppose that a space X is semi*-connected. Let A be a non-empty proper subset of X. We claim that s*Fr(A)≠ϕ. If possible, let s*Fr(A)=ϕ. Then by Theorem 2.7, A is semi*-regular. By Theorem 3.6, X is not semi*-connected which is a contradiction. Conversely, suppose that every non-empty proper subset of X has a non-empty semi*-frontier. We claim that X is semi*-connected. On the contrary, suppose that X is not semi*-connected. By Theorem 3.6, X has a non-empty proper subset, say A, which is semi*-regular. By Theorem 2.7, s*Fr(A)=ϕ which is a contradiction to the assumption. Hence X is semi*-connected.

Theorem 3.9: Let f :X⟶Y be semi*-continuous surjection and X be semi*-connected. Then Y is connected.

Proof: Let f :X⟶Y be semi*-continuous surjection and X be semi*-connected. Let V be a clopen subset of Y. By Definition 2.14(i) and by Theorem 2.15(i), f^{-1}(V) is semi*-regular in X. Since X is semi*-connected, f^{-1}(V)=ϕ or X. Hence V=ϕ or Y. This proves that Y is connected.

Theorem 3.10: Let f :X⟶Y be a semi*-irresolute surjection. If X is semi*-connected, so is Y.

Proof: Let f :X⟶Y be a semi*-irresolute surjection and let X be semi*-connected. Let V be a subset of Y that is semi*-regular in Y. By Definition 2.14(ii) and by Theorem 2.15(ii), f^{-1}(V) is semi*-regular in X. Since X is semi*-connected, f^{-1}(V)=ϕ or X. Hence V=ϕ or Y. This proves that Y is semi*-connected.

Theorem 3.11: Let f :X⟶Y be a pre-semi*-open and pre-semi*-closed injection. If Y is semi*-connected, so is X.

Proof: Let A be subset of X that is semi*- regular in X. Since f is both pre-semi*-open and pre-semi*-closed, f(A) is semi*-regular in Y. Since Y is semi*-connected, f(A)=ϕ or Y. Hence A=ϕ or X. Therefore X is semi*-connected.

Theorem 3.12: If f :X⟶Y is a semi*-open and semi*-closed injection and Y is semi*-connected, then X is connected.

Proof: Let A be a clopen subset of X. Then f(A) is semi*- regular in Y. Since Y is semi*-connected, f(A)=ϕ or Y. Hence A=ϕ or X. By Theorem 2.10, X is connected.

Theorem 3.13: If there is a semi*-totally-continuous function from a connected space X onto Y, then Y has the indiscrete topology.

Proof: Let f be a semi*-totally-continuous function from a connected space X onto Y. Let V be an open set in Y. Then by Theorem 2.5(i), V is semi*-open in Y. Since f is semi*-totally-continuous, f^{-1}(V) is clopen in X. Since X is connected, by Theorem 2.10, f^{-1}(V)=ϕ or X. This implies V=ϕ or Y. Hence Y has the indiscrete topology.

Theorem 3.14: If there is a totally semi*-continuous function from a semi*-connected space X onto Y, then Y has the indiscrete topology.

Proof: Let f be a totally semi*-continuous function from a semi*-connected space X onto Y. Let V be an open set in Y. Since f is totally semi*-continuous, f^{-1}(V) is semi*- regular in X. Since X is semi*-connected, f^{-1}(V)=ϕ or X. This implies V= ϕ or Y. Thus Y has the indiscrete topology.

Theorem 3.15: If f :X⟶Y is a strongly semi*-continuous bijection and Y is a space with at least two points, then X is not semi*-connected.

Proof: Let y∈Y. Then f^{-1}({y}) is a non-empty proper subset that is semi*-regular in X. Hence by Theorem 3.6, X is not semi*-connected.

Theorem 3.16: Let f :X⟶Y be contra-semi*-continuous surjection and X be semi*-connected. Then Y is connected.

Proof: Let f :X⟶Y be contra-semi*-continuous surjection and X be semi*-connected. Let V be a clopen subset of Y. By Definition 2.14(viii) and Theorem 2.15(iii), f^{-1}(V) is semi*-regular in X. Since X is semi*-connected, f^{-1}(V)=ϕ or X. Hence V=ϕ or Y. This proves that Y is connected.

Theorem 3.17: Let f :X⟶Y be a contra-semi*-irresolute surjection. If X is semi*-connected, so is Y.

Proof: Let f :X⟶Y be a semi*-irresolute surjection and let X be semi*-connected. Let V be a subset of Y that is semi*-regular in Y. By Definition 2.14(ix) and Theorem 2.15(iv), f^{-1}(V) is semi*-regular in X. Since X is semi*-connected, f^{-1}(V)=ϕ or X. Hence V=ϕ or Y. This proves that Y is semi*-connected.

Theorem 3.18: Let X be a locally indiscrete space. Then the following are equivalent:

(i) X is connected.
(ii) X is semi*-connected.
(iii) X is semi-connected.

Proof: Follows from Remark 2.16.

IV. Semi*-Compact and Semi*-Lindelöf Spaces

In this section we introduce semi*-compact spaces and semi*-Lindelöf spaces and study their properties.

Definition 4.1: A collection $\mathcal{A}$ of semi*-open sets in X is called a ***semi*-open cover*** of B⊆X if B⊆∪{U_α : U_α∈$\mathcal{A}$} holds.

Definition 4.2: A space X is said to be ***semi*-compact*** if every semi*-open cover of X has a finite subcover.

Definition 4.3: A subset B of X is said to be ***semi*-compact relative to X*** if for every semi*-open cover $\mathcal{A}$ of B**,** there is a finite subcollection of $\mathcal{A}$ that covers B.

Definition 4.4: A space X is said to be ***semi*-Lindelöf*** if every cover of X by semi*-open sets contains a countable subcover.

Remark 4.5: Every finite space is semi*-compact and every countable space is semi*- Lindelöf.

Theorem 4.6: (i) Every semi-compact space is semi*-compact.
(ii) Every semi*-compact space is compact.
(iii) Every semi-Lindelöf space is semi*-Lindelöf.
(iv) Every semi*-Lindelöf space is Lindelöf.
(v) Every semi*-compact space is semi*-Lindelöf.

Proof: (i), (ii), (iii) and (iv) follow from Theorem 2.5. (v) follows from Definition 2.12, Definition 2.13, Definition 4.2 and Definition 4.4.

Theorem 4.7: Every semi*-closed subset of a semi*-compact space X is semi*-compact relative to X.

Proof: Let A be a semi*-closed subset of a semi*-compact space X. Let B be semi*-open cover of A. Then B∪{X\A} is a semi*-open cover of X. Since X is semi*-compact, this cover contains a finite subcover of X, namely $\{B_1, B_2,..., B_n, X\backslash A\}$. Then $\{B_1, B_2... B_n\}$ is a finite subcollection of B that covers A. This proves that A is semi*-compact relative to X.

Theorem 4.8: A space X is semi*-compact if and only if every family of semi*-closed sets in X with empty intersection has a finite subfamily with empty intersection.

Proof: Suppose X is compact and $\{F_\alpha : \alpha\in\Delta\}$ is a family of semi*-closed sets in X such that $\cap\{F_\alpha : \alpha\in\Delta\}=\phi$. Then $\cup\{X\backslash F_\alpha : \alpha\in\Delta\}$ is a semi*-open cover for X. Since X is semi*-compact, this cover has a finite subcover, say $\{X \backslash F_{\alpha_1}, X \backslash F_{\alpha_2},..., X \backslash F_{\alpha_n}\}$ for X. That is $X=\cup\{X \backslash F_{\alpha_i} : i=1,2,...,n\}$. This implies that $\bigcap_{i=1}^{n} F_{\alpha_i}=\phi$. Conversely, suppose that every family of semi*-closed sets in X which has empty intersection has a finite subfamily with empty intersection. Let $\{U_\alpha : \alpha\in\Delta\}$ be a semi*-open cover for X. Then $\cup\{U_\alpha : \alpha\in\Delta\}=X$. Taking the complements, we get $\cap\{X\backslash U_\alpha : \alpha\in\Delta\}=\phi$. Since $X\backslash U_\alpha$ is semi*-closed for each $\alpha\in\Delta$, by the assumption, there is a finite sub family, $\{X \backslash U_{\alpha_1}, X \backslash U_{\alpha_2},..., X \backslash U_{\alpha_n}\}$ with empty intersection. That is $\bigcap_{i=1}^{n}(X \backslash U_{\alpha_i})=\phi$. Taking the complements on both sides, we get $\bigcup_{i=1}^{n} U_{\alpha_i}=X$. Hence X is semi*-compact.

Theorem 4.9: Let X be a semi*-T_2 space in which S*O(X) is closed under finite intersection. If A is a semi*-compact subset of X, then A is semi*-closed.

Proof: Suppose X is a semi*-T_2 space in which S*O(X) is closed under finite intersection. Let A be a semi*-compact subset of X. Let $x\in X\backslash A$. Since X is semi*-T_2, for each $a\in A$, there are disjoint semi*-open sets U_a and V_a containing x and a respectively. $\{V_a : a\in A\}$ is a semi*-open cover for A. Since A is semi*-compact, this cover has a finite subcover say, $\{V_{\alpha_1}, V_{a_2},..., V_{a_n}\}$. Let $U_x=\bigcap_{i=1}^{n} U_{a_i}$. Then by assumption, U_x is a semi*-open set containing x. Also $U_x\cap A=\phi$ and hence $U_x\subseteq X\backslash A$. Then by Theorem 2.17, X\A is semi*-open and hence A is semi*-closed.

Theorem 4.10: Let f :X⟶Y be a semi*-irresolute surjection and X be semi*-compact. Then Y is semi*-compact.

Proof: Let f :X⟶Y be a semi*-irresolute surjection and X be semi*-compact. Let $\{V_\alpha\}$ be a semi*-open cover for Y. Then $\{f^{-1}(V_\alpha)\}$ is a cover of X by semi*-open sets. Since X is semi*-compact, $\{f^{-1}(V_\alpha)\}$ contains a finite subcover, namely $\{f^{-1}(V_{\alpha_1}), f^{-1}(V_{\alpha_2}),...,f^{-1}(V_{\alpha_n})\}$. Then $\{V_{\alpha_1}, V_{\alpha_2},..., V_{\alpha_n}\}$ is a finite subcover for Y. Thus Y is semi*-compact.

Theorem 4.11: If f :X⟶Y is a pre-semi*-open function and Y is semi*-compact, then X is semi*-compact.

Proof: Let $\{V_\alpha\}$ be a semi*-open cover for X. Then $\{f(V_\alpha)\}$ is a cover of Y by semi*-open sets.Since Y is semi*-compact, $\{f(V_\alpha)\}$contains a finite subcover, namely $\{f(V_{\alpha_1}), f(V_{\alpha_2}),...,f(V_{\alpha_n})\}$Then $\{V_{\alpha_1}, V_{\alpha_2},..., V_{\alpha_n}\}$is a finite subcover for X. Thus X is semi*-compact.

Theorem 4.12: If f :X⟶Y is a semi*-open function and Y is semi*-compact, then X is compact.

Proof: Let $\{V_\alpha\}$ be an open cover for X. Then $\{f(V_\alpha)\}$is a cover of Y by semi*-open sets.
Since Y is semi*-compact, $\{f(V_\alpha)\}$contains a finite subcover, namely $\{f(V_{\alpha_1}), f(V_{\alpha_2}),...,f(V_{\alpha_n})\}$.
Then $\{V_{\alpha_1}, V_{\alpha_2},..., V_{\alpha_n}\}$ is a finite subcover for X. Thus X is compact.

Theorem 4.13: Let f :X⟶Y be a semi*-continuous surjection and X be semi*-compact.
Then Y is compact.

Proof: Let f :X⟶Y be a semi*-continuous surjection and X be semi*-compact. Let $\{V_\alpha\}$ be an open cover for Y. Then $\{f^{-1}(V_\alpha)\}$ is a cover of X by semi*-open sets. Since X is semi*-compact, $\{f^{-1}(V_\alpha)\}$ contains a finite subcover, namely $\{f^{-1}(V_{\alpha_1}), f^{-1}(V_{\alpha_2}),...,f^{-1}(V_{\alpha_n})\}$. Then $\{V_{\alpha_1}, V_{\alpha_2},..., V_{\alpha_n}\}$ is a cover for Y. Thus Y is compact.

Theorem 4.14: A space X is semi*-Lindelöf if and only if every family of semi*-closed sets in X with empty intersection has a countable subfamily with empty intersection.

Proof: Suppose X is compact and $\{F_\alpha : \alpha\in\Delta\}$ is a family of semi*-closed sets in X such that $\cap\{F_\alpha : \alpha\in\Delta\}=\phi$. Then $\cup\{X\backslash F_\alpha : \alpha\in\Delta\}$ is a semi*-open cover for X. Since X is semi*-Lindelöf, this cover has a countable sub cover, say $\{X \backslash F_{\alpha_i}$: i=1, 2, ...$\}$ for X. That is $X=\cup\{X \backslash F_{\alpha_i} : i=1,2,...\}$. This implies that $\bigcap_{i}(X \backslash F_{\alpha_i})=\phi$. Conversely, suppose that every family of semi*-closed sets in X which has empty intersection has a countable subfamily with empty intersection. Let $\{U_\alpha : \alpha\in\Delta\}$ be a semi*-open cover for X. Then $\cup\{U_\alpha : \alpha\in\Delta\}=X$. Taking the complements, we get $\cap\{X\backslash U_\alpha : \alpha\in\Delta\}=\phi$.

Since X\U$_\alpha$ is semi*-closed for each α∈Δ, by the assumption, there is a countable sub family, { $X \setminus U_{\alpha_i}$: i=1, 2, ...} with empty intersection. That is $\bigcap_i (X \setminus U_{\alpha_i})$ =ϕ. Taking the complements we get $\bigcup_i U_{\alpha_i}$ =X. Hence X is semi*-Lindelöf.

Theorem 4.15: Let *f*:X⟶Y be a semi*-continuous surjection and X be semi*-Lindelöf.
Then Y is Lindelöf.

Proof: Let *f* :X⟶Y be a semi*- continuous surjection and X be semi*-Lindelöf. Let {V$_\alpha$} be an open cover for Y. Then {f^{-1}(V$_\alpha$)}is a cover of X by semi*-open sets. Since X is semi*-Lindelöf, {f^{-1}(V$_\alpha$)} contains a countable subcover, namely {f^{-1}(V_{α_n})}. Then { V_{α_n} } is a countable subcover for Y. Thus Y is Lindelöf.

Theorem 4.16: Let *f* :X⟶Y be a semi*-irresolute surjection and X be semi*-Lindelöf.
Then Y is semi*-Lindelöf.

Proof: Let *f* :X⟶Y be a semi*-irresolute surjection and X be semi*-Lindelöf. Let { V_α} be a semi*-open cover for Y. Then {f^{-1}(V$_\alpha$)} is a cover of X by semi*-open sets. Since X is semi*- Lindelöf, {f^{-1}(V$_\alpha$)} contains a countable sub cover, namely {f^{-1}(V_{α_n})}. Then { V_{α_n} } is a countable subcover for Y. Thus Y is semi*-Lindelöf.

Theorem 4.17: If *f* :X⟶Y is a pre-semi*-open function and Y is semi*-Lindelöf, then X is semi*-Lindelöf.

Proof: Let {V$_\alpha$} be a semi*-open cover for X. Then {*f*(V$_\alpha$)} is a cover of Y by semi*-open sets.
Since Y is semi*- Lindelöf, {*f*(V$_\alpha$)} contains a countable subcover, namely {$f(V_{\alpha_n})$}. Then { V_{α_n} } is a countable subcover for X. Thus X is semi*- Lindelöf.

Theorem 4.18: If *f* :X⟶Y is a semi*-open function and Y is semi*-Lindelöf, then X is Lindelöf.

Proof: Let {V$_\alpha$} be an open cover for X. Then {*f*(V$_\alpha$)}is a cover of Y by semi*-open sets. Since Y is semi*- Lindelöf, {*f*(V$_\alpha$)} contains a countable subcover, namely {$f(V_{\alpha_n})$}. Then { V_{α_n} } is a countable subcover for X. Thus X is Lindelöf.

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