

Optimization of a Polynomial Seventh-Order Two-Parameter Interpolation Kernel Using the Extended Continuity Criterion

Zoran Milivojevic¹, Violeta Stojanovic², Dijana Kostic³

¹Prof. Dr. Department of Electrical and Computer Engineering, Engineering Academy of Serbia, Belgrade, Serbia.

²Prof. Department of Information-communication technology, Academy of Applied Technical and Preschool Studies, Niš, Serbia.

³M.Sc. Department of Information-communication technology, Academy of Applied Technical and Preschool Studies, Niš, Serbia.

Corresponding Author: Zoran Milivojevic

ABSTRACT: A polynomial seventh-order two-parameter (2P) interpolation kernel provides increased flexibility for approximating the ideal sinc interpolation kernel compared with the corresponding one-parameter (1P) kernel. This paper presents an optimization approach for the seventh-order polynomial 2P interpolation kernel based on the Continuity criterion. The criterion requires continuity of the sixth-order derivative of the seventh-order polynomial kernel at the interpolation node $s = 1$. Since this condition provides only one equation for the two independent parameters, α and β , the Continuity criterion is extended by imposing the same condition at the interpolation node $s = 2$. The resulting Extended continuity criterion provides a system of two equations whose solution uniquely determines the kernel parameters α_ξ and β_ξ . The performance of the optimized 2P kernel is evaluated in the spectral domain using similarity measures based on the amplitude characteristic, including the total squared error, the slope in the transition band, and ripple measures. The results are compared with those obtained for seventh-order polynomial 1P and 2P kernels optimized according to the Slope and Extended slope criteria, respectively, as well as with the ideal sinc and windowed sinc kernels. The results show that the Extended continuity criterion provides lower RMS ripple and ripple energy, whereas the Extended slope criterion provides lower total squared error and a steeper transition characteristic. The findings demonstrate that both optimization approaches can be effectively applied to the seventh-order polynomial 2P interpolation kernel, with the choice of criterion depending on the required interpolation characteristics.

KEY WORDS: Convolution; interpolation; polynomial kernel; Amplitude characteristics; Continuity

Date of Submission: 12-10-2026

Date of acceptance: 24-10-2026

I. INTRODUCTION

Interpolation is a fundamental operation in digital signal processing (DSP), enabling the estimation of signal values at locations between discrete samples [1], [2]. It is widely used in image, audio, and speech processing, including spatial and geometric transformations, image rotation, fundamental frequency estimation, and speech synthesis [3]–[5]. The increasing application of interpolation in real-time DSP systems requires both high interpolation accuracy and computational efficiency. Convolution-based interpolation addresses these requirements by using an interpolation kernel, r , to determine the contribution of neighboring discrete samples to the interpolated value [6], [7]. Therefore, the characteristics of the interpolation kernel directly influence the accuracy and spectral properties of the resulting interpolation.

The ideal interpolation kernel is represented by the sinc function, $\text{sinc}(x) = \sin(x)/x$, and is commonly referred to as the sinc kernel. Its time-domain characteristic extends over an infinite interval, while its amplitude characteristic in the frequency domain corresponds to an ideal box-shaped response with a flat passband and stopband and an infinitely large slope at the transition. Such a sinc kernel cannot be directly implemented in practical finite-length convolution-based systems, which necessitates the use of finite-support approximations [8]. One of the simplest approaches is to truncate the sinc function using a window function, w ,

resulting in a finite-length windowed, sincw, kernel. However, windowing modifies the ideal spectral characteristic, introducing passband and stopband ripple and replacing the ideal infinitely large slope at the transition with a finite slope. These limitations provide motivation for the development of finite-support interpolation kernels that can more closely reproduce the desired time- and frequency-domain characteristics of the ideal sinc kernel.

Finite-support polynomial interpolation kernels have been developed as practical approximations of the ideal sinc kernel, with increasing polynomial order generally providing improved interpolation accuracy [2], [9]. The commonly used polynomial kernels include the zeroth-order, linear, cubic, quintic, and seventh-order kernels. The introduction of one or more free parameters provides additional flexibility in adjusting the time- and frequency-domain characteristics of polynomial kernels according to a selected design criterion. In particular, one-parameter (1P, α) kernels allow the kernel characteristics to be controlled by a single parameter, while the introduction of a second independent parameter leads to greater flexibility. The third-order 2P kernel was first proposed in [10], followed by the development of a fifth-order 2P kernel in [11]. The seventh-order polynomial 2P interpolation kernel was constructed in [12] by extending the corresponding seventh-order 1P formulation [13] through the introduction of two independent parameters, α and β . Two-parameter (2P, α, β) interpolation kernels provide an additional degree of freedom for controlling the characteristics of finite-support polynomial kernels.

In [8], the Slope criterion was presented for the optimization of the seventh-order polynomial 2P interpolation kernel. The Slope criterion requires the slope of the time-domain characteristic of the 2P kernel at the interpolation node $s = 1$ to be equal to the corresponding slope of the ideal sinc kernel. This condition results in a single equation with two unknown parameters, α and β , and therefore does not provide a unique solution. To obtain a unique pair of parameter values, the Slope criterion was extended by introducing an additional condition requiring equality of the slopes at the interpolation node $s = 2$. The resulting Extended slope criterion provides a system of two equations with two unknowns, whose solution determines the parameters of the seventh-order polynomial 2P kernel [14].

In this paper, the seventh-order polynomial 2P interpolation kernel is optimized according to the Continuity criterion. The Continuity criterion is defined by requiring continuity of the $(n - 1)$ th-order derivative of the polynomial kernel at the interpolation nodes $s = \pm 1$, where n is the polynomial order. For the seventh-order kernel, this condition corresponds to the continuity of the sixth-order derivative [8]. Since the 2P kernel contains two independent parameters, α and β , the continuity condition at $s = 1$ results in a single equation with two unknown parameters and therefore does not provide a unique solution. To obtain a unique solution, the Continuity criterion is extended by imposing the same continuity condition at the interpolation node $s = 2$. The resulting Extended continuity criterion provides a system of two equations with two unknowns, whose solution determines the parameters α_ξ and β_ξ of the optimized seventh-order polynomial 2P interpolation kernel. The performance of the optimized seventh-order polynomial 2P interpolation kernel is analyzed in the spectral domain and compared with the corresponding kernels optimized according to the Slope and Continuity criteria, as well as with the non-parametric sinc and windowed sincw kernels. The comparison is performed using similarity measures based on the amplitude characteristics, including the total squared error E_T , the slope of the amplitude characteristic k_H in the transition band, and the ripple characteristics, represented by the peak-to-peak ripple $H_{r,pp}$, root-mean-square (RMS) ripple $H_{r,rms}$, and ripple energy E_r . The obtained results are presented graphically and in tabular form.

The remainder of this paper is organized as follows. Section II presents the definition of the seventh-order polynomial 2P interpolation kernel. Section III describes the optimization of the 2P kernel using the Extended continuity criterion. Section IV presents the performance analysis of the optimized 2P kernel in the spectral domain. Section V concludes the paper.

II. POLYNOMIAL SEVENTH-ORDER 2P INTERPOLATION KERNEL

The polynomial seventh-order one-parameter (1P) convolutional interpolation kernel was presented in [13]. The polynomial seventh-order two-parameter (2P) interpolation kernel was subsequently constructed in [12] based on the 1P kernel, with the aim of achieving a more accurate approximation of the ideal sinc interpolation kernel over the interval $(-5, 5)$. The 2P interpolation kernel is defined as:

$$r(s) = \begin{cases} a_{70}|s|^7 + \dots + a_{10}|s| + a_{00}, & |s| \leq 1 \\ a_{71}|s|^7 + \dots + a_{11}|s| + a_{01}, & 1 < |s| \leq 2 \\ a_{72}|s|^7 + \dots + a_{12}|s| + a_{02}, & 2 < |s| \leq 3 \\ a_{73}|s|^7 + \dots + a_{13}|s| + a_{03}, & 3 < |s| \leq 4 \\ a_{74}|s|^7 + \dots + a_{14}|s| + a_{04}, & 4 < |s| \leq 5 \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

The kernel is composed of piecewise seventh-order polynomials defined over the subintervals $(-5, -4)$, ..., $(4, 5)$. These polynomials are required to satisfy the following constraints [12]: a) $r(0) = 1$, b) $r(s) = 0$ for $|s| = 1, \dots, 5$, and c) the derivatives $r^{(\ell)}(s)$ must be continuous at $|s| = 1, \dots, 5$, where $\ell = 1, \dots, 5$. These constraints result in a system of 38 equations with 40 unknowns. To obtain a unique solution, two variables are introduced as independent parameters, defined as $\alpha = a_{73}$ and $\beta = a_{74}$. The resulting coefficients of the 2P kernel in Eq. (1) are listed in Table I.

Table I: The 40 coefficients of the polynomial seventh-order 2P kernel as functions of α and β

i	a_{7i}	a_{6i}	a_{5i}	a_{4i}
0	$245\alpha - 13909\beta + 821/1734$	$-621\alpha + 35289\beta - 1148/867$	0	$760\alpha - 43280\beta + 1960/867$
1	$301\alpha - 16855\beta + 1687/6936$	$-3309\alpha + 185593\beta - 2492/867$	$14952\alpha - 839958\beta + 32683/2312$	$-35640\alpha + 2005060\beta - 128695/3468$
2	$57\alpha - 2947\beta + 35/6936$	$-1083\alpha + 56295\beta - 175/1734$	$8736\alpha - 456654\beta + 1995/2312$	$-38720\alpha + 2035660\beta - 4725/1156$
3	α	$-27\alpha + 57\beta$	$312\alpha - 1353\beta$	$-2000\alpha + 13360\beta$
4	β	-34β	495β	-4000β
	a_{3i}	a_{2i}	a_{1i}	a_{0i}
0	0	$-384\alpha + 21900\beta - 1393/578$	0	1
1	$47880\alpha - 2696750\beta + 127575/2312$	$-36000\alpha + 2028996\beta - 13006/289$	$14168\alpha - 798714\beta + 120407/6936$	$-2352\alpha + 132628\beta - 2233/1156$
2	$101640\alpha - 5374510\beta + 1575/136$	$-157632\alpha + 8382180\beta - 5670/289$	$133336\alpha - 7127418\beta + 42525/2312$	$-47280\alpha + 2538900\beta - 8505/1156$
3	$7680\alpha - 70225\beta$	$-17664\alpha + 207174\beta$	$22528\alpha - 325119\beta$	$-12288\alpha + 211932\beta$
4	19375β	-56250β	90625β	-62500β

III. OPTIMIZATION OF THE KERNEL PARAMETERS

The Continuity criterion for the optimization of the parameters of polynomial interpolation kernels was presented in [8]. The criterion is defined to optimize the time-domain characteristic of the kernel by ensuring continuity of the $(n-1)$ -th-order derivative at the nodes $s = \pm 1$. The continuity of the derivative is achieved by selecting the optimal kernel parameters such that $\lim_{s \rightarrow 1^-} r^{(n-1)}(s) = \lim_{s \rightarrow 1^+} r^{(n-1)}(s)$, where n denotes the order of the polynomial kernel. Based on the definition of the 2P kernel given by Eq. (1), $r(s)$ is a symmetric function. Therefore, the continuity of the derivative will be analyzed only at $s = 1$. The piecewise seventh-order polynomial on the interval $(0, 1)$ is given by:

$$r_0(s) = \left(245\alpha - 13909\beta + \frac{821}{1734} \right) s^7 + \left(35289\beta - 621\alpha - \frac{1148}{867} \right) s^6 + \left(760\alpha - 43280\beta + \frac{1960}{867} \right) s^4 + \left(21900\beta - 384\alpha - \frac{1393}{578} \right) s^2 + 1 \quad (2)$$

The piecewise seventh-order polynomial on the interval $(1, 2)$ is given by:

$$r_1(s) = \left(301\alpha - 16855\beta + \frac{1687}{6936} \right) s^7 + \left(185593\beta - 3309\alpha - \frac{2492}{867} \right) s^6 + \left(14952\alpha - 839958\beta + \frac{32683}{2312} \right) s^5 + \left(2005060\beta - 35640\alpha - \frac{128695}{3468} \right) s^4 + \left(47880\alpha - 2696750\beta + \frac{127575}{2312} \right) s^3 + \left(2028996\beta - 36000\alpha - \frac{13006}{289} \right) s^2 + \left(14168\alpha - 798714\beta + \frac{120407}{6936} \right) s - 2352\alpha + 132628\beta - \frac{2233}{1156} \quad (3)$$

The sixth-order derivatives of $r_0(s)$ and $r_1(s)$ are given by:

$$\frac{d^6 r_0(s)}{ds^6} = \left(1234800\alpha - 70101360\beta + \frac{689640}{289} \right) s + 25408080\beta - 447120\alpha - \frac{275520}{289}, \quad (4)$$

and

$$\frac{d^6 r_1(s)}{ds^6} = \left(1517040\alpha - 84949200\beta + \frac{354270}{289} \right) s + 133626960\beta - 2382480\alpha - \frac{598080}{289}. \quad (5)$$

The values of the derivatives at the node $s = 1$ are:

$$\left. \frac{d^6 r_0(s)}{ds^6} \right|_{s=1} = 787680\alpha - 44693280\beta + \frac{24360}{17}, \quad (6)$$

and

$$\left. \frac{d^6 r_1(s)}{ds^6} \right|_{s=1} = 48677760\beta - 865440\alpha - \frac{243810}{289}. \quad (7)$$

The continuity of the sixth-order derivative at $s = 1$, according to the Continuity criterion, is expressed as:

$$\left. \frac{d^6 r_0(s)}{ds^6} \right|_{s=1} = \left. \frac{d^6 r_1(s)}{ds^6} \right|_{s=1} \Rightarrow 787680\alpha - 44693280\beta + \frac{24360}{17} = 48677760\beta - 865440\alpha - \frac{243810}{289}. \quad (8)$$

from which the continuity condition for the sixth-order derivative at $s = 1$ is obtained as:

$$1653120\alpha - 93371040\beta + \frac{657930}{289} = 0. \quad (9)$$

Equation (9) contains two unknown parameters, α and β , and therefore a unique solution cannot be obtained. To obtain a unique solution and perform the parameter optimization, the Continuity criterion is extended. According to the Extended Continuity criterion, continuity of the $(n - 1)$ th-order derivative must also be ensured at the nodes $s = \pm 2$. Following the procedure described above, the piecewise seventh-order polynomial on the interval $(2, 3)$ is given by:

$$\begin{aligned} r_2(s) = & \left(57\alpha - 2947\beta + \frac{35}{6936} \right) s^7 + \left(56295\beta - 1083\alpha - \frac{175}{1734} \right) s^6 \\ & + \left(8736\alpha - 456654\beta + \frac{1995}{2312} \right) s^5 + \left(2035660\beta - 38720\alpha - \frac{4725}{1156} \right) s^4 \\ & + \left(101640\alpha - 5374510\beta + \frac{1575}{136} \right) s^3 + \left(8382180\beta - 157632\alpha - \frac{5670}{289} \right) s^2 \\ & + \left(133336\alpha - 7127418\beta + \frac{42525}{2312} \right) s - 47280\alpha + 2538900\beta - \frac{8505}{1156} \end{aligned} \quad (10)$$

The sixth-order derivative is given by:

$$\frac{d^6 r_2(s)}{ds^6} = \left(287280\alpha - 14852880\beta + \frac{7350}{289} \right) s + 40532400\beta - 779760\alpha - \frac{21000}{289}. \quad (11)$$

The values of the derivatives at the node $s = \pm 2$ are:

$$\left. \frac{d^6 r_1(s)}{ds^6} \right|_{s=2} = 651600\alpha - 36271440\beta + \frac{110460}{289}, \quad (12)$$

and

$$\left. \frac{d^6 r_2(s)}{ds^6} \right|_{s=2} = 10826640\beta - 205200\alpha - \frac{6300}{289}. \quad (13)$$

The continuity condition for the sixth-order derivative at the node $s = 2$ is:

$$\left. \frac{d^6 r_1(s)}{ds^6} \right|_{s=2} = \left. \frac{d^6 r_2(s)}{ds^6} \right|_{s=2} \Rightarrow 651600\alpha - 36271440\beta + \frac{110460}{289} = 10826640\beta - 205200\alpha - \frac{6300}{289}. \quad (14)$$

Finally, the continuity condition is given by:

$$856800\alpha - 47098080\beta + \frac{116760}{289} = 0. \quad (15)$$

The Extended continuity criterion is used to determine the optimal values of the two parameters, α_ξ and β_ξ , by solving the system of equations formed by Eqs. (9) and (15), which yields:

$$\alpha_{\xi} = \frac{145}{4468}, \quad \beta_{\xi} = \frac{30}{50087}. \quad (16)$$

Fig. 1.a shows the time-domain characteristics of: a) the windowed sinc kernel, r_{sincw} , with $L_w = 10$; b) the seventh-order 1P kernel, $r_{\zeta 7th,1P}$, with $\alpha_{1P,\zeta} = -1027/452574$ [8]; c) the seventh-order 2P kernel, $r_{\zeta 7th,2P}$, with $\alpha_{\zeta} = 146/1917$ and $\beta_{\zeta} = 25/18257$ [14]; d) the seventh-order 1P kernel, $r_{\xi 7th,1P}$, with $\alpha_{1P,\xi} = -3133/2275008$ [8]; and e) the seventh-order 2P kernel, $r_{\xi 7th,2P}$, with $\alpha_{\xi} = 145/4468$ and $\beta_{\xi} = 30/50087$. Fig. 1.b shows the spectral characteristics of: a) the ideal sinc kernel, H_{sinc} ; b) the windowed sinc kernel, H_{sincw} , with $L_w = 10$; c) the seventh-order 1P kernel, $H_{\zeta 7th,1P}$; d) the seventh-order 2P kernel, $H_{\zeta 7th,2P}$; e) the seventh-order 1P kernel, $H_{\xi 7th,1P}$; and f) the seventh-order 2P kernel, $H_{\xi 7th,2P}$.

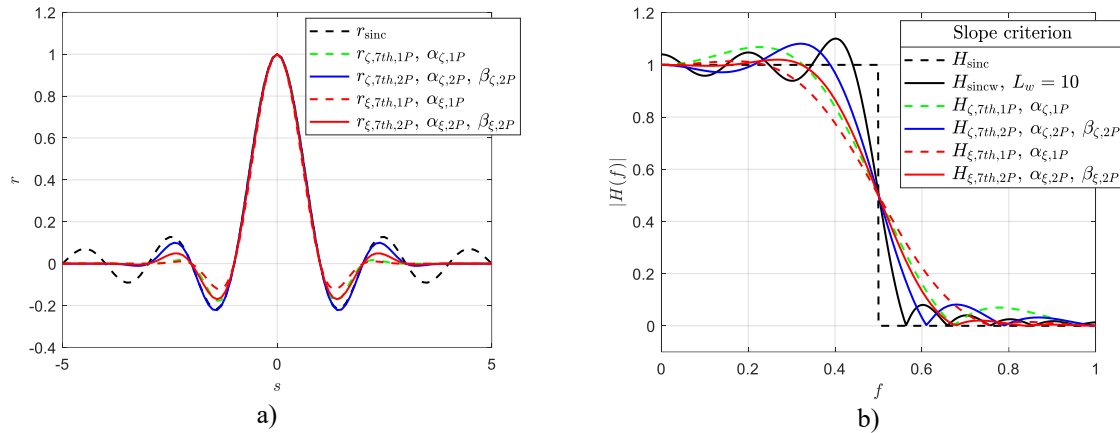


Figure 1: Characteristics of the kernels: a) time-domain characteristics of the windowed sinc kernel, the seventh-order 1P kernel $r_{\zeta 7th,1P}$, with $\alpha_{1P,\zeta} = -1027/452574$ (Slope criterion), and the seventh-order 2P kernel $r_{7th,2P}$, with $\alpha_{\zeta} = 146/1917$ and $\beta_{\zeta} = 25/18257$ (Extended slope criterion); the seventh-order 1P kernel, $r_{\xi 7th,1P}$, with $\alpha_{1P,\xi} = -3133/2275008$ (Continuity criterion); and the seventh-order 2P kernel, $r_{\zeta 7th,2P}$, with $\alpha_{\xi} = 145/4468$ and $\beta_{\xi} = 30/50087$. (Extended continuity criterion); and b) spectral characteristics of the ideal sinc kernel H_{sinc} , the windowed sinc kernel H_{sincw} , the seventh-order 1P kernel $H_{\zeta 7th,1P}$, and the seventh-order 2P kernel $H_{\zeta 7th,2P}$ (Extended slope criterion); and 1P kernel, $H_{\xi 7th,1P}$; and f) the seventh-order 2P kernel, $H_{\xi 7th,2P}$ (Extended continuity criterion).

IV. PERFORMANCE OF THE OPTIMIZED 2P KERNEL

For the seventh-order polynomial 2P interpolation kernel, whose parameters were optimized in the time domain using the Extended continuity criterion (Section III), the performance is analyzed in the spectral domain. The deviations of the amplitude characteristic from that of the ideal sinc interpolation kernel, H_{sinc} , are analyzed. The amplitude characteristic H_{sinc} is a box function. The performance is evaluated using similarity measures [14].

4.1 Similarity measures

To quantify the deviation of the spectral characteristic of the 2P kernel, $H(f)$, from the box characteristic of the ideal sinc kernel, H_{sinc} , the following similarity measures are considered: a) the error function $E(f)$; b) the total squared error E_T ; c) the slope of the amplitude characteristic $k_H(1/2)$; and d) the ripple of the amplitude characteristic $H_r(f)$.

a) **Error function $E(f)$.** The error function $E(f)$ is defined as the squared difference between the amplitude characteristics:

$$E(f) = |H_{\text{sinc}}(f) - H(f)|^2 = |1 - H(f)|^2. \quad (17)$$

b) **Total squared error E_T .** The total squared error E_T is obtained by integrating the squared difference between the amplitude characteristics over the passband frequency range:

$$E_T = \int_{-1/2}^{1/2} |1 - H(f)|^2 df = \int_{-1/2}^{1/2} E(f) df. \quad (18)$$

c) **Slope of the amplitude characteristic $k_H(1/2)$.** The slope of the amplitude characteristic k_H is defined as the derivative of the amplitude characteristic in the transition band, evaluated at $f = 1/2$:

$$k_H = \left. \frac{dH(f)}{df} \right|_{f=1/2}. \quad (19)$$

d) **Ripple of the amplitude characteristic $H_r(f)$.** The amplitude characteristic of the kernel, $H(f)$, can be decomposed into two components: a) smooth, ripple-free component, $H_s(f)$, and b) ripple component, $H_r(f)$:

$$H(f) = H_s(f) + H_r(f). \quad (20)$$

The smooth component $H_s(f)$ represents the underlying monotonically decreasing amplitude characteristic of the kernel and is obtained by zero-phase FIR filtering, $H_s(f) = \text{LPF}\{H(f)\}$. The ripple component $H_r(f)$ represents the oscillatory variations of the amplitude characteristic along the frequency axis f . It is defined as the difference between $H(f)$ and the smooth component $H_s(f)$, i.e., $H_r(f) = H(f) - H_s(f)$. To characterize the ripple of the amplitude characteristic $H(f)$, several parameters of the ripple component $H_r(f)$ are considered, primarily over the passband frequency range $(0, f_c)$:

i) Peak-to-peak ripple:

$$H_{r,pp} = \max_f H_r(f) - \min_f H_r(f). \quad (21)$$

ii) Root-mean-square (RMS) ripple:

$$R_{r,rms} = \text{RMS}\{H_r(f)\} = \sqrt{\frac{1}{2} \int_0^{1/2} H_r^2(f) df}. \quad (22)$$

iii) Ripple energy:

$$E_r = \int_0^{1/2} H_r^2(f) df. \quad (23)$$

4.2 Results

Fig. 2.a shows: a) the amplitude characteristic of the ideal sinc kernel, H_{sinc} ; b) the error functions, $E(f)$, for the seventh-order polynomial 1P kernel, $E_{\zeta,7th,1P}$, corresponding to $r_{\zeta,7th,1P}$ and $H_{\zeta,7th,1P}$, optimized using the Slope criterion [8], and for the seventh-order polynomial 2P kernel, $E_{\zeta,7th,2P}$, corresponding to $r_{\zeta,7th,2P}$ and $H_{\zeta,7th,2P}$, optimized using the Extended slope criterion [14]; and c) the error functions for the seventh-order polynomial 1P kernel, $E_{\xi,7th,1P}$, corresponding to $r_{\xi,7th,1P}$ and $H_{\xi,7th,1P}$, optimized using the Continuity criterion [8], and for the seventh-order polynomial 2P kernel, $E_{\xi,7th,2P}$, corresponding to $r_{\xi,7th,2P}$ and $H_{\xi,7th,2P}$, optimized using the Extended continuity criterion (Section III). The corresponding error functions are presented as log-plots in Fig. 2.b. Figure 3.a shows the amplitude characteristic of the ideal sinc kernel, H_{sinc} , together with the smooth components, H_s , of: a) the windowed sinc kernel, $H_{s,\text{sincw}}$, with $L_w = 10$; b) the seventh-order polynomial 1P kernel, $H_{s,\zeta,7th,1P}$; c) the seventh-order polynomial 2P kernel, $H_{s,\zeta,7th,2P}$; d) the seventh-order polynomial 1P kernel, $H_{s,\xi,7th,1P}$; and e) the seventh-order polynomial 2P kernel, $H_{s,\xi,7th,2P}$. The corresponding ripple components, $H_{r,\text{sincw}}$, $H_{r,\zeta,7th,1P}$, $H_{r,\zeta,7th,2P}$, $H_{r,\xi,7th,1P}$, and $H_{r,\xi,7th,2P}$, are shown in Fig. 3.b. Table II presents the numerical values of the similarity measures for the optimized seventh-order polynomial 1P and 2P kernels: a) the total squared error, E_T (Eq. (15)); b) the slope of the amplitude characteristic, k_H (Eq. (16)); and the ripple measures: c) peak-to-peak ripple, $H_{r,pp}$ (Eq. (18)); d) root-mean-square (RMS) ripple, $H_{r,rms}$ (Eq. (19)); and e) ripple energy, E_r (Eq. (20)). In addition, Table II presents the corresponding similarity measures for the non-parametric kernels: a) the ideal sinc kernel, r_{sinc} , and b) the windowed sinc kernel, r_{sincw} .

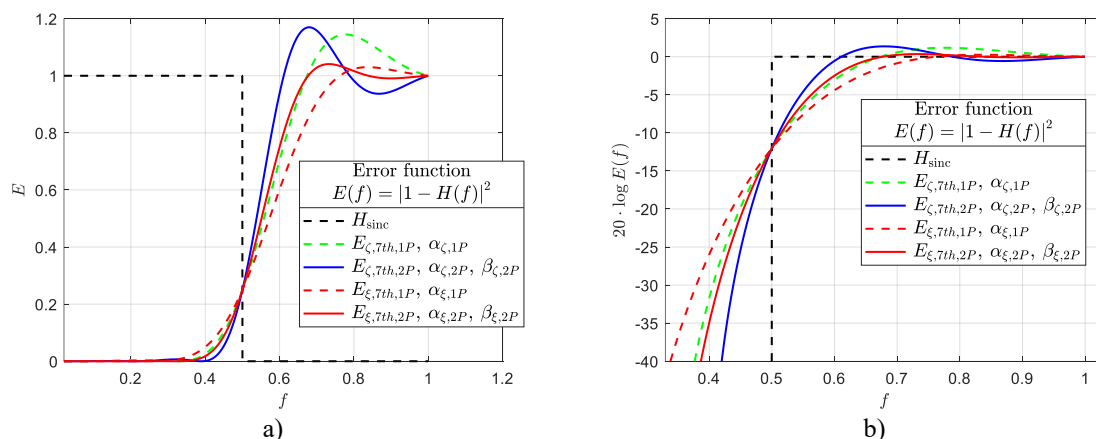


Figure 2: a) amplitude characteristic of the ideal sinckernel, H_{sinc} , and the error functions $E(f)$ of: i) the seventh-order 1P kernel ($E_{\zeta 7th,1P}, r_{\zeta 7th,1P}, H_{\zeta 7th,1P}$) (Slope criterion); ii) the seventh-order 2P kernel ($E_{\zeta 7th,2P}, r_{\zeta 7th,2P}, H_{\zeta 7th,2P}$) (Extended slope criterion); iii) the seventh-order 1P kernel ($E_{\xi 7th,1P}, r_{\xi 7th,1P}, H_{\xi 7th,1P}$) (Continuity criterion); and iv) the seventh-order 2P kernel ($E_{\xi 7th,2P}, r_{\xi 7th,2P}, H_{\xi 7th,2P}$) (Extended continuity criterion); b) log-plots of the corresponding error functions.

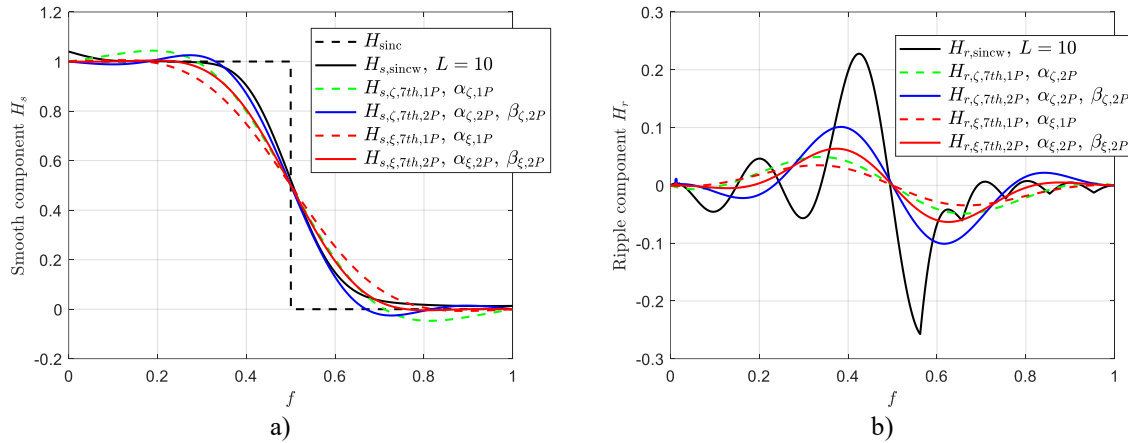


Figure 3: a) amplitude characteristic of the ideal sinckernel, H_{sinc} , and the smooth components, H_s , of the windowed sinckernel, $H_{s,\text{sincw}}$, with $L_w = 10$; the seventh-order polynomial 1P kernel, $H_{s,\zeta 7th,1P}$; the seventh-order polynomial 2P kernel, $H_{s,\zeta 7th,2P}$; the seventh-order polynomial 1P kernel, $H_{s,\xi 7th,1P}$; and the seventh-order polynomial 2P kernel, $H_{s,\xi 7th,2P}$; b) the corresponding ripple components, $H_{r,\text{sincw}}, H_{r,\zeta 7th,1P}, H_{r,\zeta 7th,2P}, H_{r,\xi 7th,1P}$, and $H_{r,\xi 7th,2P}$.

Table II: Numerical values of the similarity measures.

Kernel	α	β	E_T	$k_H(1/2)$	$H_{r,pp}$	$H_{r,rms}$	E_r
r_{sinc}	-	-	0	∞	0	0	0
r_{sincw}	-	-	0.0201	-9.9916	0.4851	0.0857	0.0074
Extended slope criterion							
	α_{ζ}	β_{ζ}	$E_{T,\zeta}$	$k_{H,\zeta}(1/2)$	$H_{r,pp,\zeta}$	$H_{r,rms,\zeta}$	$E_{r,\zeta}$
$r_{7th,1P}$	-1027/452574	-	0.0501	-3.6923	0.0980	0.0293	$8.5589 \cdot 10^{-4}$
$r_{7th,2P}$	146/1917	25/18257	0.0337	-5.6156	0.2022	0.0515	0.0026
Extended continuity criterion							
	α_{ξ}	β_{ξ}	$E_{T,\xi}$	$k_{H,\xi}(1/2)$	$H_{r,pp,\xi}$	$H_{r,rms,\xi}$	$E_{r,\xi}$
$r_{7th,1P}$	-3133/2275008	-	0.0609	-2.9654	0.0694	0.0213	$4.5379 \cdot 10^{-4}$
$r_{7th,2P}$	145/4468	30/50087	0.0428	-4.1942	0.1268	0.0336	0.0011

4.3 Analysis of the Results

Based on the results shown in Table II, it is concluded that, with the applied Slope criterion for the optimization of the parameters of the seventh-order polynomial 2P kernel, the total squared error, E_T , is smaller compared to: a) the parametric polynomial 1P kernel, $r_{7th,1P} \Rightarrow E_{T,\zeta 7rd,1P}/E_{T,\zeta 7rd,2P} = 0.0501/0.0337 = 1.4911$; and b) the windowed sinckernel: $\text{sinc} \Rightarrow E_{T,\text{sincw}}/E_{T,\zeta 7rd,2P} = 0.0201/0.0337 = 0.5947$ times. The total squared error, E_T , when optimizing according to the Continuity criterion is smaller compared to: a) the parametric polynomial 1P kernel, $r_{7th,1P} \Rightarrow E_{T,\xi 7rd,1P}/E_{T,\xi 7rd,2P} = 0.0609/0.0428 = 1.4186$; and b) the windowed sinc kernel: $\text{sinc} \Rightarrow E_{T,\text{sincw}}/E_{T,\xi 7rd,2P} = 0.0201/0.0428 = 0.4674$ times. The total squared error, E_T , for the seventh-order polynomial 2P kernel is smaller when the optimization is performed according to the Extended slope criterion compared to the optimization according to the Extended continuity criterion: $E_{T,\xi 7rd,2P}/E_{T,\zeta 7rd,2P} = 0.0428/0.0337 = 1.2722$ times.

The slope k_H of the spectral characteristic of the seventh-order polynomial 2P kernel in the transition band, when the optimization is performed according to the Slope criterion, is greater compared to: a) the parametric polynomial 1P kernel, $r_{7th,1P} \Rightarrow k_{H,\zeta 7rd,2P}/k_{H,\zeta 7rd,1P} = 5.6156/3.6923 = 1.5209$ times; and b) the

windowed sinc kernel: $\text{sinc} \Rightarrow k_{H,\zeta 7rd,2P}/k_{H,\text{sincw}} = 5.6156/9.9916 = 0.5620$ times. The slope k_H when optimizing according to the Continuity criterion is smaller compared to: a) the parametric polynomial 1P kernel, $r_{7th,1P} \Rightarrow k_{H,\xi 7rd,2P}/k_{H,\xi 7rd,1P} = 4.1942/2.9654 = 1.4144$ times; and b) the windowed sinc kernel: $\text{sinc} \Rightarrow k_{H,\xi 7rd,2P}/k_{H,\text{sincw}} = 4.1942/9.9916 = 0.4198$ times. The slope k_H of the seventh-order polynomial 2P kernel is greater when the optimization is performed according to the Extended slope criterion compared to the optimization according to the Extended continuity criterion: $k_{H,\zeta 7rd,2P}/k_{H,\xi 7rd,2P} = 5.6156/4.1942 = 1.3389$ times.

The ripple of the amplitude characteristic is analyzed using the ripple energy E_r . For the polynomial seventh-order 1P kernel optimized according to the Continuity criterion, a lower ripple energy E_r is obtained compared to: a) the 1P kernel optimized according to the Slope criterion, $r_{7th,1P} \Rightarrow E_{r,\zeta 7rd,1P}/E_{r,\xi 7rd,1P} = 8.5589 \cdot 10^{-4}/4.5379 \cdot 10^{-4} = 1.8861$; and b) the windowed sinc kernel: $\text{sinc} \Rightarrow E_{r,\text{sincw}}/E_{r,\xi 7rd,1P} = 0.0074/4.5379 \cdot 10^{-4} = 16.3071$ times. For the polynomial seventh-order 2P kernel optimized according to the Extended continuity criterion, a lower ripple energy E_r is obtained compared to: a) the 2P kernel optimized according to the Extended slope criterion, $r_{7th,2P} \Rightarrow E_{r,\zeta 7rd,2P}/E_{r,\xi 7rd,2P} = 0.0026/0.0011 = 2.3636$; and b) the windowed sinc kernel: $\text{sinc} \Rightarrow E_{r,\text{sincw}}/E_{r,\xi 7rd,2P} = 0.0074/0.0011 = 6.7273$ times.

The comparative analysis using similarity measures of the seventh-order polynomial 2P interpolation kernel optimized using the Extended slope criterion and the Extended continuity criterion indicates that the Extended continuity criterion provides better results for: a) root-mean-square (RMS) ripple, $H_{r,\text{rms}}$; b) ripple energy, E_r ; and poorer results for: a) total squared error, E_T ; b) slope of the amplitude characteristic, k_H .

The obtained results indicate that the seventh-order polynomial 2P interpolation kernel can be optimized using either the Extended slope criterion or the Extended continuity criterion, depending on the type of signal to be interpolated, such as audio, video, music, and other signals.

V. CONCLUSION

This paper presented an optimization approach for the seventh-order polynomial two-parameter (2P) interpolation kernel based on the Continuity criterion. The criterion was formulated by requiring continuity of the sixth-order derivative of the seventh-order polynomial kernel at the interpolation nodes. Since the continuity condition at $s = 1$ provides only one equation for the two independent kernel parameters, α and β , the Continuity criterion was extended by imposing the same condition at $s = 2$. The resulting Extended continuity criterion provides a system of two equations whose solution uniquely determines the parameters α_ξ and β_ξ . The performance of the optimized seventh-order polynomial 2P kernel was analyzed in the spectral domain and compared with the corresponding seventh-order polynomial 1P kernels optimized according to the Slope and Continuity criteria, as well as with the ideal sinc and windowed sinc kernels. The comparison was performed using similarity measures based on the amplitude characteristics, including the total squared error E_T , the slope of the amplitude characteristic k_H , and the ripple measures: peak-to-peak ripple $H_{r,\text{pp}}$, RMS ripple $H_{r,\text{rms}}$, and ripple energy E_r . The obtained results show that the optimization criterion significantly influences the spectral characteristics of the polynomial interpolation kernel. The Extended slope criterion provides lower values of the total squared error E_T and a steeper transition characteristic, while the Extended continuity criterion provides lower RMS ripple and ripple energy. These results indicate that different optimization criteria emphasize different spectral properties of the interpolation kernel. The presented analysis demonstrates that both the Extended slope criterion and the Extended continuity criterion can be successfully applied for the optimization of the seventh-order polynomial 2P interpolation kernel. The choice of the optimization criterion may therefore depend on the requirements of a particular interpolation application, including audio, speech, image, and video signal processing.

REFERENCES

- [1]. A. Kazuyuki, K. Shoichi and S. Hiroshi, (2022). "Spatial active noise control based on individual kernel interpolation of primary and secondary sound fields", IEEE International Conference on Acoustics, Speech, and Signal Processing (ICASSP), pp. 1056–1060.
- [2]. R. Keys, (1981). "Cubic convolution interpolation for digital image processing", IEEE Transactions on Acoustics, Speech, and Signal Processing, 29(1), pp. 1153–1160.
- [3]. B. Sun and S. Xin, (2023). "An edge-guided weighted image interpolation algorithm", 2023 IEEE 13th International Conference on Electronics Information and Emergency Communication (ICEIEC), Beijing, China, pp. 139–143.
- [4]. O. Rukundo and B. Maharaj, (2014). "Optimization of image interpolation based on nearest neighbor algorithm", International Conference on Computer Vision Theory and Applications (VISAPP), Lisbon, Portugal, pp. 641–647.
- [5]. Z. Guo, Q. Wang, J. Du, J. Pan, Q.-F. Liu and C.-H. Lee, (2024). "A variance-preserving interpolation approach for diffusion models with applications to single-channel speech enhancement and recognition", IEEE/ACM Transactions on Audio, Speech, and Language Processing, 32, pp. 3025–3038.
- [6]. M. Maczka, S. Pawłowski and G. Haldas, (2022). "Application of polynomial approximation in simulations of quantum cascade lasers", Przegląd Elektrotechniczny, 98(12), pp. 321–324.

- [7]. E. Meijering, (2002). "A chronology of interpolation: From ancient astronomy to modern signal and image processing", *Proceedings of the IEEE*, 90(3), pp. 319–342.
- [8]. J. Niessen, E. Meijering, J. Pluim and A. Viergever, (1999). "Quantitative comparison of sinc-approximating kernels for medical image interpolation", *Lecture Notes in Computer Science*, 1679, pp. 210–217.
- [9]. J. Shangquan, L. Yan and W. Yong, (2011). "Fast algorithm of modified cubic convolution interpolation", *4th IEEE International Congress on Image and Signal Processing*, pp. 1072–1075.
- [10]. E. Meijering and M. Unser, (2003). "A note on cubic convolution interpolation", *IEEE Transactions on Image Processing*, 12(4), pp. 477–479.
- [11]. N. Savić, Z. Milivojević and B. Prlinčević, (2021). "Development of the 2P fifth-degree interpolation convolutional kernel", *International Journal of Innovative Research in Advanced Engineering (IJIRAE)*, 11(8), pp. 306–311.
- [12]. Z. Milivojević, N. Savić, B. Prlinčević, V. Stojanović and D. Kostić, (2026). "Construction and Parameterization of a Polynomial Seventh-Order Two-Parameter Interpolation Kernel", *International Refereed Journal of Engineering and Science (IRJES)*, 15(4), pp. 86–92.
- [13]. K. Meijering, M. Zuiderveld and A. Viergever, (1999). "Image reconstruction by convolution with symmetrical piecewise n-th-order polynomial kernels", *IEEE Transactions on Image Processing*, 8(2), pp. 192–201.
- [14]. Z. Milivojevic, V. Stojanovic, D. Kostic, (2016) "Polynomial Seventh-Order Two-Parameter Interpolation Kernel Optimization Using the Extended Slope Criterion", *International Journal of Engineering Research and Development*, 22(8), pp. 108-114.