OPEN ACCESS

International
Journal
Of Modern Engineering Research (IJMER)

Backstepping Method and Its Application to Dc Motor Speed Control

Dao Thi My Linh, Tran Minh Hai*
Thai Binh University, Hung Yen province, Viet Nam

Abstract
Speed control of direct current (DC) motors is an important problem in electric drive systems because it directly affects the response quality and overall system stability. Under operating conditions involving disturbances and parameter uncertainties, traditional linear control methods often exhibit certain limitations, making the study and application of nonlinear control approaches necessary. In this paper, the complete electromechanical model of the DC motor is established in the form of a state-space system. Based on the natural cascade structure of the system, a backstepping controller is designed using the Lyapunov approach for the speed-tracking control problem. The effectiveness of the proposed controller is evaluated through simulations under scenarios of varying speed references and load torque disturbances, and is also compared with that of a conventional PID controller. Simulation results show that the backstepping controller guarantees asymptotic stability, improves transient response performance, reduces tracking error, and enhances disturbance rejection capability compared with the PID controller. These results confirm the feasibility and effectiveness of the backstepping method for DC motor speed control.
Keywords*:DC motor, speed control, backstepping control, Lyapunov stability.*

Date of Submission: 09-04-2026 Date of acceptance: 22-04-2026

I. Introduction

Direct current (DC) motors are among the most widely used drive systems in industry, research, and education because of their relatively simple structure, good torque characteristics in the low-speed region, and flexible controllability [1]–[3]. Owing to the clear relationship among armature voltage, armature current, and rotational speed, DC motors are particularly suitable for speed-control and position-control applications [1], [2].

When the full model is considered, a DC motor represents a typical electromechanical system with a cascade structure, in which the control voltage does not act directly on the rotational speed but rather through the armature current and the electromagnetic torque [1], [2]. In addition, the system is affected by various uncertainties and disturbances, such as armature resistance variations, changes in the moment of inertia, friction, and load torque disturbances [3]–[5]. Therefore, DC motors are often selected as benchmark systems for investigating, evaluating, and validating the effectiveness of control strategies, particularly nonlinear control methods [1], [3], [4].

In many previous studies, the speed-control problem of DC motors has mainly been addressed using linear controllers, particularly PI and PID controllers, due to their simple structure, ease of tuning, and practical implementation convenience [1], [4], [5]. However, when the system operates under load disturbances or parameter uncertainties, the performance of these controllers may deteriorate, as reflected by increased tracking error, large overshoot, or poor robustness of the response [4]–[6].

To overcome these limitations, various nonlinear control methods have been proposed and developed. Among them, sliding-mode control offers robustness against disturbances and uncertainties, but it is often accompanied by the chattering phenomenon, which adversely affects control signal quality and actuator lifespan [4], [6]. Compared with sliding-mode control, the backstepping method enables a systematic controller design based on Lyapunov functions and is particularly suitable for nonlinear systems in strict-feedback form [7]–[10]. By directly exploiting the dynamic structure of the plant, backstepping not only guarantees closed-loop stability but also improves response performance and disturbance rejection capability [7]–[9].

For DC motors, the backstepping method is especially suitable because the plant has an inherent electromechanical coupling and a natural hierarchical structure between the electrical loop and the mechanical loop [2], [7], [8]. Several studies have shown that backstepping controllers can provide good speed-tracking performance, reduce transient error, and maintain stability even in the presence of load disturbances or model parameter mismatches [7], [8], [11]. In addition, recent developments have extended backstepping to adaptive and optimal forms in order to enhance the capability of handling parameter uncertainties and improve the robustness of the control system [9]–[13].

II. Main Content

When the complete electromechanical model is considered, the speed-control problem of a DC motor is no longer a simple linear control problem but becomes the control problem of a nonlinear system with a cascade structure. In this system, the control input $u(t)$does not directly affect the angular speed $\omega(t)$; instead, it first governs the dynamics of the armature current $i(t)$, which then generates the electromagnetic torque that influences the mechanical dynamics of the rotor. Therefore, the controller design process must simultaneously take into account the dynamic relationships among the system state variables, rather than relying solely on the input-output relationship of a reduced-order model [8], [11].

From a scientific perspective, the objective is to construct a controller capable of guaranteeing asymptotic stability for the entire electromechanical system while effectively exploiting its intrinsic structure. In contrast, conventional linear controllers are typically designed on the basis of simplified models or locally linearized models around an operating point, and thus their ability to fully capture the nonlinear characteristics of the system remains limited. As a result, under disturbances, varying loads, or parameter uncertainties, ensuring both control performance and global stability becomes more challenging [7], [8].

In this context, the backstepping control method is a suitable choice because it allows a systematic design of control laws for cascade-type nonlinear systems based on Lyapunov functions. A major advantage of this method is that the design can be carried out recursively step by step, where an intermediate state variable is treated as a virtual control input at one step and then realized in the subsequent step. Consequently, the resulting controller is not only well matched to the electromechanical coupling nature of the DC motor but also provides a rigorous basis for proving the stability of the closed-loop system [7], [8], [11].

2.1 Mathematical Model and Dynamic Structure of the DC Motor

A DC motor is described by two tightly coupled dynamic subsystems: the electrical dynamics of the armature circuit and the mechanical dynamics of the rotor rotational motion [1], [2]. On the electrical side, the armature circuit consists of the armature resistance RRR, armature inductance LLL, control voltage $u(t)$, and the back electromotive force $e_b(t)$generated by the rotor rotation. According to Kirchhoff's voltage law, the armature circuit voltage equation can be written as follows [1]:

$$u(t) = Ri(t) + L\frac{di(t)}{dt} + e_b(t) \tag{2.1}$$

For a DC motor with constant field excitation, the back electromotive force is proportional to the angular speed of the rotor, as

$$e_b(t) = k_e\omega(t) \tag{2.2}$$

where k_e is the back electromotive force constant. Substituting (2.2) into (2.1) yields the dynamic equation of the armature current as follows:

$$L\dot{i}(t) = -Ri(t) - k_e\omega(t) + u(t) \tag{2.3}$$

On the mechanical side, the rotational motion of the rotor follows Newton's second law for rotational systems. The total torque acting on the motor shaft includes the electromagnetic torque generated by the armature current, the viscous friction torque, and the external load torque. Therefore, the torque balance equation is written as follows [1], [2]:

$$\begin{aligned} J\dot{\omega}(t) &= \sum \tau \\ &= k_t i(t) - b\omega(t) - \tau_L(t) \end{aligned} \tag{2.4}$$

where J is the equivalent moment of inertia referred to the motor shaft, $\tau_L(t)$is the external load torque, and the electromagnetic torque and viscous friction torque are respectively given by [1], [2]:

$$\tau_e(t) = k_t i(t) \tag{2.5}$$

$$\tau_f(t) = b\omega(t) \tag{2.6}$$

From (2.5) and (2.6), the complete electromechanical model of the DC motor can be represented in the form of a first-order state-space system:

$$\begin{cases} \dot{\omega}(t) = \frac{k_t}{J} i(t) - \frac{b}{J}\omega(t) - \frac{1}{J}\tau_L(t) \\ \dot{i}(t) = -\frac{R}{L} i(t) - \frac{k_e}{L}\omega(t) + \frac{1}{L} u(t) \end{cases} \quad (2.7)$$

Model (2.7) clearly reflects the electromechanical nature of the DC motor: the control voltage u(t)u(t)u(t) first governs the dynamics of the armature current i(t)i(t)i(t), and the current then generates the electromagnetic torque that affects the angular speed dynamics ω(t)\omega(t)ω(t). In other words, the control signal does not act directly on the speed, but through an intermediate state variable, namely the armature current. Therefore, the DC motor drive system has a natural cascade structure, which is highly suitable for the design of a backstepping controller using the recursive Lyapunov-based design approach.

2.2 Designof Backstepping Controller

The control objective is to design the control law *u* such that the motor speed*ω(t)* tracks the reference speed signal $\omega_d(t)$ [7], [8].

2.2.1 Virtual Controller for the Speed Loop

Consider the speed tracking error:

$$e_\omega(t) = \omega(t) - \omega_d(t) \quad (2.8)$$

Taking the time derivative of (2.8) yields

$$\dot{e}_\omega(t) = \dot{\omega}(t) - \dot{\omega}_d(t) \quad (2.9)$$

Substituting the speed dynamic equation $\dot{\omega}(t)$ from (2.7) into (2.9) gives

$$\dot{e}_\omega(t) = \left[\frac{k_t}{J} i(t) - \frac{b}{J}\omega(t) - \frac{1}{J}\tau_L(t)\right] - \dot{\omega}_d(t) \quad (2.10)$$

Choose the Lyapunov candidate function for the speed loop in the form:

$$V_1 = \frac{1}{2} e_\omega^2(t) \quad (2.11)$$

Then, the time derivative of V_1 is:

$$\dot{V}_1 = e_\omega(t).\dot{e}_\omega(t) \quad (2.12)$$

Substituting (2.10) into (2.12) yields

$$\dot{V}_1 = e_\omega(t)\left[\frac{k_t}{J} i(t) - \frac{b}{J}\omega(t) - \frac{1}{J}\tau_L(t) - \dot{\omega}_d(t)\right] \quad (2.13)$$

To ensure that $\dot{V}_1$ is negative definite, choose the virtual reference current $i_d(t)$ such that the expression in brackets in (2.13) satisfies:

$$\frac{k_t}{J} i_d(t) - \frac{b}{J}\omega(t) - \frac{1}{J}\tau_L(t) - \dot{\omega}_d(t) = -k_1.e_\omega(t), \quad (k_1 > 0) \quad (2.14)$$

Accordingly, the reference current is determined as:

$$i_d(t) = \frac{J}{k_t}\left[\dot{\omega}_d(t) + \frac{b}{J}\omega(t) + \frac{1}{J}\tau_L(t) - k_1.e_\omega(t)\right] \quad (2.15)$$

If the armature current *i(t)*exactly tracks the desired value $i_d(t)$, then the speed error dynamics becomes:

$$\dot{e}_\omega(t) = -k_1 e_\omega(t) \quad (2.16)$$

Then, the derivative of the Lyapunov function is simplified as:

$$\dot{V}_1 = -k_1 e_\omega^2(t) \quad (2.17)$$

Since $k_1 > 0$, it follows that $\dot{V}_1 \leq 0$ for all e_ω, and $\dot{V}_1 < 0$ when $e_\omega \neq 0$. Therefore, e_ω asymptotically converges to zero in the Lyapunov.

2.2.2 Controller Design for the Current Loop

To design the controller for the current loop, the current tracking error is first defined as follows [7], [8]:

$$e_i(t) = i(t) - i_d(t) \tag{2.18}$$

It follows that:

$$i(t) = e_i(t) + i_d(t) \tag{2.19}$$

Taking the time derivative of $e_i(t)$, we obtain:

$$\dot{e}_i(t) = \dot{i}(t) - \dot{i}_d(t) \tag{2.20}$$

Substituting the armature current dynamic equation in (2.8) into the above expression yields:

$$\dot{e}_i(t) = -\frac{R}{L} i(t) - \frac{k_e}{L}\omega(t) + \frac{1}{L}u(t) - \dot{i}_d(t) \tag{2.21}$$

The objective of this design step is to construct the control input $u(t)$such that the current error $e_i(t)$converges to zero.

From the previous speed-loop design step, the derivative of the Lyapunov function V_1 can be rewritten by substituting (2.19) into (2.13) and performing some straightforward manipulations, yielding:

$$\begin{aligned}
\dot{V}_1 &= e_\omega(t)\left[\frac{k_t}{J}\left[e_i(t) + i_d(t)\right] - \frac{b}{J}\omega(t) - \frac{1}{J}\tau_L(t) - \dot{\omega}_d(t)\right] \\
&= e_\omega(t)\left[\frac{k_t}{J} i_d(t) - \frac{b}{J}\omega(t) - \frac{1}{J}\tau_L(t) - \dot{\omega}_d(t) + \frac{k_t}{J} e_i(t)\right] \\
&= e_\omega(t)\left[-k_1 e_\omega + \frac{k_t}{J} e_i(t)\right] \\
&= -k_1 e_\omega^2 + \frac{k_t}{J} e_i.(t).e_\omega(t)
\end{aligned} \tag{2.22}$$

To incorporate the current dynamics into the stability analysis, an extended Lyapunov function is selected in the following form [8]:

$$V = V_1 + \frac{1}{2} e_i^2(t) \tag{2.23}$$

Taking the time derivative of V yields:

$$\begin{aligned}
\dot{V} &= \dot{V}_1 + e_i \dot{e}_i \\
&= \dot{V}_1 + e_i\left(\dot{i} - \dot{i}_d\right)
\end{aligned} \tag{2.24}$$

Substituting (2.21) and (2.22) into (2.24) yields

$$\begin{aligned}
\dot{V} &= -k_1 e_\omega^2 + \frac{k_t}{J} e_i.e_\omega + e_i\left(-\frac{R}{L} i - \frac{k_e}{L}\omega + \frac{1}{L}u - \dot{i}_d\right) \\
&= -k_1 e_\omega^2 + e_i\left[\frac{k_t}{J} e_\omega - \frac{R}{L} i - \frac{k_e}{L}\omega + \frac{1}{L}u - \dot{i}_d\right]
\end{aligned} \tag{2.25}$$

To ensure that $\dot{V}$ is negative definite, choose the control input $u(t)$ such that the expression in brackets in (2.25) satisfies:

$$\left[\frac{k_t}{J} e_\omega - \frac{R}{L} i - \frac{k_e}{L}\omega + \frac{1}{L}u - \dot{i}_d\right] = -k_2 e_i \tag{2.26}$$

where $k_2 > 0$is a design gain. Accordingly, the control law for the current loop is determined as:

$$\begin{aligned}
u &= L.\dot{i}_d + R.i + k_e\omega - L\frac{k_t}{J} e_\omega - L k_2 e_i \\
&= L\left(\dot{i}_d - k_2 e_i - \frac{k_t}{J} e_\omega\right) + R.i + k_e\omega
\end{aligned} \tag{2.27}$$

Substituting (2.27) into (2.25) yields:

$$\dot{V} = -k_1 e_\omega^2 - k_2 e_i^2 \tag{2.28}$$

Thus, $\dot{V} < 0$ for all e_ω and e_i, except at the equilibrium point. Therefore, according to Lyapunov stability theory, the errors e_ω and e_i asymptotically converge to zero. This indicates that the designed backstepping controller guarantees asymptotic stability for both the speed loop and the current loop of the DC motor system [7], [8].

III. Simulation Results and Evaluation

In the simulation section, the plant under study is a low-power DC motor whose characteristic electromechanical parameters are listed in Table I. Based on this model, the backstepping controller is designed with a two-loop structure, consisting of an inner current loop and an outer speed loop. The controller parameters are selected such that the closed-loop system achieves a fast response, small tracking error, and asymptotic stability.

Table I. Electromechanical Parameters of the DC Motor

Symbol	Value	Unit	Physical Meaning
R	1.2	Ω	Armature resistance
L	0.0008	H	Armature inductance
K_e	0.02	V·s/rad	Back electromotive force constant
K_t	0.02	N·m/A	Torque constant
J	0.00012	kg·m²	Rotor moment of inertia
b	0.0001	N·m·s/rad	Viscous friction coefficient
u_{max}	24	V	Supply voltage limit

Based on the parameter set given in Table I, simulation scenarios are constructed to evaluate the speed-tracking capability and response performance of the drive system under the backstepping controller. To clearly demonstrate the effectiveness of the proposed method, a classical PID controller is also developed using the same motor model and the same test scenarios for comparison. The simulation results are analyzed in terms of speed response, tracking error, control voltage, and armature current.

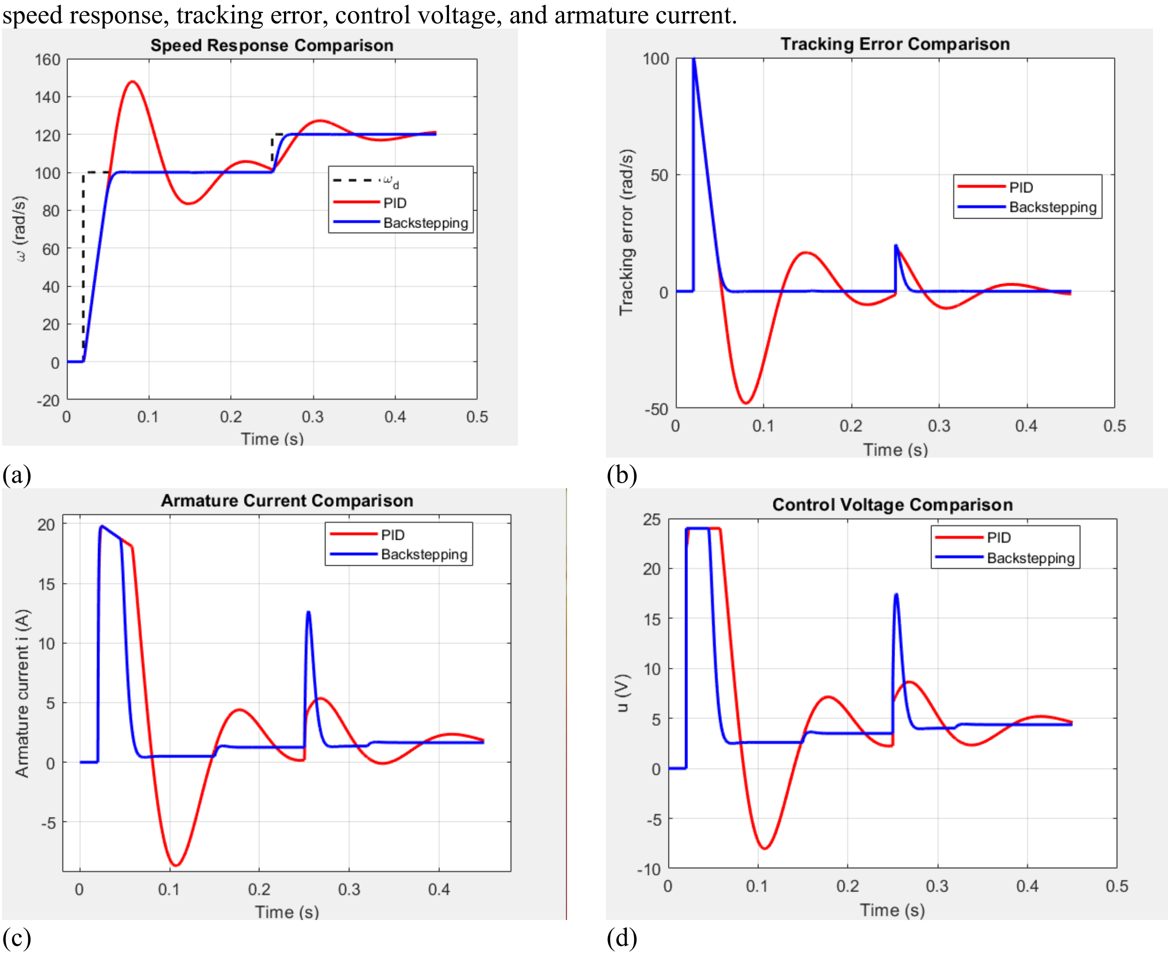

Fig. 1. Comparison results between the PID controller and the backstepping controller for the DC motor: (a) speed response, (b) tracking error, (c) control voltage, and (d) armature current.

Fig.1a shows that the backstepping controller provides significantly better speed-response performance than the PID controller. At the first reference step from 0 to 100 rad/s at t≈0.02s, the backstepping controller converges rapidly to the desired value with almost negligible overshoot. In contrast, the PID controller exhibits a large peak overshoot, reaching approximately 148rad/s, which corresponds to nearly 48% overshoot, followed by oscillations before settling. At the second reference step from 100 to 120 rad/s at t≈0.25s, the backstepping controller continues to track rapidly with only slight oscillation, whereas the PID controller still overshoots to about 127 rad/s, corresponding to approximately 5.8% overshoot.

This trend is more clearly illustrated by the tracking-error plot in Fig.1b. With the backstepping controller, the initial error of about 100 rad/s decreases rapidly to nearly zero with almost no residual oscillation; at the second reference step, the error increases only to about 20 rad/s and is then quickly eliminated. By contrast, the PID controller produces considerably larger error oscillations: after the first reference step, the negative error reaches nearly −48rad/s, then changes sign several times before gradually decaying; at the second reference step, the positive error still reaches nearly 20 rad/s and continues to oscillate around the equilibrium. This indicates that the backstepping controller achieves a faster convergence rate and better tracking performance.

The difference is also reflected in the control input and armature current shown in Fig.1c and Fig.1d. For the backstepping controller, the control voltage quickly rises to the saturation limit of approximately 24V in the initial stage, and then rapidly returns to a stable range of about 3-5V; at the second reference change, only a short pulse of approximately 17-18V appears. In contrast, the PID control voltage oscillates more strongly; after saturating at 24V, it drops to nearly −8V before stabilizing. Similarly, the armature current under backstepping reaches a peak of about 20A during startup and 12-13A during the speed transition, then quickly returns to the range of 1-2A. By contrast, the PID current exhibits larger oscillations and even reverses sign to nearly −9A, indicating less stable regulation of the electromagnetic torque.

Overall, the simulation results demonstrate that backstepping outperforms PID under the same test conditions. Specifically, the proposed controller significantly reduces overshoot, shortens the settling time, decreases the amplitude of tracking-error oscillations, and improves the quality of both the control voltage and the armature current. These results confirm that backstepping is more suitable for DC motors with an electromechanical cascade structure, since it directly exploits the relationship among voltage, current, and rotational speed, whereas the PID controller is more prone to oscillation and larger overshoot.

IV. Conclusion

This paper has presented a Lyapunov-based backstepping controller for DC motor speed control by considering the full electromechanical dynamics and input voltage saturation. Comparative simulation results with a conventional PID controller have shown that the proposed method achieves consistently better control performance in transient operation. In particular, the backstepping controller provides fast convergence, negligible overshoot, and significantly reduced tracking error under step changes in the reference speed.

The results also indicate that the proposed controller improves not only the speed response but also the internal electromechanical behavior of the system. Compared with PID control, the backstepping approach yields smoother control voltage and armature current responses, with smaller oscillations and better torque coordination during transients. These characteristics confirm that the proposed design is well suited to the cascade structure of the DC motor, where the control input affects the speed indirectly through the armature current and electromagnetic torque.

Overall, the study demonstrates that backstepping is a more effective solution than conventional PID control for DC motor speed regulation when high accuracy, fast response, and low oscillation are required. Future work will focus on extending the proposed scheme toward robust and adaptive backstepping designs to improve performance under parameter uncertainties, load disturbances, and practical operating conditions.

REFERENCES

[1] M. Kuczmann, "Review of DC Motor Modeling and Linear Control: Theory with Laboratory Tests," *Electronics,* vol. 13, no. 11, 2024.

[2] E. Molina-Santana, L. A. Iturralde Carrera, J. M. Álvarez-Alvarado, M. Aviles, and J. Rodríguez-Resendiz, "Modeling and Control of a Permanent Magnet DC Motor: A Case Study for a Bidirectional Conveyor Belt's Application," *Eng,* vol. 6, no. 3, 2025.

[3] G. Saravanan, C. Pazhanimuthu, and P. Naveen, "Performance improvement of DC motor control system using PID controller with Kookaburra and Red Panda optimization algorithm," *Sci Rep,* vol. 15, no. 1, p. 20021, Jun 6 2025.

[4] M. J. Hussein, O. T. Khazraji, and A. M. Almawla, "Simulation and Experimental Evaluation of DC Motor Control Strategies Using MATLAB and Arduino Mega," *Journal Européen des Systèmes Automatisés,* vol. 58, no. 1, pp. 55-64, 2025.

[5] P. Proaño, M. Pozo, C. Gallardo, and O. Camacho, "Non-linear PID control of AC current and DC voltage for a photovoltaic system operating on a microgrid," *Results in Control and Optimization,* vol. 18, 2025.

[6] D. S. M. B. G. K. G. Parashard, "Comparison of Fuzzy-PID and PID Controller for Speed Control of DC Motor using LabVIEW," presented at the International Conference on Pervasive Computing Advances and Applications – PerCAA 2019, 2019.

[7] R. Afifa, S. Ali, M. Pervaiz, and J. Iqbal, "Adaptive Backstepping Integral Sliding Mode Control of a MIMO Separately Excited DC Motor," *Robotics,* vol. 12, no. 4, 2023.

[8] A. H. Hameed, S. A. Al-Samarraie, A. J. Humaidi, and N. Saeed, "Backstepping-Based Quasi-Sliding Mode Control and Observation for Electric Vehicle Systems: A Solution to Unmatched Load and Road Perturbations," *World Electric Vehicle Journal,* vol. 15, no. 9, 2024.

[9] L. Wang, X. Wang, and Z. Wang, "Event-Triggered Optimal Tracking Control for StrictFeedback Nonlinear Systems With Non-A," 2023.

[10] M. Zhou, X. Zhang, and X. Deng, "Tracking control problem of nonlinear strict-feedback systems with input nonlinearity: An adaptive neural network dynamic surface control method," *PLoS One,* vol. 19, no. 10, p. e0312345, 2024.

[11] L. H. Abood, B. Kadhim Oleiwi, and E. Hussain Ali, "Optimal backstepping controller for controlling DC motor speed," *Bulletin of Electrical Engineering and Informatics,* vol. 11, no. 5, pp. 2564-2572, 2022.

[12] X. Tang and H. Jiang, "Prescribed Performance Back-Stepping Tracking Control for a Class of High-Order Nonlinear Systems via a Disturbance Observer," *Entropy,* vol. 25, no. 1, 2023.

[13] L. Zhang, L. Han, and H. Liu, "Event-Triggered Backstepping Control of Fractional-Order Chaotic Systems with Dead Zone Via Disturbance Observer," *Journal of Nonlinear Mathematical Physics,* vol. 31, no. 1, 2024.