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Assessing Student Success through Fuzzy Logic

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Abstract
The traditional methods of assessing student success often rely on rigid grading systems and quantitative metrics that may not effectively capture the nuanced nature of educational achievements. This paper presents a novel approach utilizing fuzzy logic to assess student success, accommodating the inherent uncertainty and ambiguity in educational environments. Fuzzy logic, a mathematical framework that handles imprecision and vagueness, is leveraged to develop a comprehensive assessment model that integrates multiple qualitative and quantitative factors influencing student success. By employing a fuzzy inference system, the proposed model translates linguistic variables into a structured evaluative process. The study demonstrates the model's robustness and flexibility through a case study involving diverse student data from a higher education institution. The results indicate that the fuzzy logic approach provides a more holistic and adaptable evaluation of student success compared to conventional assessment methods. This paper highlights the potential of fuzzy logic as a transformative tool in educational assessment, advocating for its broader adoption to foster more personalized and equitable education systems.

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I. INTRODUCTION

The landscape of educational assessment has evolved dramatically over the past few decades, driven by an imperative to better understand and improve student success. Traditional assessment models, deeply rooted in rigid numerical and categorical evaluations, have often struggled to fully capture the diverse and nuanced nature of student performance. This has spurred interest in alternative methodologies that can address the complexities of educational settings. Among these innovations, fuzzy logic has emerged as a robust framework for dealing with the inherent uncertainties and subjectivities associated with student assessment [1, 2].

Fuzzy logic, conceptualized by Lotfi A. Zadeh in the mid-1960s, extends classical binary set theory by incorporating the concept of partial truth—where values can range between totally true and completely false [1]. This flexibility allows fuzzy logic to adeptly handle imprecise information, making it particularly suitable for applications in environments characterized by ambiguity and complex decision-making processes. Over the years, fuzzy logic has been successfully applied across various domains, including control systems, pattern recognition, and artificial intelligence, providing insightful models that cope with vagueness and ambiguity [3].

In the realm of education, the application of fuzzy logic to assess student success is gaining traction, offering transformative potentials for addressing widely acknowledged limits of traditional grading systems [4]. Conventional methods often rely on absolute numerical grades that may not fully represent the breadth of a student's capabilities or the multifaceted nature of learning outcomes. Fuzzy logic provides a framework that allows for the integration of qualitative and quantitative aspects of student performance, paving the way for a more holistic assessment [5].

Fuzzy logic employs linguistic variables, such as "poor," "average," and "excellent," which can be expressed in mathematical terms with degrees of membership within a defined range. This can encompass intricate dimensions of student success, such as creativity, critical thinking, and collaboration—skills that are challenging to quantify with traditional numerical grades. By embracing these variables, educators are equipped to evaluate student performance in a manner that respects individual learning paths and diverse competencies [6].

Furthermore, fuzzy logic's ability to synthesize multifactorial data is particularly advantageous for creating integrated evaluation systems that consider variables like class participation, project work, and self-assessment alongside examination scores. This enables educational institutions to tailor their teaching strategies and interventions more effectively, fostering a learning environment that better accommodates personal growth and educational achievement [7].

Recent studies underscore the efficacy of fuzzy logic in educational assessments. For example [8], demonstrated the robustness of fuzzy logic in evaluating academic performance at Alexandria University, highlighting its adaptability and ease of integration with existing educational technologies. In addition [9], explored the application of fuzzy logic in evaluating academic recollection methods, showcasing its potential to improve accuracy and personalization in assessment procedures.

Despite its advantages, the implementation of fuzzy logic in the education sector is not without challenges. Critically, the process of defining membership functions and linguistic variables must be carefully managed to reflect educational goals and learning objectives accurately. Additionally, educators and institutions may face resistance to change, requiring comprehensive training and a shift in assessment paradigms to adopt fuzzy logic-based methodologies effectively [10].

Moreover, the success of fuzzy logic applications in education demands a collaborative approach among educators, administrators, and technology developers to ensure methodologies remain aligned with pedagogical objectives. This involves ongoing evaluation and refinement of fuzzy systems, as well as fostering a culture of innovation and openness within educational practices.

This paper addresses the application of fuzzy logic in assessing student success, exploring how this approach can provide more inclusive and comprehensive methodologies that encapsulate the full spectrum of student learning outcomes. By analyzing case studies and empirical research, we seek to elucidate the transformative potential of fuzzy logic, offering insights into future directions for educational assessment practices that prioritize accuracy, inclusivity, and adaptability.

II. METHODOLOGY

Fuzzy logic is a sophisticated mathematical system designed to model uncertainty, vagueness, and imprecision—characteristics prevalent in real-world scenarios. This approach extends classical logic by introducing multivalued logic systems, where truth values range continuously between 0 (completely false) and 1 (completely true). This section addresses the fundamental principles and mathematical structures of fuzzy logic.

Fuzzy logic was introduced by Lotfi A. Zadeh in 1965 through the seminal paper "Fuzzy Sets" (Zadeh, 1965) [1]. Unlike binary logic, which operates with crisp boundaries, fuzzy logic allows for degrees of membership, facilitating decision-making processes in ambiguous environments. This flexibility makes fuzzy logic particularly valuable in fields such as control systems, artificial intelligence, and decision-making algorithms.

2.1. Basic Concepts and Definitions

At the heart of fuzzy logic is the concept of fuzzy sets. A fuzzy set A in a universe of discourse X is characterized by a membership function $\mu_A: X \rightarrow [0,1]$, where $\mu_A(x)$ indicates the degree of membership of element x in fuzzy set A.

i. Membership Functions

Membership functions are fundamental in depicting fuzzy sets. They transform input data into grades of membership ranging from 0 to 1. Common types of membership functions include:

Triangular: Defined by three parameters (a, b, c), which form a triangle.

$$\mu_A(x) = \begin{cases} 0, & x \le a \\ \frac{x-a}{b-a}, & a < x \le b \\ \frac{c-x}{c-b}, & b < x \le c \\ 0, & x > c \end{cases}$$

(1)

Trapezoidal: Defined by four parameters (a, b, c, d), which form a trapezoid.

$$\mu_A(x) = \begin{cases} 0, & x \le a \\ \frac{x-a}{b-a}, & a < x \le b \\ 1, & b < x \le c \\ \frac{d-x}{d-c}, & c < x \le d \\ 0, & x > d \end{cases} \quad (2)$$

Gaussian: Utilizes a bell-shaped curve characterized by mean c and standard deviation σ.

$$\mu_A(x) = e^{-\frac{(x-c)^2}{2\sigma^2}} \quad (3)$$

ii. Fuzzy Logic Operations

Fuzzy logic extends classical set operations using specific rules:

Fuzzy Union (∪): $\mu_{A \cup B}(x) = max(\mu_A(x), \mu_B(x))$ (4)

Fuzzy Intersection (∩): $\mu_{A \cap B}(x) = min(\mu_A(x), \mu_B(x))$ (5)

Fuzzy Complement: $\mu_{\neg A}(x) = 1 - \mu_A(x)$ (6)

These operations enable complex logical reasoning by defining how fuzzy sets interact, mirroring human-like inference.

III. FUZZY INFERENCE SYSTEM (FIS)

At the core of a Fuzzy Inference Systems (FIS) are fuzzy sets and linguistic variables, which allow for the representation of data not as binary or discrete values but as degrees of truth. The system operates through a series of steps: fuzzification, where crisp inputs are transformed into fuzzy values; the application of a rule base, composed of if-then statements that mimic expert knowledge and decision-making processes; aggregation, which combines the outputs of all individual rules; and defuzzification, where the aggregated fuzzy outcomes are converted back into a precise, actionable result. This process enables FIS to handle complex, real-world problems by incorporating human-like reasoning, thus offering a flexible and intuitive approach for dealing with uncertainty and subjectivity in various applications, ranging from control systems and data analysis to decision-making in uncertain environments. The steps in the FIS structure include:

a. **Fuzzification**: Converts crisp inputs into fuzzy sets.

b. **Inference**: Uses a set of fuzzy rules in the form of "IF-THEN" constructs.

c. **Composition**: Combines the results of multiple rules.

d. **Defuzzification**: Converts fuzzy output into a crisp action, often using methods like the centroid or maximum.

3.1 Fuzzification

Fuzzification is a crucial process in fuzzy logic systems, where it serves as the initial step in transforming crisp numerical inputs into fuzzy sets. This transformation allows a fuzzy system to handle imprecise and uncertain data, enabling more human-like reasoning and decision-making.

The primary role of fuzzification is to map crisp input values to fuzzy sets, where each input value is expressed in terms of membership degrees to language descriptors such as "low," "medium," and "high." This process allows a system to work with vague concepts and linguistic terms rather than precise values, thereby matching human reasoning more closely. The following are the steps in the fuzzification process.

a. **Identify Input Variables:**

Determine the set of input parameters that need to be evaluated. These could be temperature, speed, or any other quantifiable measure pertinent to the system.

b. **Define Universe of Discourse:**

Establish the range of values over which each input variable will be evaluated. For instance, if the input is temperature, the universe of discourse could be from 0°C to 100°C.

c. **Design Membership Functions:**

Create membership functions for each linguistic term associated with the input variables. Common membership functions include triangular, trapezoidal, Gaussian, and bell-shaped curves. The choice of function type depends on the specific application and desired sensitivity. Triangular and trapezoidal functions are often used for simplicity, while Gaussian functions are used for smooth transitions.

d. **Calculate Membership Degrees:**

For a given crisp input value, calculate its degree of membership in each predefined fuzzy set using the respective membership function. For example, if "medium" is one of the fuzzy sets defined for temperature, the membership function calculates how much a specific input temperature belongs to this "medium" category.

Example of Fuzzification Process

Consider a system controlling room temperature with the following fuzzy sets: "Cold," "Comfortable," and "Hot." Assume that we use triangular membership functions, Eq. (1), as indicated in Fig. 1:

$$\mu_A(x) = \begin{cases} 0, & x \le a \\ \frac{x-a}{b-a}, & a < x \le b \\ \frac{c-x}{c-b}, & b < x \le c \\ 0, & x > c \end{cases}$$

Where the coefficients are defined as follows:

- "Cold": *a*=15, *b*=20, *c*=25
- "Comfortable": *a*=20, *b*=25, *c*=30

- "Hot": a=25, b=30, c=35

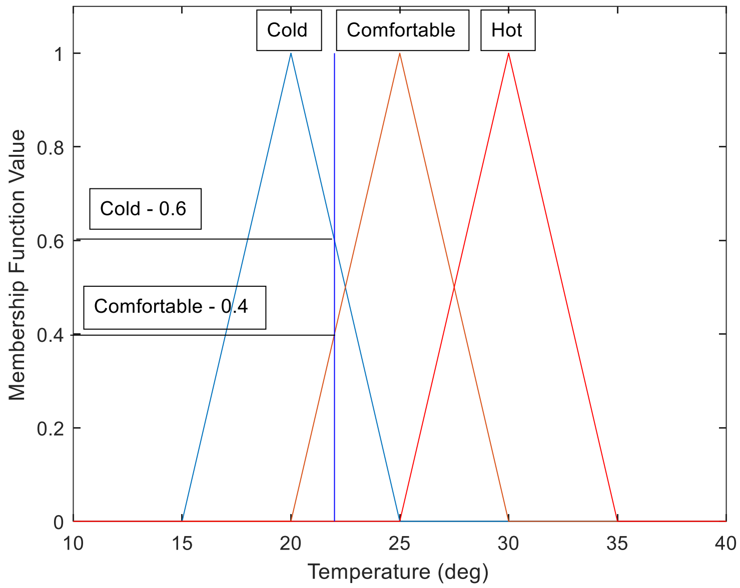

Fig. 1 Membership functions.

Assume the input temperature is 22°C. The degree of membership for 22°C is calculated as follows:

- $\mu_{Cold}(22) = \frac{25-22}{25-20} = \frac{3}{5} = 0.6$
- μComfortable(22)=25−2022−20=52=0.4 $\mu_{Comfortable}(22) = \frac{22-20}{25-20} = \frac{2}{5} = 0.4$
- $\mu_{Hot}(22) = 0$ (since 22°C is less than 25, it has no membership in "Hot")

Thus, 22°C is 0.6 "Cold" and 0.4 "Comfortable," reinforcing fuzzy logic's handling of uncertainties by expressing partial memberships.

Significance of Fuzzification

- **Handling Uncertainty:** By transforming precise data into fuzzy sets, fuzzification allows the system to incorporate the inherent uncertainty and imprecision of real-world information, making it exceptionally valuable for control systems, pattern recognition, and decision-making.
- **Enhanced Decision-Making:** Through fuzzification, systems can make decisions based on the resemblance to different fuzzy sets, rather than being constrained to rigid boundaries. This results in more flexible and adaptive behavior.
- **Interface with Human Reasoning:** Since humans often think in terms of qualitative rather than quantitative descriptors, fuzzification creates a bridge between human-like reasoning and computational processes.

3.2 Inference

Inference in fuzzy logic is the process through which fuzzy logic systems apply rules to input data to derive conclusions or make decisions. This mechanism forms the core of fuzzy systems, enabling them to emulate human decision-making processes by handling imprecision and uncertainty inherent in real-world information. Here's a detailed exploration of the inference process in fuzzy logic.

Fuzzy inference is the step that takes fuzzified inputs, applies a series of logical rules, and produces fuzzy outputs. This process is composed of several stages: fuzzification, rule evaluation, aggregation of rule outputs, and defuzzification. The core inference mechanism focuses on the middle two stages. The components of the fuzzy interface include:

a. **Fuzzified Inputs:**

As discussed in fuzzification section, crisp inputs are transformed into degrees of membership across different fuzzy sets. These fuzzified inputs are then used to evaluate the fuzzy rules.

b. **Rule Base:**

A rule base consists of a set of "IF-THEN" rules that form the decision-making foundation of a fuzzy system. These rules are usually derived from expert knowledge and are expressed using linguistic variables.

c. **Rule Evaluation:**

This step involves computing the degree to which each rule applies to the current fuzzified inputs. Techniques like the min-max method are commonly used:

i. **Antecedent Calculation:** Determine the degree of truth for each condition in the rule antecedent. For example, if a rule's antecedent contains the AND operator, compute the minimum membership value, and if a rule's antecedent contains the OR operator, compute the maximum membership value.

ii. **Consequent Activation:** Apply the result of the antecedent to the rule's consequent fuzzy set. Typically, the implication method modifies the consequent set based on the antecedent's result.

d. **Aggregation of Rule Outputs:**

After evaluating all rules, the outputs are aggregated to form a single fuzzy set. This stage involves combining the membership functions of all activated rule outputs into a single output fuzzy set.

The aggregation process often uses the max method, where the membership function of the final output is given by the maximum truth value of all the rules:

$$\mu_C(x) = max\{\mu_1(x), \mu_2(x), \ldots\ldots, \mu_n(x)\} \quad (7)$$

In certain cases the aggregation process uses the min method, where the membership function of the final output is given by the minimum truth value of all the rules:

$$\mu_C(x) = min\{\mu_1(x), \mu_2(x), \ldots\ldots, \mu_n(x)\} \quad (8)$$

Here, $\mu_i(x)$ represents the membership function for each of the individual rule outputs.

e. Inference Methods:

Commonly used inference methods include Mamdani [11] and Sugeno [12] methods. Mamdani method is the most common inference method and is characterized by its intuitive and straightforward approach to combining fuzzy sets. Sugeno method is often used in control systems and differs by producing outputs that are either constants or linear functions, providing crisp output without a defuzzification step.

The advantages of the Fuzzy Inference include adaptability, robustness, and transparency. In terms of adaptability Fuzzy inference systems can tackle complex problems by easily incorporating expert knowledge through linguistic rules and are able to adjust to new data patterns without needing a complete redesign. In terms of robustness**,** the fuzzy systems can handle noisy and uncertain data, maintaining effective operation even when precise data is unavailable. And in terms of transparency the rule-based structure of fuzzy inference systems provides clear insight into how input data translates to decisions, which enhances understanding and trust in system decisions.

3.3 Composition

Composition in fuzzy logic refers to a critical step within the fuzzy inference process where the outputs of individual fuzzy rules are combined or aggregated to form a final fuzzy output set. This step is crucial for synthesizing the partial conclusions drawn from each rule into a cohesive solution to the problem at hand. Here's a breakdown of how composition works in fuzzy logic systems:

Within a fuzzy inference system, multiple rules can be activated simultaneously, each producing a partial fuzzy conclusion. The composition stage aggregates these outputs into a single fuzzy set that represents the overall response of the system to the input conditions. This aggregated set will then undergo defuzzification to produce a crisp output, used for decision-making or control actions. The steps in the composition process include:

a. **Rule Evaluation:**

Each fuzzy rule is evaluated individually, producing a fuzzy output set. The output set from each rule is a fuzzy set defined over the output variable's universe of discourse, such as "fan speed" in this example.

b. **Implication:**

The implication process modifies the consequent of a fuzzy rule based on the degree of truth of its antecedent. The degree to which the input satisfies the rule's condition scales the consequent fuzzy set. This is often done using multiplication (for scaling) or minimum (for truncation).

c. **Composition or Aggregation:**

During composition, all the modified fuzzy sets generated from each rule's implication are aggregated to form a single fuzzy output set. Common methods for composition include:

i. **Max Method (Union):** Combines the output fuzzy sets by taking the maximum of all the membership values for each point on the output set's universe of discourse. This method captures the strongest response suggested by the rules. The formula used is:

$$\mu_C(x) = max\{\mu_{C1}(x), \mu_{C2}(x), \ldots\ldots, \mu_{Cn}(x)\} \quad (9)$$

where $\mu_{Ci}(x)$ is the membership function of the *i*-th rule's consequent.

ii. **Sum Method:** Aggregates the membership values by summing them at each point. This can, however, lead to membership values exceeding 1, which disrupts the interpretation of membership degrees as probabilities or normalized measures.

iii. **Probabilistic OR:** A probabilistic approach adjusting overlap by combining membership grades using a probabilistic sum formula:

$$\mu_C(x) = \mu_1(x) + \mu_2(x) - \mu_1(x) \cdot \mu_2(x) \quad (10)$$

Composition ensures that the final output set reflects the collective implications of all relevant rules, giving a holistic result that considers multiple perspectives on the input conditions. The composition in fuzzy logic harmonizes disparate rule effects into a unified output fuzzy set, enabling the system to mimic nuanced, human-like judgment. By appropriately aggregating rule outputs, fuzzy systems achieve enhanced robustness and flexibility in decision support, control, and inference applications.

3.4 Defuzzification

Defuzzification is the final step in the fuzzy inference process, where the resulting fuzzy output set is transformed into a single crisp value used for decision-making or control actions. This stage is crucial for converting the insights provided by fuzzy logic into practical, actionable outputs.

Fuzzy logic systems operate by using linguistic and imprecise input data to generate fuzzy sets as outputs. While these fuzzy sets provide valuable information about the degree of truth behind various conditions, translating these into a specific numeric output is essential for systems that require precise control or decision-making. Defuzzification bridges this gap, ensuring the fuzzy logic system produces a concrete result that can be implemented within the application.

Common Methods of Defuzzification

Several techniques are used for defuzzification, each offering unique approaches to deriving crisp values from fuzzy sets. Here are some widely used methods:

a. Centroid Method (Center of Gravity):

This is one of the most popular defuzzification techniques due to its intuitive interpretation and reliability. The centroid method calculates the "center of gravity" of the fuzzy set, effectively finding the balance point of the area under the curve.

$$z^* = \frac{\int z \cdot \mu_C(z) dz}{\int \mu_C(z) dz} \quad (10)$$

where $\mu_C(z)$ is the membership function of the output fuzzy set, and *z* represents points within the universe of discourse.

b. Mean of Maximum (MOM):

This approach computes the average of the values at which the membership function reaches its maximum level. It is simple and quick, although it may disregard substantial portions of the fuzzy set.

$$z^* = average\ of \{z: \mu_C = max(\mu_C(z))\} \quad (11)$$

c. Largest of Maximum (LOM) and Smallest of Maximum (SOM):

These methods focus on the extreme values at which the membership function achieves its peak:

i. **LOM** selects the highest value.

ii. **SOM** chooses the lowest value with maximum membership.

d. Weighted Average Method:

Often used in systems where evenly distributed weights are assigned, this method calculates the average considering these weights. It is particularly suited for systems requiring a balance of multiple criteria.

$$z^* = \sum_i w_i \cdot z_i \quad (12)$$

where w_i represents weights, and z_i are the points of consideration, often obtained from specific rules or criteria.

The choice of defuzzification method depends largely on the system requirements and specific characteristics of the fuzzy output set. Systems requiring smooth and balanced decisions benefit from the centroid method due to its comprehensive evaluation. Applications needing quick decisions might prefer MOM for its computational simplicity. User preference or specific requirements in performance might dictate the choice between LOM or SOM in specialized applications.

Defuzzification plays an essential role in fuzzy logic systems, bridging the gap between fuzzy reasoning and actionable results. Through various techniques, defuzzification enables the practical implementation of fuzzy logic across multiple domains, enhancing the flexibility and robustness of automated decision-making processes.

Example of Fuzzy Inference System

Consider a decision-making process to evaluate student success in a class based on the evaluation of quizzes and homework assignments. Assume that we use trapezoidal membership functions, Eq. (2), as indicated in Fig. 2 and 3. The coefficients (*a*, *b*, *c*, *d*) for each category "Excellent," "Very good," and "Good," and "Excellent" are: Excellent (88, 92, 100, 102), Very good (78, 82, 88, 92), and Good (68, 72, 78, 82). We shall consider two FIS structures: (a) *min* aggregation method, Fig. 2, which represents AND antecedent calculation method, and (2) *max* aggregation method, Fig. 3, which represents OR antecedent calculation method, Fig. 3.

The crisp inputs "Quiz" and "Homework", with values of 79 and 90 respectively, are initially fuzzified, as indicated in the first two columns in Fig.2 and 3. It can be seen that "Quiz" of value 79 is 0 "Excellent," 0.25 "Very good," and 0.75 "Good," whereas "Homework" of value 90 is 0.5 "Excellent," 0. 5 "Very good," and 0 "Good."

Applying AND rule on both antecedents which states that: (1) IF both quiz AND homework are excellent THEN the final score is excellent, (2) IF both quiz AND homework are very good THEN the final score is very good, and (3) IF both quiz AND homework are very good THEN the final score is very good, and applying aggregation method indicated in Fig. 2 results in the consequent at the bottom right of the Fig. 2. The result is another trapezoid with height determined by the applied fuzzy operation, implication method, and aggregation method. Applying the Centroid method results in defuzzied value for the student success of 85.

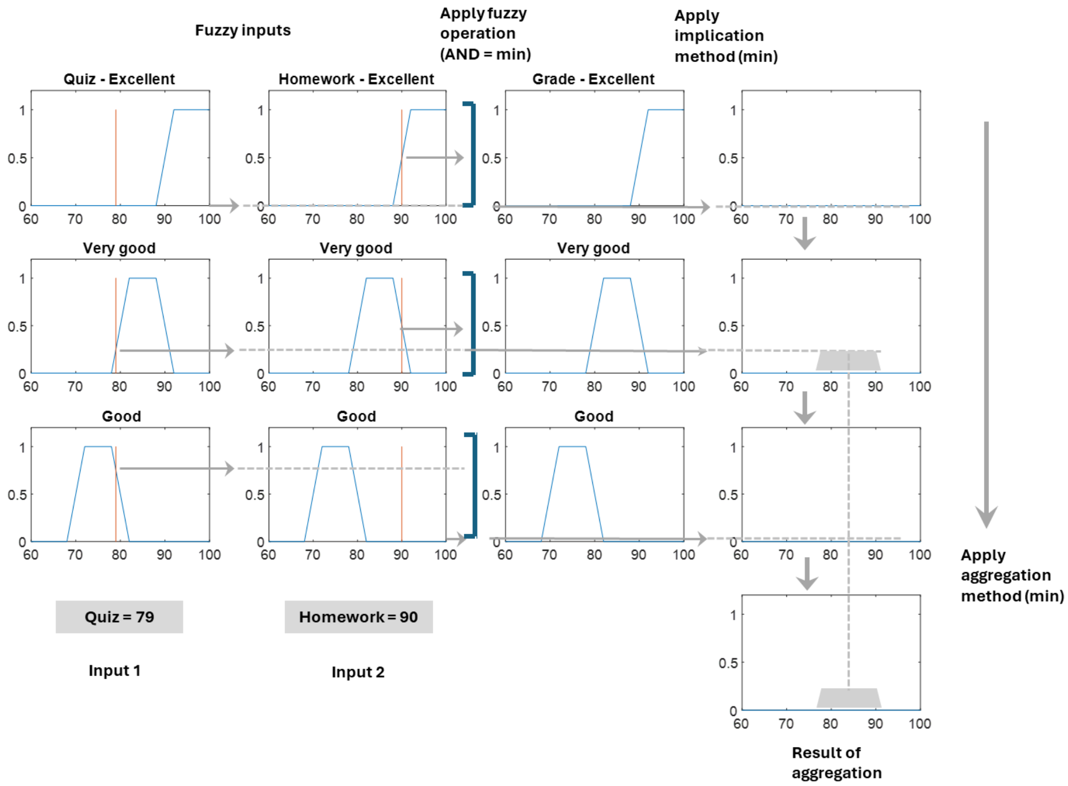

Fig. 2 FIS structure for evaluation of student performance based on the AND antecedent calculation.

Applying OR rule on both antecedents which states that: (1) IF either quiz OR homework are excellent THEN the final score is excellent, (2) IF either quiz OR homework are very good THEN the final score is very good, and (3) IF either quiz OR homework are very good THEN the final score is very good, and applying aggregation method indicated in Fig. 3 results in the consequent at the bottom right of the Fig. 3. The result is the sum of resulting trapezoids with height determined by the applied fuzzy operation, implication method, and aggregation method. Applying the Centroid method results in defuzzied value for the student success of around 84.

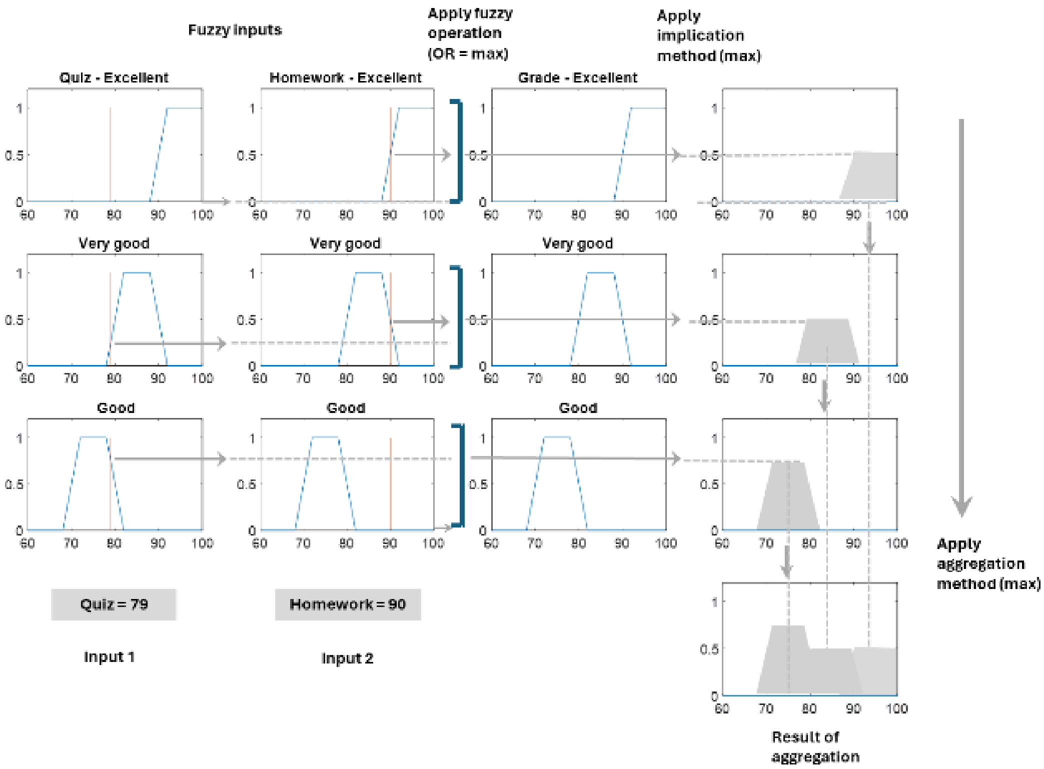

Fig. 3 FIS structure for evaluation of student performance based on the OR antecedent calculation.

3.5 Aggregated Tree Structure

The aggregated tree structure is an innovative approach used in Fuzzy Inference Systems (FIS) to streamline the processing of fuzzy logic rules. In this method, fuzzy rules are organized into a hierarchical tree format, wherein each node represents a condition or an operation, while branches signify pathways determined by specific criteria or input values. This arrangement allows for a structured, visual representation of the inference process, making it easier to comprehend and analyze the interactions between different rules. One of the primary benefits of the aggregated tree structure is enhanced interpretability; stakeholders can more intuitively understand how inputs navigate through the tree to produce an output. Furthermore, this structure supports computational efficiency by enabling parallel processing of branches, thereby speeding up the inference process. It also optimizes memory usage, as shared pathways can reduce redundancy in rule evaluation. Another advantage is the ease of scalability and modification; new rules or variables can be incorporated into the tree without significant overhauls, facilitating incremental learning and adaptation. Overall, the aggregated tree structure not only enhances the clarity and efficiency of FIS operations but also provides a flexible framework for managing complexity and continuous system updates.

Fuzzy Inference Systems (FIS) employ various methodologies to model complex patterns of reasoning similar to human decision-making. One promising approach is the aggregated tree structure for organizing the inference process. This structure arranges fuzzy rules and logical operations into a hierarchical format resembling a tree, where nodes represent decision points and branches embody pathways based on specific criteria. The aggregated tree structure enhances interpretability, making it easier to visualize and understand how different inputs interact and culminate in an output decision. Additionally, this hierarchical organization benefits computational efficiency by enabling parallel processing of rule evaluations and optimizing memory usage through shared pathways for similar inputs. By arranging rules hierarchically, the system can also facilitate incremental modifications, allowing for effortless adaptation and learning as new rules or datasets are introduced. The aggregated tree structure thus provides a robust framework for managing complexity in FIS, ensuring adaptability, computational efficiency, and enhanced clarity in system operation.

The comparison between traditional Fuzzy Inference Systems (FIS) and the aggregated tree structure reveals notable differences in how each handles rule processing, complexity, and system performance.

a. Traditional Fuzzy Inference System

Traditional FISs, such as the Mamdani and Sugeno models, utilize a flat structure where all fuzzy rules are evaluated in parallel. In these systems, each rule is typically processed individually, regardless of the presence of shared inputs or conditions. Traditional FISs often excel in applications where the rule base is relatively small and straightforward due to their straightforward implementation and strong theoretical foundation. Benefits of the traditional FIS structure include:

i. *Simplicity and Interpretability.* The flat structure provides a clear mapping from rules to outputs, making the system easier to design and troubleshoot.

ii. *Widespread Application and Support.* Being the standard, traditional FISs are well-documented and have extensive support in software tools.

b. Aggregated Tree Structure in Fuzzy Inference Systems

The aggregated tree structure organizes the fuzzy rules into a hierarchical framework resembling a decision tree [13]. This hierarchical arrangement allows for conditions (nodes) to be evaluated only when necessary, reducing computational redundancy. The tree structure is particularly beneficial when handling complex systems with numerous rules and shared input conditions. Benefits of the aggregated tree structure in FIS systems include:

i. *Computational Efficiency*. By streamlining rule evaluations and processing only relevant pathways, the system can operate more efficiently, particularly in systems with a large number of rules [14].

ii. *Scalability and Flexibility.* The tree structure easily accommodates new rules and modifications without requiring a complete redesign, supporting dynamic environments and evolving datasets [15].

iii. *Enhanced Interpretability*. The visual aspect of the tree structure aids in understanding how different inputs affect outcomes, providing clearer insights into the system's decision-making process [16].

The aggregated tree structure of the Fuzzy Logic system applied in this study is shown in the Fig. 4, where each Fuzzy Inference System (FIS) has two inputs (antecedents) and one output (consequent). The entire set of antecedents is divided into subsets consisting of two antecedents that have some similarities, e.g. mid-term exams and final exam, and quizzes and homework assignments. These antecedents are then used to determine the consequent. The consequent is then used as the antecedent for the next FIS and the process is repeated till there is only one consequent. In this example the final exam is compared with mid-term exams, homework assignments are compared with quizzes, and term project is compared with project presentation. It is recognized that some of the elements are valued more than others, e.g. homework assignments are valued more than quizzes, and term project is valued more than presentation. Alternately, it is assumed that final exam is valued the same as average between mid-term exams. In this case rules in the fis1 system consider that both antecedents are of the same value, whereas the fis2 system considers the first antecedent to be more valuable than the second antecedent.

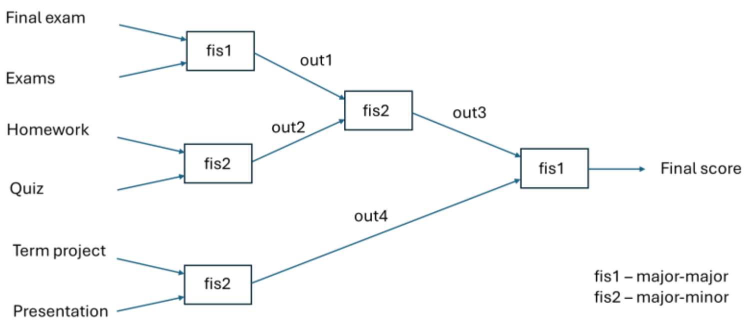

Fig. 4. Aggregated tree structure

The analysis is performed using Matlab software and its Fuzzy Logic Designer toolbox. The initial step was to design two different FIS systems: (1) the FIS where both antecedents are of equal value, and (2) the FIS where one of the antecedents is of higher value than the other one. The structure of both FIS systems, Fig. 5, was assumed to be the same except for the inference rules.

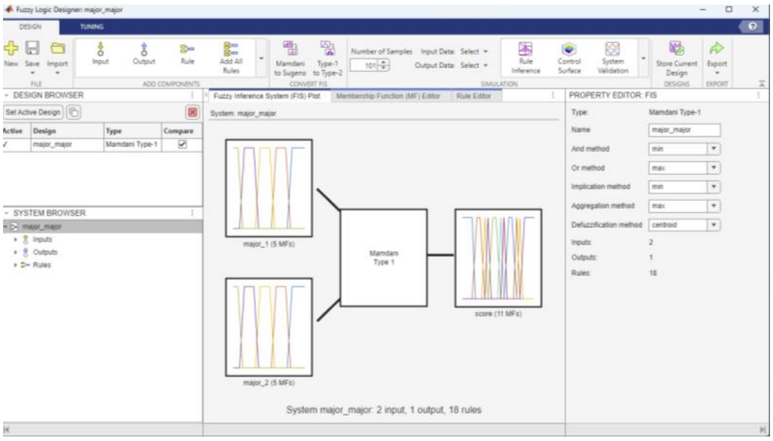

Fig. 5. The FIS structure for both fis1 and fis2 systems.

The membership functions for the antecedents are represented in Fig. 6, where the scale is from 55 – 100 indicating grade levels between A and F. The membership functions are trapezoidal with breakout points indicated on the figure.

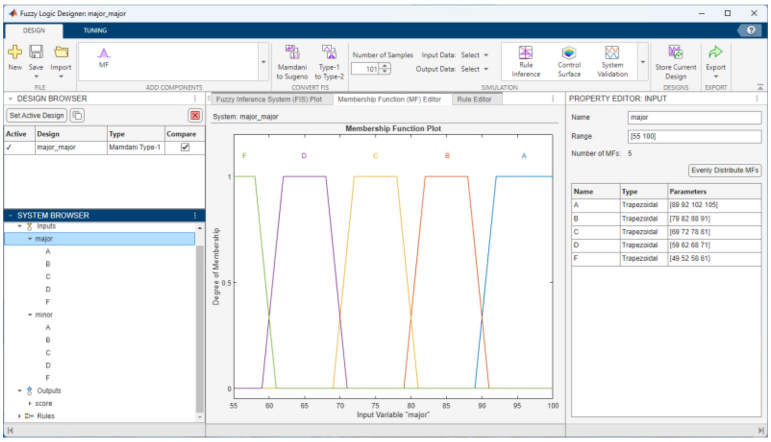

Fig. 6. The FIS structure for both fis1 and fis2 systems.

The membership functions for the consequent are represented in Fig 7, where the scale is from 55 – 100 indicating grade levels between A and F. The membership functions are combination of triangular and trapezoidal with breakout points indicated on the figure. The membership functions allow for partial grades, e.g. B+ and B-. This coincides with the grading scale currently used for grading courses at the institution.

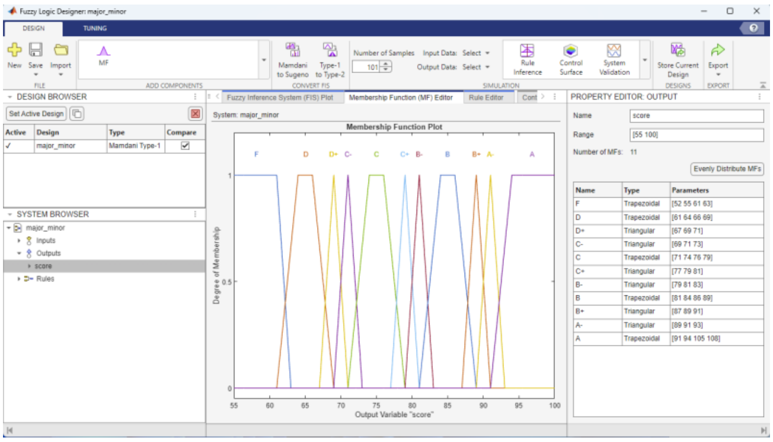

Fig. 7. The FIS structure for both fis1 and fis2 systems.

The fuzzy rules applied in the case of fis1 system, where both antecedents are perceived to be of equal value, are represented in Table 1. Table indicates the relationship between antecedents and impact on the consequent, e.g. when antecedent 1 is "B" and antecedent 2 is "A" then the consequent will be "A-," indicating that the consequent will be somewhere between "A" and "B" with slight benefit towards the higher score. Since both antecedents are of perceived equal value than changes in either one of them will have significant influence on the consequent. The resulting control surface for the specified fuzzy rules used in fis1 system is indicated in Fig. 8.

Table 1. Fuzzy rules for fis1 system where both antecedents are of equal/similar value.

		Antecedent 1				
		A	B	C	D	F
Antecedent 2	A	A	A-	B	C+	F
	B	A-	B	B-	C	F
	C	B	B-	C	D+	F
	D	C+	C	D+	D	F
	F	F	F	F	F	F

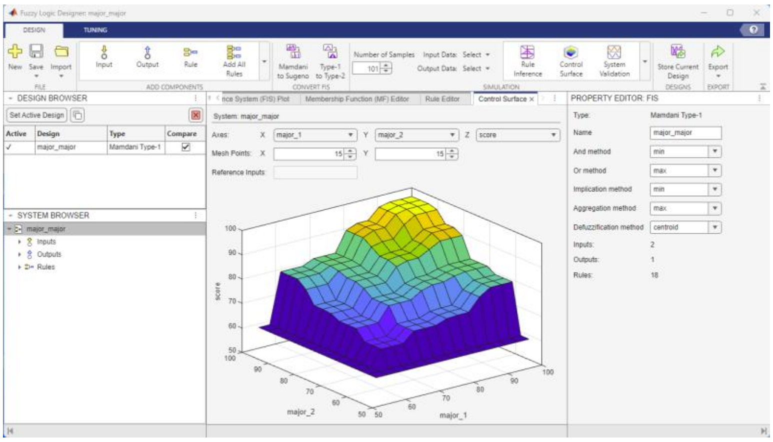

Fig 8. The control surface for the fis1 system.

The fuzzy rules applied in the case of fis2 system, where antecedent 1 is of perceived higher value than antecedent 2, are represented in Table 2. Table represents impact of antecedents on the consequent, e.g. when antecedent 1 is "B" and antecedent 2 is "A" then the consequent will be "B+" instead of "A-" as in fis1 system, indicating the consequent will be between "A" and "B" and slanted towards the lower score. Since antecedent 1 has higher perceived value than antecedent 2 than changes in antecedent1 1 will have higher impact on consequent than antecedent 2. The resulting control surface for the specified fuzzy rules used in fis2 system is indicated in Fig. 9.

Table 2. Fuzzy rules for fis2 system where antecedent 1 is valued more than antecedent 2.

		Antecedent 1				
		A	B	C	D	F
Antecedent 2	A	A	B+	B-	C	F
	B	A-	B	C+	C-	F
	C	B+	B-	C	D+	F
	D	B	C+	C-	D	F
	F	F	F	F	F	F

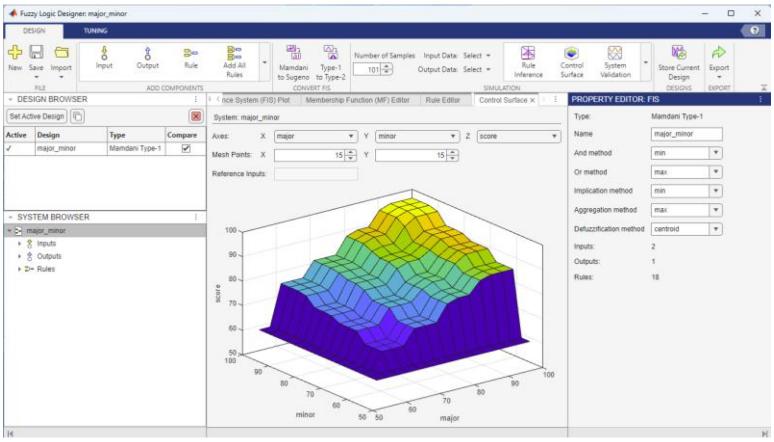

Fig 9. The control surface for the fis2 system.

IV. RESULTS

The study was performed on data collected from two engineering courses – "Dynamics," and "Advanced Technical Analysis." Data represents student performance over several semesters. The following sections present the application of the Fuzzy Logic approach to evaluation of student success in each of these two courses.

Course 1 - Dynamics

The FIS structure for evaluating student performance is shown in Fig. 10.

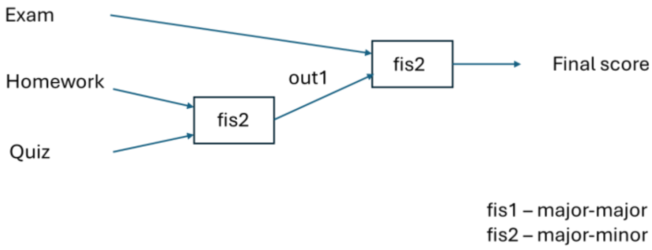

Fig. 10. Aggregated tree structure for assessment of student success in MET 310 course.

The student performance assessment is performed based on 43 quizzes, 8 homework assignments, and 3 exams, and no final exam. The final evaluation is based on the averages within each assessment group (quiz, homework, and exams). The formula used for establishing the final score is:

$$Final\_score=0.1*Quiz\_ave+0.3*Homework\_ave+0.6*Exam\_ave \quad (13)$$

Data in this study was collected over 6 semesters starting with Fall 2022 till Fall 2024. The total number of students included in this study is 128.

The aggregated tree structure applied for this course is represented in Fig. 10. It is postulated that homework has higher value than quizzes, thus fis2 was applied for processing these two antecedents. It is also postulated that exams have higher value than the consequent of homework and quiz pair, and therefore fis2 was applied for processing antecedents Exam and out1.

The results from applying Fuzzy Logic approach are indicated in Fig. 11. The figure compares the results obtained using Eq. (13) and FIS structure shown in Fig. 8. It is apparent that outcomes of application of the FIS structure and regular formula result in the similar overall evaluation of student success.

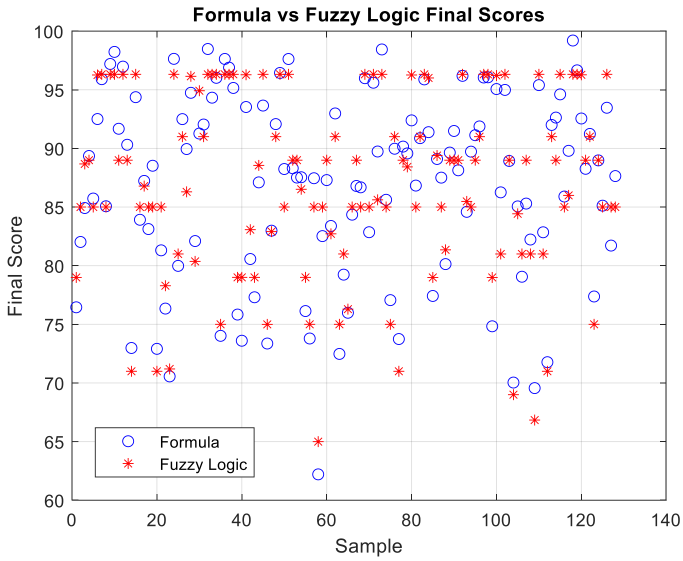

Fig. 11. The comparison between Fuzzy Logic and formula approach for evaluating student performance.

The results shown in Fig. 12 indicate difference between appropriate pairs of scores, i.e. between formula, Eq. (13), and Fuzzy Logic structure. The results are as follows:
Average of differences = -0.1366
RMS of differences = 2.1495
84.38% of results are within +/- 3 points from each other – formula vs Fuzzy Logic.
95.31% of results are within +/- 4 points from each other – formula vs Fuzzy Logic.

It is apparent that difference between the two approaches is within "reasonable/expected" assessment error between two grade levels, e.g. "A-" and "B+".

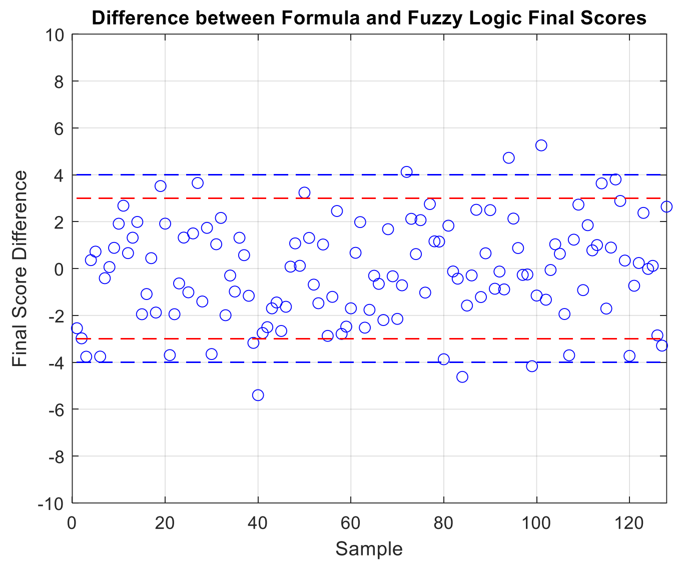

Fig. 12. The comparison between Fuzzy Logic and formula approach evaluating student performance.

The histogram representing difference between formula approach, Eq. (1), and Fuzzy Logic system approach is shown in Fig. 13.

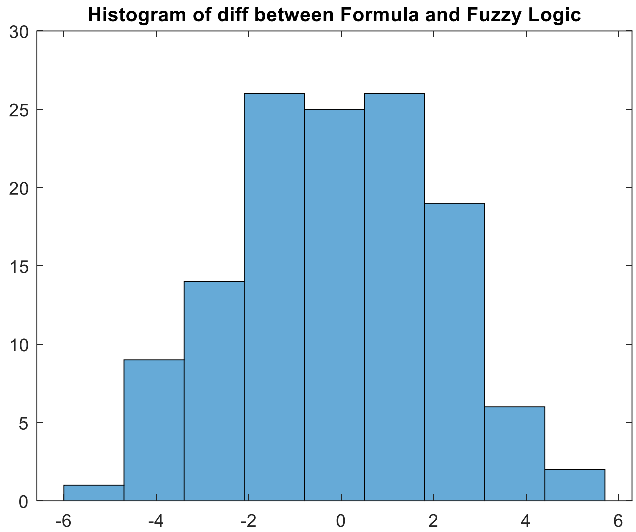

Fig. 13. The histogram of difference between FIS and formula approach.

Course 2 - Advanced Technical Analysis

The aggregated FIS structure for evaluating student performance in this course is shown in Fig. 14. It is postulated that homework has higher value than quizzes, thus fis2 was applied for processing these two antecedents. Also, it is postulated that exam 1 and exam 2 are equally valued resulting in application of fis2 system for these two antecedents. However, it is postulated that consequent of exam 1 and exam 2 has higher value than exam 3, in which case fis2 is applied to evaluate consequent out2. It is also postulated that exams have higher value than the consequent of homework and quiz pair, out3, and therefore fis2 was applied for processing consequent out2 for exams and consequent for homework and quizzes, out3.

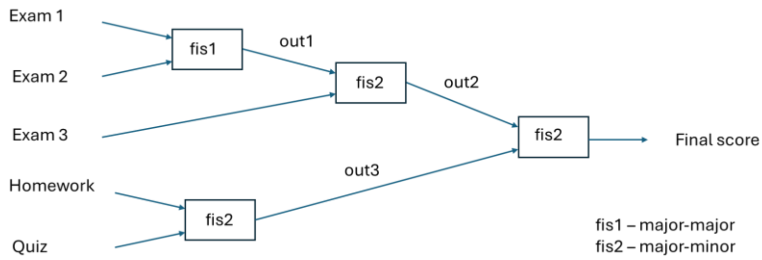

Fig. 14. Aggregated tree structure for assessment of student success in ENGT 305 course.

The student performance assessment is performed based on 78 quizzes, 14 homework assignments, and 3 exams. The formula used for establishing the final score is:

Final_score=0.1*Quiz_ave+0.4*Homework_ave+0.2*Exam1+0.2*Exam2+0.1*Exam3 (14)

Data in this study was collected over 8 semesters starting with Summer 2022 till Fall 2024. The total number of students over this period, included in this study, is 155.

The results from applying Fuzzy Logic approach are indicated in Fig. 15. The figure compares the results obtained using Eq. (14) and FIS structure shown in Fig. 14. It is apparent that outcomes of application of the FIS structure and regular formula result in the similar overall evaluation of student success.

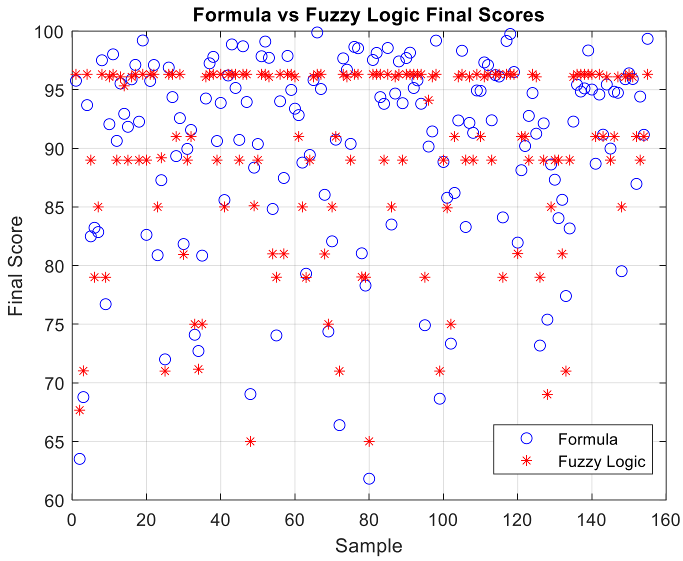

Fig. 15. The comparison between FIS and formula approach for evaluating student performance.

The results shown in Fig. 16 indicate difference between appropriate pairs of scores, i.e. between formula, Eq. (14), and FIS structure. The results are as follows:
Average of differences = 0.0349
RMS of differences = 2.9006
69.03% of results are within +/- 3 points from each other – formula vs Fuzzy Logic
81.29% of results are within +/- 4 points from each other – formula vs Fuzzy Logic.
It is apparent that difference between the two approaches is within "reasonable/expected" assessment error between two grade levels, e.g. "A-" and "B+".

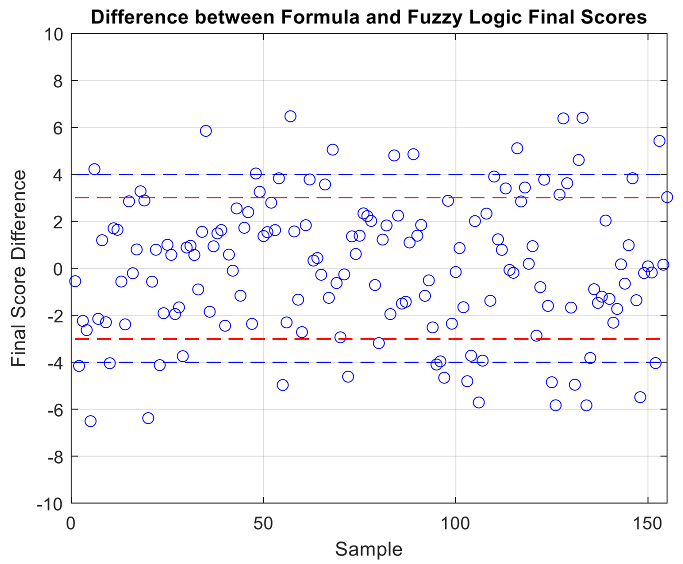

Fig. 16. The difference between FIS and formula approach for evaluating student performance.

The histogram representing difference between formula approach, Eq. (14), and Fuzzy Logic system approach is shown in Fig. 17.

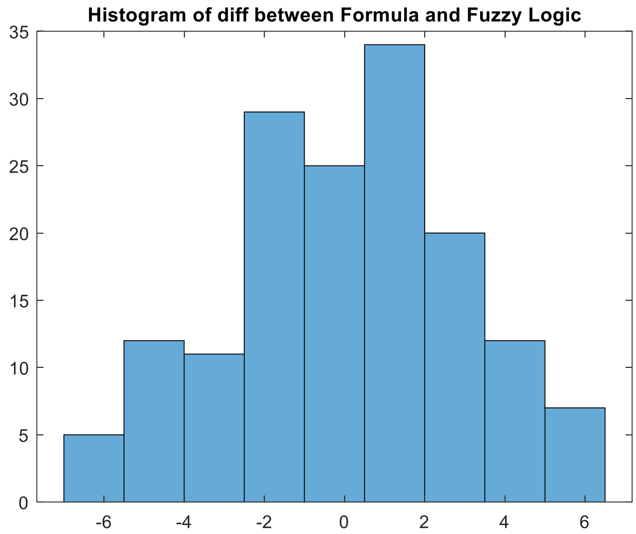

Fig. 17. The histogram of difference between Fuzzy Logic and formula approach.

V. DISCUSSION

The results indicate that the Fuzzy Logic approach for determining student success has similar outcome to traditional approach of applying formulas. The results indicate that the 90% of the score differences between the two methods are within +/- 4%. This is reasonably good considering that the membership functions (shapes and break points), for FIS method, and the weighting factors, for traditional method, are arbitrarily set. The results could be further improved, i.e. difference between the two methods can be reduced by tunning the FIS structure. This can be accomplished by adjusting membership functions (shape and break points) to closely match the results obtained by formula. That is, assuming that the formula results are taken as correct values, which is not necessarily correct one since weighting factors in formula are arbitrarily selected.

Although traditional formula approach is easier to understand than Fuzzy Logic approach, its major deficiency is its binary nature which does not recognize uncertainty between different success levels which is captured with Fuzzy Logic method. This is further complicated with integration of multiple inputs and their inter-relationships. Fuzzy Logic captures uncertainty of evaluating student success (consequent) based on multiple inputs (antecedents) and acknowledges that evaluation is not a binary process.

VI. CONCLUSION

In conclusion, measuring student performance is a multifaceted task that requires the use of diverse methods to capture the full range of students' abilities. Traditional methods such as standardized tests remain valuable but are often limited in their ability to evaluate the complexity of student learning. Alternative methods, including project-based assessments, peer reviews, and portfolios, provide a more holistic view of student performance. The incorporation of fuzzy logic into the assessment process further enhances the ability to evaluate students by accounting for uncertainty, subjectivity, and multiple criteria.

However, while fuzzy logic offers numerous advantages, its application in education is not without challenges. One of the key obstacles is the complexity of developing fuzzy inference systems that can accurately model the diverse factors influencing student performance. The design of fuzzy rules and membership functions requires expertise and a deep understanding of the educational context, which can be time-consuming and resource intensive. Moreover, the integration of fuzzy logic with existing educational systems and technologies may require significant changes to infrastructure and pedagogical practices. Nevertheless, as technology continues to evolve and educational institutions become more receptive to data-driven approaches, fuzzy logic holds the potential to significantly enhance the way student performance is assessed and understood.

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Competing interests: There are no competing interests.
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Author's contribution: Authors share equal contribution to the research and writing of the paper.
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References

[1]. Zadeh, L. A. (1965). *Fuzzy Sets*. Information and Control, 8(3), 338-353.
[2]. Chen, L. H. (2013). The fuzzy MCDM model for evaluating student performance. Annals of Operations Research, 206(1), 239-259.
[3]. Klir, G. J., Yuan, B. (1995). Fuzzy Sets and Fuzzy Logic: Theory and Applications. Prentice Hall.
[4]. Tomovic M., Tomovic C., Jovanovic V., "Application of Fuzzy Logic for evaluation of student performance." 19th Annual International Technology, Education and Development Conference (INTED 2025), March 3-5, 2025, Valencia, Spain.
[5]. Bakar, N. A., Othman, N. F. (2017). Fuzzy logic approach in evaluating students' performance for Nilai community college courses. Journal of Fundamental and Applied Sciences, 9(5S), 386-397.
[6]. Chen, L. H., Wang, T. C. (2013). A fuzzy multi-criteria model for evaluating a student's learning performance. International Journal of Distance Education Technologies, 2, 78-88.
[7]. Khan, R. A., AlOtaibi, N. E. (2019). Integration of fuzzy logic in education: A case study of student evaluation in eLearning environments. Education and Information Technologies, 24(2), 1183-1202.
[8]. El-Henawy, I. M., Shehab, A., Abdellatif, S. S. (2022). The robustness of educational performance evaluation using fuzzy logic: A case study at Alexandria University. International Journal of Emerging Technologies in Learning (iJET) , 17(8), 125-142.
[9]. Maleki, A., Alihossein, S., Samiee, M. (2020). Application of fuzzy logic to assess academic achievement recollection method in universities. The Journal of Supercomputing, 76(4), 2719-2736.
[10]. Zat, H. R., Adiyatma, W. (2021). The utilization of fuzzy logic in assessing tertiary students' academic performance: A practical implementation. Indonesian Journal of Electrical Engineering and Computer Science, 21(2), 948-955.
[11]. Verma, B. K., et al. (2020). Application of fuzzy logic in determining and predicting students' academic performance: A review. Asia Pacific Journal of Advances in Education and Social Sciences, 12(1), 12-22.
[12]. Mamdani, E. H., & Assilian, S. (1975). An experiment in linguistic synthesis with a fuzzy logic controller. International Journal of Man-Machine Studies.
[13]. Takagi, T., & Sugeno, M. (1985). Fuzzy identification of systems and its applications to modeling and control. IEEE Transactions on Systems, Man, and Cybernetics.
[14]. Ross, T. J. (2010) . Fuzzy Logic with Engineering Applications. John Wiley & Sons. Offers practical insights into implementing tree structures in fuzzy systems
[15]. Mendel, J. M. (2001). *Fuzzy Logic Systems for Engineering: A Tutorial*. IEEE Transactions on Systems, Man, and Cybernetics, 31(3), 263-277.
[16]. Jang, J. S. R., Sun, C. T., & Mizutani, E. (1997). Neuro-Fuzzy and Soft Computing: A Computational Approach to Learning and Machine Intelligence. Prentice Hall.
[17]. Melin, P., Castillo, O. (2013). Hybrid Intelligent Systems for Pattern Recognition Using Soft Computing: An Evolutionary Approach for Neural Networks and Fuzzy Systems. Springer.