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Stability And Bifurcation Analysis For The Dynamical Model Of Energy Storage Unit In Energy Storage Process

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ABSTRACT—*Nonlinear dynamic characteristics of storage process in energy storage unit are investigated in this paper. All the equilibrium points of the system and their stabilities are studied. Using Hopf bifurcation theorem and the first Lyapunov coefficient, the conditions and the type of Hopf bifurcations for the system are investigated. With the Runge-Kutta method, the phase portraits of the system are given, which verify the analytical results.*
KEY WORDS—*Energy Storage, Stability, Hopf Bifurcations*

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I. INTRODUCTION

The electric power industry, which is the foundation of the national economy industry, is an important guarantee of economic development and social progress in modern society[1].

In recent years, sustained economic growth and electricity consumption increases year by year in China, the power shortage problem also gradually revealed[2]. It is extremely significant to develop Electricity energy storage technology to ensure the supply of electrical energy balance and solve the problem of power shortage. Energy storage refers to the process of storing energy through some equipment or medium and releasing it when needed[3]. The development of its technology, which can not only solve the problems of the energy large-scale access and the conventional electric power peeling and valley filling, but also improve the efficiency of conventional electric power generation and transmission, can effectively reduce emissions and ease environmental problem so that we can implement chemical clean efficient use of energy. At the same time, the development of energy storage technology, can effectively improve development speed of renewable energy power generation that restricted of its own volatility, intermittent and randomness[4] to reduce coal proportion and achieve the large scale development of renewable energy. Therefore, energy storage has been listed in hundreds of major projects of “the 13th Five-Year Plan”, also is the first official into national development planning.

At present, the energy storage technology is divided into physical form and the chemical form according to working principle. Physical energy storage contains mechanical energy storage and the electromagnetic field energy storage[5]. For example, pumping energy storage[6], the compressed air energy storage[7], the flywheel energy storage[8], superconducting coil storage[9], the super capacitor energy storage[10], the pulse energy storage[11] are involved in physical energy storage. Chemical energy storage technology mainly contain: the battery energy storage[12], hydrogen storage[13], fuel cells[14], etc. The continuous improvement of the existing energy storage technology and research of new energy storage technology have become the urgent need of the present social production. Accordingly, it is very meaningful in in theory and practice to research energy storage model of dynamics. Based on the literature [15], which proposed a kind of mechanical elastic energy storage unit and analysed its stability and chaos, Hopf bifurcation parameter conditions and types for the system are investigated and the results of theoretical analysis are verified with the Runge-Kutta method in this paper.

II. DYNAMIC MODEL OF MESS UNIT IN ENERGY STORAGE PROCESS

Permanent magnet motor running in motor state in storage process, while the mathematical model for the synchronous d,q rotation axis[15]

$$\begin{cases} \dfrac{di_d}{d\tilde{t}} = \dfrac{1}{L_d}(-R_s i_d + \omega_r L_d i_q), \\ \dfrac{di_q}{d\tilde{t}} = \dfrac{1}{L_q}(-R_s i_q - \omega_r L_q i_d - \omega_r \psi_r), \\ \dfrac{d\omega_r}{d\tilde{t}} = \dfrac{1}{JL_q}(p_n \psi_r i_q + p_n (L_d - L_q) i_d i_q - T_L - \beta \omega_r), \end{cases} \tag{1}$$

where i_d , i_q represents the stator current on d , q rotation axis, ω_r represents angular velocity of rotor machinery, L_d , L_q represents inductor of stator winding resistance d , q axis. R_s represents stator winding resistance, ψ_r represents flux linkage produced by rotor permanent magnet, p_n represents polar numbers, β represents Viscous damping coefficient, T_L represents load torque .

With affine transformation[16] $x = \lambda \tilde{x}$ and time scale transformation[17] $t = \tau \tilde{t}$, assuming that the air gap of permanent magnet synchronous motor is uniform, equation (1) can be transformed into a dimensionless equation of state[18]:

$$\begin{cases} \dfrac{d\tilde{i}_d}{d\tilde{t}} = -\tilde{i}_d + \tilde{\omega}_r \tilde{i}_q + \tilde{u}_d, \\ \dfrac{d\tilde{i}_q}{d\tilde{t}} = -\tilde{i}_q - \tilde{\omega}_r \tilde{i}_d - \gamma \tilde{\omega}_r + \tilde{u}_q, \\ \dfrac{d\tilde{\omega}_r}{d\tilde{t}} = \sigma(\tilde{i}_q - \tilde{\omega}_r) - \tilde{T}_L, \end{cases} \tag{2}$$

where

$$\lambda = \begin{bmatrix} bk & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & \dfrac{1}{\tau} \end{bmatrix}, \ \tilde{u}_d = \frac{u_d}{R_s k}, \ \tilde{u}_q = \frac{u_q}{R_s k}, \ b = \frac{L_q}{L_d}, \ k = \frac{\beta}{p_n \tau \psi_r}, \ \tau = \frac{L_q}{R_1}, \ \gamma = -\frac{\psi_r}{kL_q}, \ \sigma = \frac{\beta\tau}{J},$$

$$\tilde{T}_L = \frac{\tau^2 T_L}{J}.$$

III. STABILITY AND BIFURCATION ANALYSIS

We discuss the equilibrium point of system (2) and Hopf bifurcation respectively.

Case 1 : $\tilde{u}_q = \tilde{u}_d = \tilde{T}_L = 0$

A. Stability analysis

When $-\gamma > 1$, $(0,0,0)$, $(-\gamma-1, \sqrt{-\gamma-1}, \sqrt{-\gamma-1})$, $(-\gamma-1, -\sqrt{-\gamma-1}, -\sqrt{-\gamma-1})$ are equilibrium points.

Jocabian matrix

$$J = \begin{pmatrix} -1 & \tilde{\omega}_r & \tilde{i}_q \\ -\tilde{\omega}_r & -1 & -\tilde{i}_d - \gamma \\ 0 & \sigma & -\sigma \end{pmatrix}, \tag{3}$$

Characteristic polynomial

$$\begin{aligned} f(\lambda) = {} & \lambda^3 + (2+\sigma)\lambda^2 \\ & + (\sigma\gamma + \sigma\tilde{i}_d + \tilde{\omega}_r^2 + 2\sigma + 1)\lambda \\ & + \sigma(\tilde{i}_q \tilde{\omega}_r + \tilde{\omega}_r^2 + \gamma + \tilde{i}_d + 1) \end{aligned} \tag{4}$$

With Routh–Hurwitz criterion, equilibrium point is stable if and only if the following conditions are satisfied:

$$\begin{cases} (2+\sigma)>0 \\ (2+\sigma)(\sigma\gamma+\sigma\tilde{i}_d+\tilde{\omega}_r^2+2\sigma+1) \\ -\sigma(\tilde{i}_q\tilde{\omega}_r+\tilde{\omega}_r^2+\gamma+\tilde{i}_d+1)>0 \\ \sigma(\tilde{i}_q\tilde{\omega}_r+\tilde{\omega}_r^2+\gamma+\tilde{i}_d+1)[(2+\sigma) \\ (\sigma\gamma+\sigma\tilde{i}_d+\tilde{\omega}_r^2+2\sigma+1) \\ -\sigma(\tilde{i}_q\tilde{\omega}_r+\tilde{\omega}_r^2+\gamma+\tilde{i}_d+1)]>0 \end{cases} \tag{5}$$

B. Hopf bifurcation analysis

According to the Hopf bifurcation theorem[19], Hopf bifurcation may appear when the system has a pair of complex eigenvalues which satisfies the following conditions:

$$\operatorname{Re}(\lambda)\big|_{\gamma=\gamma_0}=0, \operatorname{Im}(\lambda)\big|_{\gamma=\gamma_0}\neq 0, \frac{d}{dc}\operatorname{Re}(\lambda)\Big|_{\gamma=\gamma_0}\neq 0.$$

where γ_0 is the critical value of γ for Hopf bifurcation.

Obviously, Hopf bifurcation can not appear at zero point because of no eigenvalue such as $\lambda=\pm i\omega(\omega>0)$.

Letting $\lambda_{1,2}=\pm i\omega$, we obtain

$$-i\omega^3-(2+\sigma)\omega^2+(\sigma\gamma+\sigma\tilde{i}_d+\tilde{\omega}_r^2+2\sigma$$
$$+1)i\omega+\sigma(\tilde{i}_q\tilde{\omega}_r+\tilde{\omega}_r^2+\gamma+\tilde{i}_d+1)=0$$

Parameters must satisfy

$$\begin{cases} (2+\sigma)>0 \\ (\sigma\gamma+\sigma\tilde{i}_d+\tilde{\omega}_r^2+2\sigma+1)>0 \\ \sigma(\tilde{i}_q\tilde{\omega}_r+\tilde{\omega}_r^2+\gamma+\tilde{i}_d+1)>0 \\ (2+\sigma)(\sigma\gamma+\sigma\tilde{i}_d+\tilde{\omega}_r^2+2\sigma+1) \\ =\sigma(\tilde{i}_q\tilde{\omega}_r+\tilde{\omega}_r^2+\gamma+\tilde{i}_d+1) \end{cases}$$

When $\gamma=\gamma_h=-\sigma(\sigma+4)/(\sigma-2)$, the eigenvalues of two nontrivial equilibrium points will be

$$\lambda_1=-(\sigma+2), \lambda_{2,3}=\pm i\sqrt{\frac{2\sigma(\sigma+1)}{\sigma-2}},$$

Therefore, Hopf bifurcation may appear when $\gamma=\gamma_h=-\sigma(\sigma+4)/(\sigma-2)$, and all of the equilibrium points will become unstable when $\gamma<\gamma_h$.

The following, we discuss the supercriticality or subercriticality of the Hopf bifurcation. Let C^n be a linear space defined in a complex domain C with inner product, for $x=(x_1,x_2,\ldots,x_n)^T$, $y=(y_1,y_2,\ldots,y_n)^T$, especially $x_i, y_i\in C(1,2,\cdots,n)$. $\langle x,y\rangle=\sum_{i=1}^n \bar{x}_i y_i$.

Introduce norm $\|x\|=\sqrt{\langle x,y\rangle}$ and C^n is a Hilbert space. We discuss the following nonlinear system:

$$\dot{x}=Ax+F(x), x\in R^n$$

where $F(x)=O(\|x\|^2)$ is a smooth function, and it can be expanded into

$$F(x)=\frac{1}{2}B(x,x)+\frac{1}{6}C(x,x,x)+O(\|x\|^4)$$

in which $B(x,x)$ and $C(x,x,x)$ are bilinear and trilinear functions, respectively.

In equation (2), when the matrix J has a pair of pure imaginary eigenvalues $\lambda_{1,2} = \pm i\omega \ (\omega > 0)$, let $q \in C^n$ be a complex eigenvector corresponding to the eigenvalue λ_1, we will obtain $Jq = i\omega q, J\bar{q} = -i\omega\bar{q}$ .At the same time, we introduce the adjoint vector $p \in C^n$ which satisfied $J^T p = i\omega p, J^T \bar{p} = -i\omega\bar{p}$ and $\langle p, q\rangle = 1$.

Now, we introduce the following transformation

$$I_d = \tilde{i}_d - \tilde{i}_{d_{eq}}, I_q = \tilde{i}_q - \tilde{i}_{q_{eq}}, W_r = \tilde{\omega}_r - \tilde{\omega}_{r_{eq}}$$

then, the equilibrium point $(\tilde{i}_{d_{eq}}, \tilde{i}_{q_{eq}}, \tilde{\omega}_{r_{eq}})$ can be moved to the origin $(0,0,0)$.

According to the first Lyapunov coefficient theorem[20]

$$\begin{aligned} l_1(0) = \frac{1}{2\omega} \mathrm{Re}(&\langle p, C(q,q,\bar{q})\rangle \\ &-2\langle p, B(q, J^{-1}B(q,\bar{q}))\rangle) \\ &+\langle p, B(\bar{q}, (2i\omega E - J)^{-1} B(q,q))\rangle) \end{aligned}$$

When $\sigma = 4, \gamma_0 = -16, \tilde{i}_d = 15, \tilde{i}_q = \sqrt{15}, \tilde{\omega}_r = \sqrt{15}$, the Jacobian matrix of system (4) is:

$$J = \begin{pmatrix} -1 & \sqrt{15} & \sqrt{15} \\ -\sqrt{15} & -1 & 1 \\ 0 & 4 & -4 \end{pmatrix},$$

Next, we calculate the corresponding vectors p, q of the matrix J, which satisfies $Jq = i\omega q, J^T p = -i\omega p$ and $\langle p, q\rangle = 1$.For:

$$q = \left(\frac{7\sqrt{15}(1+\sqrt{5}i)}{(2\sqrt{5}i+6)(2\sqrt{5}i+1)}, \frac{1+5\sqrt{5}i}{2\sqrt{5}i+6}, 1 \right)^T$$

$$\bar{q} = \left(\frac{7\sqrt{15}(1-\sqrt{5}i)}{(-2\sqrt{5}i+6)(-2\sqrt{5}i+1)}, \frac{1-5\sqrt{5}i}{-2\sqrt{5}i+6}, 1 \right)^T$$

$$p = \begin{pmatrix} -\dfrac{\sqrt{15}(\sqrt{5}i-3)(2\sqrt{5}i-1)}{28(4\sqrt{5}i-5)} \\ -\dfrac{(\sqrt{5}i-3)(2\sqrt{5}i-1)^2}{28(4\sqrt{5}i-5)} \\ \dfrac{(\sqrt{5}i-3)^2(2\sqrt{5}i-1)^2}{196(4\sqrt{5}i-5)} \end{pmatrix}$$

For this system, bilinear and three linear functions are

$$B(X, X') = (\tilde{\omega}_r \tilde{i}_q', -\tilde{\omega}_r \tilde{i}_d', 0)$$

$$C(X, X', X'') = (0,0,0)$$

Then

$$B(q,q) = \left(\frac{1+5\sqrt{5}i}{2\sqrt{5}i+6}, -\frac{7\sqrt{15}(1+\sqrt{5}i)}{(2\sqrt{5}i+6)(2\sqrt{5}i+1)}, 0\right)^T$$

$B(q,\bar{q})=(\frac{1-5\sqrt{5}i}{-2\sqrt{5}i+6},-\frac{7\sqrt{15}(1-\sqrt{5}i)}{(-2\sqrt{5}i+6)(-2\sqrt{5}i+1)},0)^T$ The inverse matrix of the coefficient matrix is

$$J^{-1}=\begin{pmatrix} 0 & -\frac{\sqrt{15}}{15} & -\frac{\sqrt{15}}{60} \\ \frac{\sqrt{15}}{30} & -\frac{1}{30} & \frac{7}{60} \\ \frac{\sqrt{15}}{30} & -\frac{1}{30} & -\frac{2}{15} \end{pmatrix}$$

Let

$$S=J^{-1}B(q,\bar{q})=\begin{pmatrix} -\frac{7(\sqrt{5}i-1)}{2(\sqrt{5}i-3)(2\sqrt{5}i-1)} \\ -\frac{7\sqrt{15}(\sqrt{5}i+3)}{30(\sqrt{5}i-3)(2\sqrt{5}i-1)} \\ -\frac{7\sqrt{15}(\sqrt{5}i+3)}{30(\sqrt{5}i-3)(2\sqrt{5}i-1)} \end{pmatrix} \quad (24)$$

That is

$$B(q,S)=\begin{pmatrix} -\frac{7\sqrt{15}(\sqrt{5}i+3)}{30(\sqrt{5}i-3)(2\sqrt{5}i-1)} \\ \frac{7(\sqrt{5}i-1)}{2(\sqrt{5}i-3)(2\sqrt{5}i-1)} \\ 0 \end{pmatrix}$$

Therefore

$$\langle p,B(q,s)\rangle=-\frac{(\sqrt{5}i+3)(2\sqrt{5}i+1)}{(4\sqrt{5}i+5)(\sqrt{5}i-3)(2\sqrt{5}i-1)}$$

Let

$$S^{'}=(2i\omega E-J)^{-1}B(q,q)$$

$$=\begin{pmatrix} \frac{7\left(5\sqrt{5}i-8\right)}{3\left(2\sqrt{5}i+3\right)\left(\sqrt{5}i+3\right)\left(2\sqrt{5}i+1\right)} \\ \frac{7\sqrt{15}\left(\sqrt{5}i+1\right)\left(3\sqrt{5}i-13\right)}{30\left(2\sqrt{5}i+3\right)\left(\sqrt{5}i+3\right)\left(2\sqrt{5}i+1\right)} \\ \frac{7\sqrt{15}\left(3\sqrt{5}i-13\right)}{30\left(2\sqrt{5}i+3\right)\left(\sqrt{5}i+3\right)\left(2\sqrt{5}i+1\right)} \end{pmatrix}$$

Then

$$B(\bar{q},S^{'})=\begin{pmatrix} \dfrac{7\sqrt{15}\left(\sqrt{5}i+1\right)\left(3\sqrt{5}i-13\right)}{30\left(2\sqrt{5}i+3\right)\left(\sqrt{5}i+3\right)\left(2\sqrt{5}i+1\right)} \\ -\dfrac{7\left(5\sqrt{5}i-8\right)}{3\left(2\sqrt{5}i+3\right)\left(\sqrt{5}i+3\right)\left(2\sqrt{5}i+1\right)} \\ 0 \end{pmatrix}$$

Thus

$$\left\langle p,B(\bar{q},S^{'})\right\rangle=-\frac{13\sqrt{5}i+50}{6\left(4\sqrt{5}i+5\right)\left(2\sqrt{5}i+3\right)}$$

Consequently, we obtain

$$\begin{aligned} l_1(0)&=\frac{1}{2\omega}\mathrm{Re}(\left\langle p,C(q,q,\bar{q})\right\rangle \\ &-2\left\langle p,B(q,J^{-1}B(q,\bar{q}))\right\rangle \\ &+\left\langle p,B(\bar{q},(2i\omega E-J)^{-1}B(q,q))\right\rangle) \\ &=-\frac{1}{4\sqrt{5}}\frac{122}{609}=-0.02240<0 \end{aligned}$$

Therefore, the Hopf bifurcation is supercritical on the base of these parameters.
The first Lyapunov coefficient of another extraordinary equilibrium point can also be obtained under this set of parameters in the same way: $l_1(0)=-0.0944<0$ .Thus, the Hopf bifurcation is supercritical.

The following cases can be discussed in the above method

Csae 2 : $\tilde{u}_q=\tilde{T}_L=0,\tilde{u}_d\neq 0$

When $-\gamma-\tilde{u}_d>1$, the equilibrium points are

$$(0,0,0),(-\gamma-1,\sqrt{-\gamma-1-\tilde{u}_d},\sqrt{-\gamma-1-\tilde{u}_d}),$$
$$(-\gamma-1,-\sqrt{-\gamma-1-\tilde{u}_d},-\sqrt{-\gamma-1-\tilde{u}_d})\ .$$

Let $\sigma=4,\gamma=-1,\tilde{u}_{d_0}=-15,\tilde{i}_d=0,\tilde{i}_q=\sqrt{15},$ $\tilde{\omega}_r=\sqrt{15}.$ The first Lyapunov coefficient of it is $l_1(0)=-0.0224<0$. Thus, the Hopf bifurcation is supercritical on the base of these parameters.

The first Lyapunov coefficient of another extraordinary equilibrium point can also be obtained under this set of parameters: $l_1(0)=-0.0944<0$ .Therefore, the Hopf bifurcation is supercritical.

Case 3 : $\tilde{u}_q\neq 0,\tilde{T}_L\neq 0,\tilde{u}_d\neq 0$

This situation is the general situation of the unit storing up energy when the external power supply and the vortex spring are loaded. According to equation (3), we can determine the equilibrium point of the operation of the unit with numerical method

If $\sigma yz = 2\left(1+\sigma\right)^2+\left(\sigma^2+\sigma\right)\left(x+\gamma\right)+2z^2$ is got, the limit cycle will appear in (2) equation of the corresponding linearization on the base of case 1. If $\tilde{T}_L$ is got, the system will run in chaos for appropriate $\tilde{u}_d$ and $\tilde{u}_q$.

IV. NUMERICAL SIMULATIONS

Numerical results are obtained with Runge-Kutta method in the fllowing.

Taking case 1 as an example, we let $\tilde{i}_d(0)=15, \tilde{i}_q(0)=\sqrt{15}, \tilde{\omega}_r(0)=\sqrt{15}.$

（1）When $\sigma=4, \gamma=-13>\gamma_0$, the image is shown below:

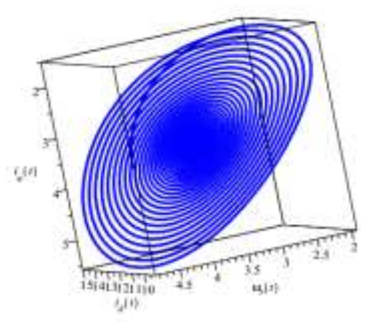

Fig.1. The phase portrait for $\sigma=4, \gamma=-13>\gamma_0$.

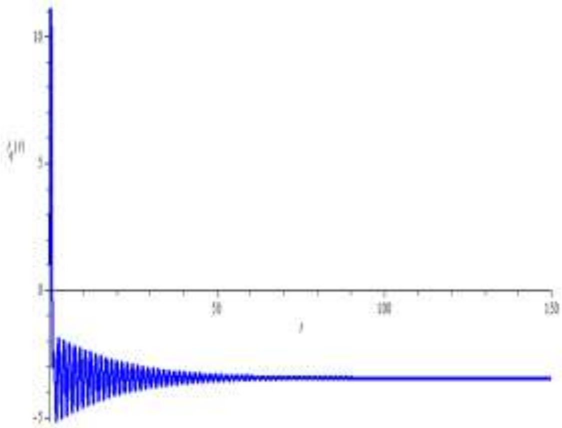

Fig.2. Trajectories of $\tilde{i}_q(t)$ for $\sigma=4, \gamma=-13>\gamma_0$

From the above picture, it can be seen that $\tilde{i}_d(t)$, $\tilde{i}_q(t)$ and $\tilde{\omega}_r(t)$ tend to $(15, \sqrt{15}, \sqrt{15})$ finally with the increase of t.

（2）When $\sigma=4, \gamma=-16.000001<\gamma_0$, the image is shown below:

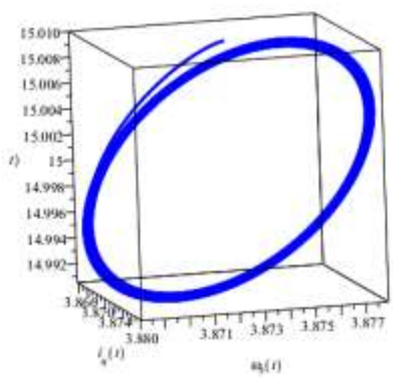

Fig.3. The phase portrait for $\sigma=4, \gamma=-16.0<\gamma_0$

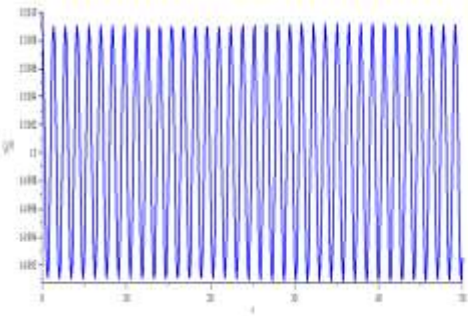

Fig.4. Trajectories of $\tilde{i}_d(t)$ for $\sigma = 4, \gamma = -16.0 < \gamma_0$

From the above picture, it can be seen that a periodic solution appears near the equilibrium point firstly , then $\tilde{i}_d(t)$ ， $\tilde{i}_q(t)$ and $\tilde{\omega}_r(t)$ do tend to closed orbit finally with the increase of t.

In summary（1）（2）,the two cases are consistent with the theoretical analysis.

V. CONCLUSION

Nonlinear dynamic characteristics of storage process in energy storage unit are investigated with analytical and numerical methods in this paper. The stability for equilibrium points of the system in energy storage process is discussed. the conditions and the type of Hopf bifurcations for the system are investigated with Hopf bifurcation theorem and the first Lyapunov coefficient. Using the Runge-Kutta method, we obtain the numerical simulation of the system in energy storage process.

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REFERENCES

[1]. Y. Xiao, D. L. Du, B. Zhao, "Energy storage technology to support a new generation of smart grid," *China Strategic Emerging Industry*, 17th ed. 2015, pp. 84-86.
[2]. W. J. Chen, G. He, "Electricity consumption and its impact factors:Based on nonparametric model," *Journal of Industrial Engineering and Engineering Management*, 23rd ed. vol 4. 2009, pp, 170-173.
[3]. Y. Ji, *Research on Key Technologies of energy efficient modular multilevel converter*, Nanjing: Southeast University, 2015.
[4]. H. M. Li, J. Y. Luo, Q. L. Wan, "Manual coplementariness optimization of electrical source involving intermittent power," *Power System Protection and Control*, 9th ed. 2013, pp, 34-39.
[5]. J. Shi, K. Gong, Y. Liu, et al, "Application of Hybrid Energy Storage," *Southern Energy Construction*, 2nd ed. vol 2. 2015, pp, 28-34.
[6]. R. X. Li, F. Wang, W. G. Shi, et al, "Refrigeration System with Pumping Compressed Gas Energy Storage Device," *Fluid Machinery*, 6th ed. 2015, pp, 69-75.
[7]. X. J. Zhang, H. S. Chen, J. C. Liu, et al, "Research progress in compressed air energy storage system:A reviw," *Energy Storage Science and Technology*, 1st ed. vol 1. 2012, pp, 26-40.
[8]. S. Y. Jiang, H. G. Wei, Z. P. Shen, "The situation of the flywheel energy storage research," *Acta Energiae Solaris Sinica*, 21st ed. vol 4. 2000, pp, 427-433.
[9]. W. Yuan, W. Xian, M. Ainslie, et al, "Design and Test of a Superconducting Magnetic Energy Storage (SMES) Coil," *IEEE Transactions on Applied Superconductivity*, 20th ed. vol 3. 2010, pp, 1379-1382.
[10]. J. Wang, J. Zhang, Y. Zhong, "Study on a Super Capacitor Energy Storage system for improving the operating stability of Distributed Generation system," *International Conference on Electric Utility Deregulation and Restructuring and Power Technologies*, New York: IEEE, 2008, pp, 2702—2706.
[11]. H. W. Yu, M. J. Xu, W. T. Duan, et al, "Investigation on pumping dynamics and pulsed energy storage performance of Yb ionsYb," *Acta Physica Sinica*, 56th ed. vol 7. 2007, pp, 4158-4168.
[12]. E. Smith, "Battery energy storage," *Proceedings of the IEEE*, 81st ed. vol 3. 1988, pp, 475-479
[13]. X. X. Huo, J. Wang, L. Jiang, et al. "Review on key technologies an applications of hydrogen enrgy storage system," *Energy Storage Science and Technology*, 5th ed. vol 2. 2016, pp, 197-203.
[14]. J. K. Nørskov, J. Rossmeisl, A Logadottir, et al, "Origin of the Overpotential for Oxygen Reduction at a Fuel-Cell Cathode," *Journal of Physical Chemistry B*, 108th ed. vol 46. 2004, pp, 17886-17892.
[15]. Y. Yu, Z. Q. Mi, "Nonlinear dynamic model and chaotic characteristics of mechanical thastic energy storage unit in energy storage process," *Acta Physica Sinica*, 62nd ed. vol 3. 2013, pp, 443-450.
[16]. W. Parry, "Ergodic Properties of Affine Transformations and Flows on Nilmanifolds," *American Journal of Mathematics*, 91st ed. vol 3. 1969, pp, 757-771
[17]. Scale Transformation. "Encyclopedia of Quality of Life & Well Being Research," 2014, pp, 5665-5665.
[18]. X. G. Qiu, "erivation of dimnesionless equations from method of analysis of directional unit system," *Journal of Aerospace Power*, 3rd ed. 1989, pp, 241-244.
[19]. K. T. Alligood, T. D. Sauer, "Yorke J A. Chaos," *Berlin: Springer-Verlag*, 1997, pp, 105-147.
[20]. Y. Yu, S Zhang, "Hopf bifurcation analysis of the Lü system," *Chaos Solitons & Fractals*, 21st ed. vol 5. 2004, pp, 1215-1220.

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