

## Cross Section for the Reaction $P+P$ Into $H+K^++K^-$ By Feynmann Rules

Sonali Pattnaik<sup>1</sup> Anjana Acharya<sup>2</sup>

*Department Of Physics , Veer Surendra Sai University Of Technology Burla , Orissa*

**ABSTRACT:** *The cross section for a particular reaction by applying Feynmann rules, Fermi golden rules is calculated and seen that there is no experimental comparison to it. Hence it is a new finding to us which has applications in production engineering, astrophysics, robotic industry and so on.*

**Keywords:** *Feynmann rules, Transition matrix, Fermi golden rules*

### I. INTRODUCTION TO ELEMENTARY PARTICLES

The scattering cross-section is the key observable in particle physics experiments such as the Large Hadron Collider in CERN. In quantum field theory the cross-section is expressed in terms of the scattering amplitudes which is calculated as the sum of Feynman diagrams. This paper also aims to investigate differential cross-section of scattered particles at each angle in proton-proton collision with the angular interval ranging between 15-30 degree. Datas are presented here with constant energy of proton 938.29MeV and momentum  $3.127 \times 10^{-6}$  kg m/sec inclusive. When many particles are involved in a process the Feynman diagram approach becomes very difficult.

1. The first elementary particle discovered was the negatively charged electron
2. The second scattering experiment indicates presence of positively charged protons inside nucleus.
3. With the discovery of neutron, the number of elementary particles became three:

#### Electron (e), proton (p) and neutron (n)

4. Cosmic ray research caused many discovery in elementary particles, the discovery of positron or positive electron ( $e^+$ ).
5. Radiation of Einstein introduced photon ( $\gamma$ ), a quantum of radiation.
6. Pauli postulated the existence of a neutral particle of negligible mass neutrino ( $\nu$ ) and anti neutrino ( $\bar{\nu}$ )[1]
7. For stability of nucleus against coulomb repulsion, Yukawa discovered the track of positively charge cosmicray particle , the positive muon ( $\mu^+$ ) followed by negative muon ( $\mu^-$ ). These are not Yukawa particles. They are weakly interacted with nucleus.
8. Powell and Occhialin discovered another cosmic ray particle, it was called pi-meson or pion ( $\pi$ ). By then three pi-mesons were discovered ( $\pi^+$ )( $\pi^-$ )( $\pi^0$ ). Both muons and pions are short lived[2]
9. Soon other short lived heavy mesons were discovered  $\kappa^+\kappa^-\kappa^0$  where  $\kappa^+$  and  $\kappa^0$  behaved as strange particles[3]
10. Later no of short lived particles were discovered lambda ( $\Lambda^0$ ) sigma ( $\Sigma^+\Sigma^-\Sigma^0$ ) Xi ( $\Xi^-\Xi^0$ ) Omega ( $\Omega^-$ )[4]

Here, we begin the quantitative formulation of elementary particles dynamics, which amounts, in practice to the calculation of decay rates, and scattering cross-section. This procedure involves two distinct parts:

1. Evaluation of the relevant Feynman diagram to determine the amplitude (M) for the process.
2. Insertion of M into the Fermi's Golden rule to compute decay rates or the cross-section.

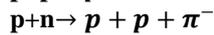
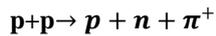
#### 1.1 Properties Of Elementary Particles:

##### Production Of Mesons:

Pions are particles with mass intermediate with that of an electron and proton. Their existence had been predicted by Yukawa, to explain the strong short range inter nucleon force. Yukawa predicted that pions are unstable.

The pions are produced by hitting a suitable target with a beam of high energy protons on alpha particles obtained from acceleration. The pions extracted from the acceleration are detected by nuclear emulsion plates suitably shielded. From their radius of curvatures in the magnetic field of the accelerator their momenta are determined. Their range in the nuclear emulsion plates gives their kinetic energy. From these quantities the mass of positive pion is determined. The mass of negative pion is determined from the measurement of mesic.

Thus  $\pi^\pm$  mesons are produced in their reactions



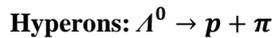
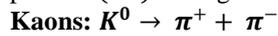
The accurately accepted values of pions mass is  $(273.123 \pm 0.002)m_e$

$$m_\pi c^2 = (139.5693 \pm 0.0007) \text{Mev}$$

The charged pion decays accordingly  $\pi^\pm \rightarrow \mu^\pm + \nu_\mu$

### Discovery Of K-Mesons And Hyperons

The identities of the decaying particle were established by measuring the radii of curvature of the tracks in magnetic field of the observable particles, their ranges and the densities of the grains on droplets along the tracks. It was found that two types of particles were involved. According to the current notations the neutral particle ( $\nu^0$ ) belong to two classes known as K-mesons or Kaons or hyperons which have the following schemes

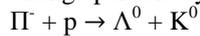


The mean life of decay were found to be about  $10^{-10}$  sec..

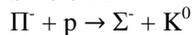
The kaons have mass of 494Mev. While the mass of  $\Lambda^0$  is about 1115Mev which is somewhat higher than nucleon mass. Later charged kaons and heavier hyperons ( $\Sigma$   $\Omega$   $\Xi$ ) were discovered. After their discovery these particles behaved in puzzling manner. So they produce in strong interaction. Further they are not produced in singly but pairs of them are produced in association with each other. The characteristic of new particles were described in terms of new property called "Strangeness" a concept developed by Gellman & Nishijima. Hence these particles are known as Strange particles.

### Production And Decay Of Strange Particles

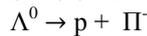
According to Gellman and Nishijima a new quantum number S known as the strangeness quantum no, is to be associated with the different strange particle to distinguish them from the non strange particle such as nucleons and the pions for which S=0. Pais proposed that strangeness is conserved during the production of the strange particle by strong interaction[5]



$$S = 0 \ 0 \ -1 \ 1$$

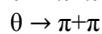
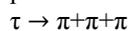


$$S = 0 \ 0 \ -1 \ 1 \quad (\text{However in weak interaction strangeness is not conserved}).$$



$$S = -1 \ 0 \ 0$$

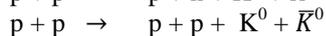
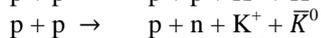
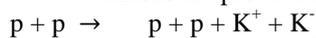
Two different modes of decay of the particle of the mass about  $1000m_e$  were observed soon after their discovery. One type decayed into the three pions and was called  $\tau$ - mesons while the other decayed into two pions and was called  $\theta$ -mesons.



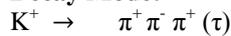
It was found that the decay product for both  $\theta$  and  $\tau$  had zero orbital angular momentum. So that the orbital parity for them must be even. Parity of  $\tau$  is odd (-ve) and parity of  $\theta$  is even (+ve). Parity is conserved in strong interaction.

### K-Mesons Or Kaons.

Kaons are produced in strong interaction either in N-N Collision or in  $\pi$ -N Collision.



### Decay Mode:



### Mass of Kaons

From the Q- values of the different modes of decay the mass of  $K^+$  has been determined

$$M(K^+) = 966.078m_e$$

$$M(K)c^2 = 493.668 \text{MeV}$$

Mean life of decay of  $K^+$  has been estimated to be  $\tau(K^+) = 1.2371 \times 10^{-8}$  sec

The spin-parity of  $K^+$  is  $0^-$  same as that of the pion. So it is a pseudo scalar meson like the pi-meson.

K<sup>-</sup> Decay modes are similar to that of K<sup>+</sup> mode. And K<sup>-</sup> is the antiparticle of K<sup>+</sup>

**Neutral Mesons:**

These were the first K mesons to be discovered. Both τ and θ modes of decay are observed. In addition the χ mode is also measured. The first two are non-leptonic while the last is leptonic decay. All are weak decays. The antiparticle of K<sup>0</sup> is  $\bar{K}^0$ . Together they constitute charge conjugate pair. The θ<sup>0</sup> meson is only the neutral kaon observed in nature. When it is produced in the following reaction:  $\pi^- + p \rightarrow \Lambda^0 + \theta^0$ . It resembles the K<sup>0</sup>. Both its two pions decay scheme  $\theta^0 \rightarrow (\pi^+ \pi^-)$  or  $(\pi^0 \pi^0)$  shows that this is not a simple K<sup>0</sup> decay. It is called K<sub>S</sub><sup>0</sup> meson having a very short mean-life (~ 10<sup>-10</sup>s). There is another neutral meson known as K<sub>L</sub><sup>0</sup> with a relatively longer mean life (~10<sup>-8</sup> sec) and different decay modes. Thus neutral K-mesons have dual character

**1.2 Conservation Laws:**

The occurrence of all physical processes is governed by a set of conservation laws, such as the conservation of energy, conservation of momentum, conservation of angular momentum, and conservation of electric charge. In the case of reactions involving the elementary particles, some other conservations laws, in addition to the above, are believed to hold, in order to account for the non-occurrence of certain physical processes though these may not violate the more well known conservation laws cited above, The different conservation principles are intimately related to the invariance of the physical laws under different types of symmetry operations some of which are well understood. However the symmetry operations related to some of the recently discovered conservation laws are not so well understood.

**Conservation Of Energy:** This holds for all type of interaction It is related to the invariance of the physical laws under translation along the time axis. This means that laws of interaction do not depend on time measurement.

**Conservation Of Linear Momentum:** This also holds for all types of interaction.It is related to the invariance of all the physical laws under translational space. Then the laws of interaction do not depend on the place of measurement so that space is homogeneous. This can be seen in Hamilton’s canonical equations.

$\frac{\partial H}{\partial p} = \dot{q}$ ,  $\frac{\partial H}{\partial q} = -\dot{p}$  where q and p are the generalized coordinate and the conjugate momentum and H is the Hamiltonian for the interacting particles. If the momentum is constant of motion  $\dot{p} = 0$  so that  $\frac{\partial H}{\partial q} = 0$  and the Hamiltonian is invariant under translation , here q is assumed to be the linear coordinate.

**Conservation Of Angular Momentum:** This law is also general validity for all types of interaction. It is related to the invariance of all types of physical laws under rotation. This follows from Hamilton’s equations in classical mechanics, if we write in terms of angular coordinate (θ<sub>i</sub>) and conjugate momenta (L<sub>i</sub>). In quantum mechanics angular momentum J must commute with H.  $[J, H] = 0$ . Here if angular momentum is constant of motion than H remains invariant under the operations.. It may noted that total angular momentum is conserved but the orbital and spin angular momentum may not be separately conserved..

**Conservation Of Parity.** This holds for strong nuclear and electromagnetic interactions but is violated in weak interactions. It is related to the invariance of physical laws under inversion of space coordinates so that x, y, z can be replaced by -x, -y, -z. This is equivalent to combined reflection and rotation. For Parity to be conserved, the parity operator must commute with the Hamiltonian. Spatial parity depends on the orbital angular momentum and is given by  $P = (-1)^l$  where l is azimuthal quantum number. Thus the states of even l has even parity (P = +1) and the states of odd l has odd parity. (P = -1). The two successive parity operations bring the system back to its initial state so that  $P^2 = 1$  which gives us the two possible eigen values  $\pm 1$  for P. The parity of a particle is usually designated by the symbols (+) for even and (-) for odd written as superscripts above the total angular momentum, thus (J<sup>P</sup>). Example for nucleons we write (J<sup>P</sup>) = ( 1/2<sup>+</sup>) for pions ( J<sup>P</sup>) = (0)

**Conservation Of Charge..** The net electric charge is conserved for all the interactions. This is related to the gauge invariance of the electromagnetic field. Like:  $n \rightarrow p^+ + e^- + \nu_e$ . In this equation the total initial charge is equal to total final charge. If we consider the decay of electron into photons and neutrinos then it would violate the law of conservation of charge.. Electron be the lightest charge particle, the conservation of charge implies that the electron must be a stable particle.

**Conservation Of Baryon Number:** The baryonic charge or the baryon number is an additive quantum no. All baryons (p, n, Σ<sup>+</sup>, Σ<sup>-</sup>, Σ<sup>0</sup>, Ξ<sup>-</sup>, Ξ<sup>0</sup>, Λ<sup>0</sup>) are assigned a baryon quantum number B = +1. Anti baryons (  $\bar{p}$   $\bar{n}$   $\bar{\Sigma}^+$   $\bar{\Sigma}^-$   $\bar{\Xi}^-$   $\bar{\Xi}^0$   $\bar{\Lambda}^0$ ) are assigned a baryon number B = -1, while the non-baryons photon

leptons and mesons are assigned a baryon number  $B = 0$ . Since the baryon number is an additive quantum number the baryon number of a set of particles is the sum of the baryon number of each particle in the set. According to this law of conservation in any reaction the initial baryon number is the same as the final baryon number. So the total number of baryons and anti-baryons should remain constant for all the interaction.

**Conservation Of Lepton Number:** The leptonic charge a lepton number is also an additive quantum number. The leptons ( $e^- \mu^- \nu_e \nu_\mu$ ) and the anti-leptons ( $e^+ \mu^+ \bar{\nu}_e \bar{\nu}_\mu$ ) are respectively assigned  $L = +1$  and  $L = -1$ . The non leptons ( baryons and mesons) are assigned  $L = 0$  According to the law of conservation of lepton number the initial lepton number must be equal to the final lepton number.

**Isospin Conservation:** In any reaction the total electric charge is conserved. Some elementary particles carry a charge  $\pm e$  or  $0 e$ . Heisenberg was first to apply a quantum number to charge and referred to the concept as “isospin”.

A multiplet number  $M$  is therefore assigned to such particles to indicate the number of their different charge states. For instance, for nucleons (protons and neutrons)  $M = 2$ , for pions  $M = 3$ , for kaons  $M = 2$  ets. As in multiplicity of atomic energy states due to spin the total number of states is  $(2s+1)$  where  $s$  is spin quantum number of electron. Now isospin quantum number  $I$  from the relation  $M = 2I+1 \Rightarrow I = (M - 1) / 2$ . Isospin is treated as vector of magnitude  $\sqrt{I(I + 1)}$  but it is dimensionless. Its component in  $Z$  axis is given as  $I_z$ . which has the allowed values  $I, (I - 1), (I - 2), (I - 2), \dots -I$ . For nucleons ( $M = 2$ )  $I = (M - 1)/2 = 1/2$ , with the two components  $I_3 = +1/2$  assigned to proton and  $I_3 = -1/2$  assigned to neutron. Isospin is conserved in strong interaction, but is violated in electromagnetic and weak interaction. The  $Z$ - components of isospin  $I_3$  is conserved in strong and electromagnetic interactions, but not in weak interaction.

| Particles     | Mass (Mev/c <sup>2</sup> ) | J   | B | L | I   | S  | G |
|---------------|----------------------------|-----|---|---|-----|----|---|
| $\gamma$      | 0                          | 1   | 0 | 0 | 0   | 0  | 0 |
| $\square_e$   | 0                          | 1/2 | 0 | 1 | 0   | 0  | 0 |
| $\square_\mu$ | 0                          | 1/2 | 0 | 1 | 0   | 0  | 0 |
| $e^-$         | 0.5                        | 1/2 | 0 | 1 | 0   | 0  | 0 |
| $\mu^-$       | 106                        | 1/2 | 0 | 1 | 0   | 0  | 0 |
| $\square^0$   | 135                        | 0   | 0 | 0 | 1   | 0  | 0 |
| $K^-$         | 494                        | 0   | 0 | 0 | 1/2 | -1 | 0 |
| $\square^0$   | 770                        | 1   | 0 | 0 | 1   | 0  | 0 |
| $P$           | 938                        | 1/2 | 1 | 0 | 1/2 | 0  | 0 |
| $\square^0$   | 1116                       | 1/2 | 1 | 0 | 0   | -1 | 0 |
| $\square^-$   | 1321                       | 1/2 | 1 | 0 | 1/2 | -2 | 0 |
| $\square^-$   | 1672                       | 3/2 | 1 | 0 | 0   | -3 | 0 |
| $D^0$         | 1865                       | 0   | 0 | 0 | 1/2 | 0  | 1 |

**Strangeness ( S):** S is defined as the difference of the hypercharge Y and the baryon number B.  $S = Y - B \Rightarrow Y = B + S$ . The hypercharge is the sum of the baryon number and the strangeness number. Strangeness is conserved in strong and electromagnetic interactions but not in weak decay. For an event involving a weak interaction, S however can change but by not more than  $\pm 1$ .

**Charge conjugation:** It means reversal of signs of all types of charges i.e. electronic, baryonic, and leptonic of the particle. If a physics law holding on a particle then it holds for corresponding antiparticle also then the principle of charge conjugation is said to be valid.. The principle of charge conjugation implies that the cross-section of the reactions of a given energy must be the same. Strong and electromagnetic interactions are invariant under charge conjugation but weak interaction does not obey charge conjugation.

**Time Reversal (T):** The operation T means time-reversal means replacing the time  $t$  by  $-t$  in all equation of motion. i.e. reflection of time axis at the origin of time coordinates in relativistic space-time continuum. If T is conserved i.e. time reversal invariance occurs, then reversed equation of motion is also a valid equation of motion of the system concerned.. All the known fundamental equation of motion are invariant in time reversal. Strong and electromagnetic interactions are invariant under time reversal transformation, but the weak interaction is not.

### 1.3 Fundamental Interaction In Nature:

There are four different types of fundamental interactions in nature, which govern the behaviors of all observable physical system. These are gravitational, electromagnetic, weak and strong nuclear interactions. Gravitational force acts between all bodies having masses. It is described by the long range inverse square type Newtonian laws of gravitation, later broadened by Einstein in his General Theory of Relativity, which describes the gravitational interaction in terms of the curvature of space. The dimension less quantity  $\frac{Gm_e M_p}{hc} \sim 3 * 10^{-42}$  is usually taken as a constant, characterizing the interaction between two bodies is believed to be mediated through the quantum of this interaction called the graviton . So far as the elementary particles are concerned, the gravitational interaction between them can be entirely left out of consideration, because it is very weak compared to other interactions.

Electromagnetic interaction is much stronger than the gravitational interaction. It is also a long range inverse square type interaction. Its strength is determined by Sommerfeld's fine structure constant  $\alpha = e^2/4\pi\epsilon_0\hbar c \sim 1/137$ . The electromagnetic interaction between a proton and an electron is about  $10^{37}$  times stronger than the gravitational force between them at the same distance. In modern quantum field theory, the electromagnetic interaction between the charged particles is described in terms of the exchange of virtual photons, which are quanta of this field. The electromagnetic interaction is manifested in the chemical behaviors of the atoms and molecules, Rutherford scattering and so forth.

The third fundamental interaction is the weak nuclear interaction, which is responsible for the nuclear beta-decay and the weak decay of certain elementary particles like the muons, pions, k-mesons and some hyperons. Unlike gravitational or electromagnetic interaction the weak interaction is a very short range force, the range being given by  $\frac{\hbar}{m_w c} \sim 2.4 * 10^{-18} m$ . Here  $m_w$  is the mass of the W bosons mediating the weak force. The coupling constant of the weak interaction has a value  $g = 1.4 * 10^{-62} J \cdot m^3$ . When expressed in dimensionless form the weak interaction constant  $g_w$  is found to be small compared to the fine structure constant  $\alpha \sim 1/137$ . The weak interaction is mediated through the heavy vector bosons, known as  $W^\pm$  and  $Z^0$  bosons discovered by G Amison et al., using p  $\bar{p}$  colliding beam. An important characteristic of the weak interaction which distinguishes it from the other fundamental interactions is that parity is not conserved in weak interaction.

The fourth fundamental interaction is the **strong nuclear interaction** between protons and neutrons. The range being  $\hbar/m_\pi c \sim 10^{-15} m$ .

Within this range it predominates over all the other forces between the neutron and the proton with the characteristic strength parameter  $\alpha \sim 1/137$  of the electromagnetic interaction. The time for the operation of the strong interaction is of the order of the characteristic nuclear time ( $\sim 10^{-23} s$ ). The strong inter nucleon force is mediated through the exchange of pi-mesons ( $\pi^\pm, \pi^0$ ) which is the quantum of the nucleon field.

| Interaction     | Characteristic Constant      | Strength   | Range of interaction | Typical cross section | Typical lifetime |
|-----------------|------------------------------|------------|----------------------|-----------------------|------------------|
| Strong          | $g^2\hbar/hc$                | 1~10       | $10^{-13} cm$        | $10^{-26} cm^2$       | $10^{-23} s$     |
| Electromagnetic | $e^2/hc$                     | 1/137      | $\infty$             | $10^{-29} cm^2$       | $10^{-16} s$     |
| Weak            | $g_w^2/hc = G_F m_p^2 c/h^3$ | $10^{-5}$  | $10^{-16} cm$        | $10^{-38} cm^2$       | $10^{-10} s$     |
| Gravitational   | $Gm_p^2/hc$                  | $10^{-39}$ | $\infty$             |                       |                  |

## II. FEYNMAN THEORY

Three experimental probes of elementary particle interactions bound states decays and scattering. Non-relativistic quantum mechanics is well adapted to handle bound states. Relativistic is especially to describe decays and scattering (Feynman formulation).

### 2.1 Decay Rate:

In case of decay, the most important factor is the lifetime. The critical parameter is the decay rate, the probability per unit time that any given muon will disintegrate. If we have large collection of muons say  $N(t)$  at time  $t$  the  $N \Gamma dt$  of them would decay in the next instant  $dt$ .

$dN = -\Gamma N dt$ . It follows that

$$N(t) = N(0)e^{-\Gamma t}$$

The no. of particles left decreasing exponentially with time. The mean lifetime is simply the reciprocal of the decay rate.

$$\tau = 1/\Gamma$$

Now the total decay rate is the sum of the individual decay rate

$$\Gamma_{tot} = \sum \Gamma_i$$

And the lifetime of the particle is the reciprocal of  $\Gamma_{tot}$   
 $\tau=1/\Gamma_{tot}$

### 2.2 Cross Section:

For elementary particle scattering, if we fire a stream of electrons into a tank of hydrogen, the parameter which is to be used is the size of the proton- the cross-sectional area  $\sigma$  it presents to the incident beam, the closer we come the greater is the deflection.

Secondly, the cross-section depends on the structure of the target. Electrons scatter off hydrogen more sharply than neutrinos and less so than pions, because different interactions are involved. It depends too on the outgoing particles. If the energy is high enough we cannot have only elastic scattering, but also a variety of inelastic scattering. Each one has its own scattering cross-section  $\sigma_i$ .

As the final products are not determined so we will find only the total cross-section:

$$\sigma_{tot} = \sum \sigma_i$$

Suppose a particle comes along, encounters some kind of potential, and scatters off at an angle  $\theta$ . This scattering angle is the function of the impact parameter  $b$ , the distance by which the incident particle would have meets the scattering center on its original trajectory. The smaller the impact parameter, the larger the deflection, but the actual functional form of  $\theta(b)$  depends on the particular potential involved.

### 2.3 Golden Rule:

Here we need to calculate decay rates and cross-sections. There are two factors involving in this,

- 1- Amplitude for the process
- 2- The phase space available

The amplitude contains all the dynamical information; We calculate it by evaluating the relevant Feynman diagrams, using the Feynman rules appropriate to the interaction. The phase space factor is purely kinematic; it depends on the masses, energies and momenta of the participants and reflects the fact that a given process is more likely to occur the more in the final state. For example, the decay of the heavy particle into light secondaries involves a large phase space factor. By contrast the decay of the neutron ( $n \rightarrow p + e + \nu_e$ ) in which there is almost no extra mass to spare, is tightly constrained and the phase space factor is very small.

Fermi's golden rule says that a transition rate is given by the product of the phase space and the absolute square of the amplitude. Statistical factor is a factor that corrects for double counting when there are identical particles in the final state: for each group of  $s$  particles,  $S$  gets a factor of  $(1/s!)$ . For instance, if  $a \rightarrow b + b + c + c + c$ , then  $S = (1/2!)(1/3!) = 1/12!$ . If there are no identical particles in the final state, the

### 2.4 Feynman Rules To Calculate Scattering Amplitude (M)

Our problem is to find the scattering amplitude  $M$  associated with the respective Feynman diagrams.

- **Notation:** Label the internal momenta as  $q_1, q_2, \dots$  and the incoming and outgoing four momenta  $p_1, p_2, p_3, \dots$ . Put an arrow beside each line as per the positive direction, so that it is forward in time for external lines and arbitrary for internal lines.
- **Vertex Factors:** For each vertex there will be a factor  $-ig$  where  $g$  is the coupling constant, and it specifies the strength of interaction between A, B and C. Here,  $g$  has the dimension of momentum but in real world theories the coupling constant is always dimensionless.
- **Propagators:** There is a factor for each internal line

$$\frac{i}{q_j^2 - m_j^2 c^2}$$

Where  $q_j$  is the four momentum of the line and  $m_j$  is the mass of the particle that the line describes.

- **Conservation Of Energy And Momentum:**

For each vertex there is a delta function of the form

$$(2\pi)^4 \delta^4(k_1 + k_2 + k_3)$$

Where the  $k$ 's are the three four momenta coming into the vertex. This factor imposes conservation of energy and momentum at each vertex. Since the delta function is zero unless the sum of the incoming momenta equals the sum of the outgoing momenta.

### Integration Over Internal Momenta:

There is a factor for each internal line,  $\frac{1}{(2\pi)^4} d^4 q_j$  and integrate over all internal momenta.

- **Erase Delta Function:** The result will include delta function

$$(2\pi)^4 \delta^4(p_1 + p_2 + \dots - p_n)$$

Reflecting overall conservation of energy and momentum. Cancel this factor and multiply by  $i$ . Then we get the result of our scattering amplitude  $M$ .

### III. Outline of the problem

(a) An experiment is performed to search for evidence of the reaction  $pp \rightarrow H + K^+ + K^-$

The values of electric charge, strangeness and baryon number and quark composition of the particle  $H$

Solution:  $K^+$  has strangeness  $S=1$ , so RHS becomes  $S=2$ , Proton has strangeness  $S=0$

LHS becomes  $S=0$ . So to conserve the strangeness  $H$  must be having  $S=-2$

Now  $k^+$  has  $B=0$  so RHS becomes  $B=0$ , Proton has  $B=1$  hence LHS becomes  $B=2$ . Thus to satisfy baryon no conservation,  $H$  must be having  $B=2$ .

Electric charge( $q$ ) of  $K^+=1$ , hence RHS is  $Q=2$ . Electric charge( $Q$ ) of  $P=1$ , thus LHS is  $Q=2$ . So to satisfy charge conservation  $H$  must have electric charge  $Q=0$ .

To satisfy the above conservation laws  $H$  must contain at least six quarks. Because Up quark( $U$ ) has  $B=1/3$ , Down quark( $d$ ) has  $B=1/3$ , and strange quark( $s$ ) has  $B=1/3$ . To satisfy this condition  $H$  has six quarks i.e  $UUDDSS$ .

(b) A theoretical calculation for the mass of this state  $H$  yields a predicted value of  $m_H = 2150$  MeV. Then the question arises what is the minimum value of incident-beam proton momentum necessary to produce this state? (Assume that the target protons are at rest)

Solution: At minimum incident energy, the particles are produced at rest in the center-of-mass frame.

As  $(\Sigma E)^2 - (\Sigma p)^2$  is invariant, we have

$$(E_0 + mp)^2 - p_0^2 = (m_H + 2m_K)^2$$

$$E_0 = (m_H + 2m_K)^2 - 2m_p^2$$

$$= (2.15 + 2 \times 0.494)^2 - 2 \times 0.938^2$$

$$= 4.311 \text{ GeV}^2$$

and

hence the minimum incident momentum  $p_0 = \sqrt{E_0^2 + m_p^2} = 4.208 \text{ GeV}/c$

(c) If the mass prediction is correct, what can you say about the possible decay modes of  $H$  considering both strong and weak decays[6]

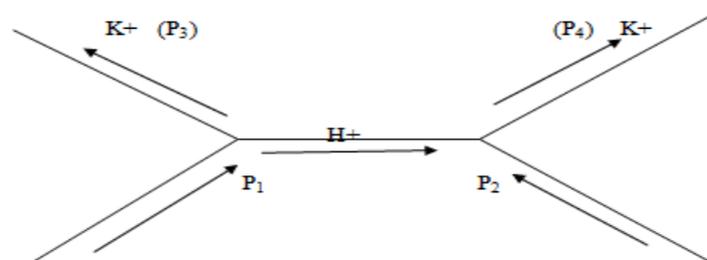
Solution: As for strong decays,  $\Delta S = 0$ ,  $\Delta B = 0$ , the possible channels are  $H \rightarrow \Lambda_0 \Lambda_0, \Lambda_0 \Sigma_0, \Xi^- p, \Xi^0 n$ . However they all violate the conservation of energy and are forbidden. Consider possible weak decays.

The possible decays are non-leptonic decays  $H \rightarrow \Lambda + n, \Sigma_0 + n, \Sigma^- + p$ , and semi-leptonic decays

$H \rightarrow \Lambda + p + e^- + \bar{\nu}$

### IV. Differential Cross section of Fermion particle when decays into kaons

To calculate the differential cross section we need to draw the Feynman diagram for the respective reaction.



The lowest order contribution to the process  $P + P \rightarrow H + K^+ + K^-$  is shown in the above fig. In this case there are two vertices (hence two factors of  $-ig$ ) one internal line with the propagator

$$\frac{i}{q_j^2 - m_j^2 c^2}$$

delta functions  $(2\pi)^4 \delta^4(P_1 - P_3 - q)$  and  $(2\pi)^4 \delta^4(q + P_2 - P_4)$  and one integration

$$\frac{1}{(2\pi)^4} d^4 q_j$$

Then summing up the Feynman rules from 1 – 5 we have

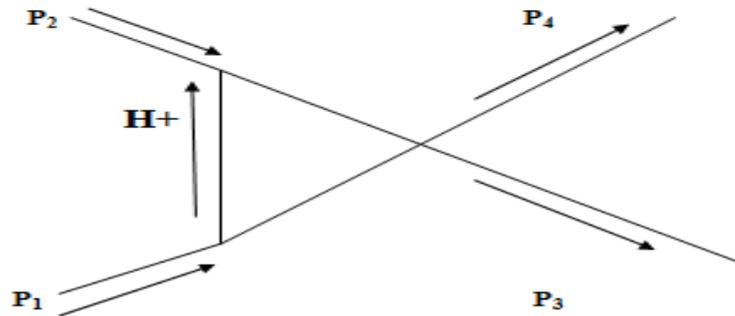
#### 4.1 Calculations:

$$M = -i(2\pi)^4 g^2 \int 1/q^2 - m_c^2 \delta^4(P1 - P3 - q) \delta^4(q + P2 - P4) d^4q \text{ -----(1)}$$

Doing the integral for second delta function we have  $q \rightarrow P_4 - P_2$   
 According to the rule 6 we erase the delta function and multiplying by I we get,

$$M = g^2 / (P_4 - P_2)^2 - m_c^2 c^2 \text{ ----- (2)}$$

Now there is another diagram of order  $g^2$  obtained by twisting the K lines.



Since this diagram is different from the first one only by interchange  $P_3 \rightarrow P_4$ . Summing up the Feynman rules for this fig, the amplitude for this process is

$$M = g^2 / (P_3 - P_2)^2 - m_c^2 c^2 \text{ ----- (3)}$$

Combining the two amplitudes for this reaction we have to get the total amplitude we have.

$$M = g^2 / (P_4 - P_2)^2 - m_c^2 c^2 + g^2 / (P_3 - P_2)^2 - m_c^2 c^2 \text{ ----- (4)}$$

Now to calculate differential cross-section for this process, we assume  $m_A = m_B = m$  and  $m_c = m$ . Then

$$(P_4 - P_2)^2 - m_c^2 c^2 = P_4^2 + P_2^2 - 2P_2 \cdot P_4 = -2P^2 (I - \cos\theta) \text{ ----- (5)}$$

$$(P_3 - P_2)^2 - m_c^2 c^2 = P_3^2 + P_2^2 - 2P_3 \cdot P_2 = -2P^2 (I + \cos\theta) \text{ -----(6)}$$

Solving the above equation and substituting the values in eq 4 we get

$$M = g^2 / -p^2 \sin^2\theta \text{ ----- (7)}$$

The differential cross-section reaction

$$\frac{d\sigma}{d\Omega} = \left(\frac{\hbar c}{8\pi}\right)^2 \frac{S |M|^2 |p_f|}{(E_1 + E_2) |p_i|} \text{ -----(8)}$$

Where  $|p_f|$  is magnitude of either out going momentum and  $|p_i|$  is the magnitude of either incoming momentum

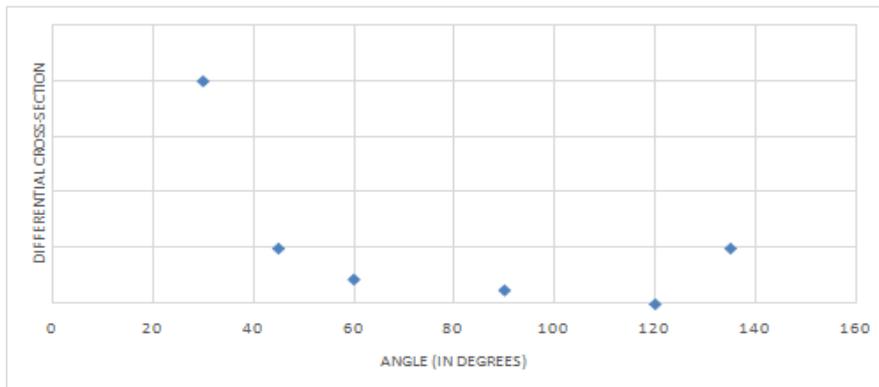
Substituting the value of M from eq(7) in eq (8) we get

$$\frac{d\sigma}{d\Omega} = \frac{1}{2} \left( \frac{g^2}{16\pi E P^2 \sin^2 \theta} \right)^2 \text{ ----- (9)}$$

taking  $|P_f| = |P_i| = P$  and  $E_1 = E_2 = E$ . This is the required differential cross-section for the given reaction.

Now substituting the energy of proton is  $P = 938.28\text{MeV}$  and momentum of proton is  $312.76 * 10^{-8} \text{ kg m/sec}$  we got the differential cross-section versus angle .

**4.2 Graph Plot**  
**4.3**



**Figure 1** The differential cross-section versus angle of scattering for the given reaction

**V. CONCLUSION**

This is the required plotting for this reaction which gives the highest value of  $\frac{d\sigma}{d\Omega} = 4.97867 \mu\text{b}$  at  $\theta = 45$  degree and the lowest value of  $\frac{d\sigma}{d\Omega} = 1.24467 \mu\text{b}$  at  $\theta = 90$  degree and we see that the plot is increasing and decreasing and maintaining the same value at certain values of  $\theta$  which is due to the sine parameter in the denominator and which is the new finding for this reaction where fermion particle shows radical increase and decrease properties which we can conclude from our theoretical calculations.

## References

- [1]. Martinus Veltman, Diagrammatic' The Path to Feynman Diagrams, Cambridge Lecture Notes in Physics' ISBN 0-521-45692-4 (expanded, updated version of above)
- [2]. Mark Srednicki, Quantum Field Theory, online Script (2006)
- [3]. Introduction to Elementary particles by David Griffiths
- [4]. Nuclear physics by Dr, S.N. Ghosal
- [5]. Dibakar Roychoudhury 'Holographic thermalization from non relativistic branes'
- [6]. arXiv:1601 :00136v2[hep-th] May 2016