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International Journal Of Modern Engineering Research (IJMER)

The Dynamics of Two Harvesting Species with variable Effort Rate with the Optimum Harvest Policy

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ABSTRACT: The dynamics of two ecologically independent species which are being harvested with variable effort have been discussed. The dynamics of effort is considered separately. The local and global dynamics of the system is studied. The existence of Hopf bifurcation with respect to the total cost of fishing is investigated. The co-existence of species in the form of stable equilibrium point and limit cycle is possible. The quasi-periodic behavior is also possible for some choices of parameters. An expression for Optimum population level has been obtained.

I. INTRODUCTION

Exploitation of biological resources as practiced in fishery, and forestry has strong impact on dynamic evolution of biological population. The over exploitation of resources may lead to extinction of species which adversely affects the ecosystem. However, reasonable and controlled harvesting is beneficial from economical and ecological point of view. The research on harvesting in predator-prey systems has been of interest to economists, ecologists and natural resource management for some time now.

The optimal management of renewable resources has been extensively studied by many authors [1, 2, 3, 7, 8, 12]. The mathematical aspects of management of renewable resources have been discussed by [10]. He had investigated the optimum harvesting of logistically growing species. The problem of combined harvesting of two ecologically independent species has been studied [10, 13]. The effects of harvesting on the dynamics of interacting species have been studied Measterton-Gibbons [14], Chaudhuri et.al. [6-9] with constant harvesting, the prey predator model is found to have interesting dynamical behavior including stability, Hopf bifurcation and limit cycle [4, 5, 11, 15].

The multi species food web models have found to have rich dynamical behavior [16, 18]. S Kumar et. al. [17] have investigated the harvesting of predator species predating over two preys.

In this paper the dynamics of two ecologically independent species which are being harvested have been discussed when the dynamics of effort is considered separately.

II. THE MATHEMATICAL MODEL

Consider two independent biological species with densities X_1 and X_2 with logistic growth. The Mathematical model of two harvesting prey species with effort rate is given by the following system of ordinary differential equations:

$$\begin{aligned}
\frac{dX_1}{dT} &= r_1X_1\left(1-\frac{X_1}{K_1}\right)-\frac{A_1q_1X_1E}{1+B_1X_1+B_2X_2} = X_1f_1(X_1,X_2,E)\\
\frac{dX_2}{dT} &= r_2X_2\left(1-\frac{X_2}{K_2}\right)-\frac{A_2q_2EX_2}{1+B_1X_1+B_2X_2} = X_2f_2(X_1,X_2,E)\\
\frac{dE}{dT} &= E\left(h(X_1,X_2)-C\right) = Ek\left(\frac{p_1q_1A_1X_1+p_2q_2A_2X_2}{1+B_1X_1+B_2X_2}-C\right) = Ef_3(X_1,X_2)
\end{aligned} \tag{1}$$

The logistic growth is considered for the two preys. The model does not consider any direct competition between the two populations. The constants $K_i, r_i, A_i,$ and B_i, are model parameters assuming only positive values. The effort E is applied to harvest both the species and C is total cost of fishing. The harvesting is proportional to the product of effort E and the fish population density X_i.The catch-ability coefficients q_i are assumed to be different for the two species. In the model, the third equation considers the dynamics of effort *E*. The constants p_1 and p_2 are the price of the per unit prey species. The last equation of (1) implies that the rate of increase of the effort is proportional to the rate of net economic revenue. The constant k is the proportionality constant.

Let the constant M_0 is the reference value of E. Introduce the following dimensionless transformations:

$$t = r_1 T,\ y_i = X_i / K_i\,(i = 1,2),\ x = E/M_0, w_1 = A_1 q_1 E_0 / r_1,\ w_2 = B_1 K_1, w_3 = B_2 K_2$$
$$w_4 = r_2 / r_1,\ w_5 = A_2 q_2 M / r_{10}, w_6 = kK_1 / M_0, w_7 = kK_2 / M_0;$$

The dimensionless nonlinear system is obtained as:

$$\frac{dy_1}{dt} = y_1\left(1 - y_1 - \frac{w_1 x}{1 + w_2 y_1 + w_3 y_2}\right) = y_1 f_1(y_1, y_2, x)$$
$$\frac{dy_2}{dt} = y_2\left((1 - y_2) w_4 - \frac{w_5 x}{1 + w_2 y_1 + w_3 y_2}\right) = y_2 f_2(y_1, y_2, x) \tag{2}$$
$$\frac{dx}{dt} = x\left(\frac{p_1 w_1 w_6 y_1 + p_2 w_5 w_7 y_2}{1 + w_2 y_1 + w_3 y_2} - C\right) = x f_3(y_1, y_2)$$

III. EXISTENCE OF EQUILIBRIUM POINTS

Since $0 \le y_i \le 1;\ i = 1,2$, the underlying non-linear model (2) is bounded and has a unique solution. There are at most seven possible equilibrium points of the nonlinear harvesting model:

$$E_0 = (0,0,0),\ E_1 = (1,0,0),\ E_2 = (0,1,0),\ E_3 = (1,1,0),$$
$$E_4 = (y_1^*, 0, x^*),\quad x^* = (1 - y_1^*)(1 + w_2 y_1^*)/w_1;\quad y_1^* = C/(w_1 p_1 w_6 - C w_2)$$
$$E_5 = (0, y_2^*, x^*),\quad x^* = \left(w_4(1 - y_2^*)(1 + w_3 y_2^*)\right)/w_5;\quad y_2^* = C/(w_5 p_2 w_7 - C w_3),$$
$$E_6 = (y_1^*, y_2^*, x^*).$$

Theorem 3.1 The equilibrium point $E_4 = (y_1^*, 0, x^*)$ is feasible only when

$$C < w_1 p_1 w_6 / (1 + w_2) \tag{3}$$

Theorem 3.2 The equilibrium point $E_5 = (0, y_2^*, x^*)$ is feasible only when

$$C < w_5 p_2 w_7 / (1 + w_3) \tag{4}$$

The proofs of the two theorems are straightforward as $0 < y_i^* < 1;\ i = 1,2$.

Theorem 3.3 The positive non-zero equilibrium E_6 of nonlinear harvesting model (2) exists provided the following conditions are satisfied:

$$C < w_1 p_1 w_6 / w_2;\quad C < w_5 p_2 w_7 / w_3 \tag{5}$$

Proof: Proof is given in [6]; the equilibrium points are

$$x^* = (1 - y_1^*)(1 + w_2 y_1^* + w_3 y_2^*)/w_1$$
$$y_1^* = \frac{(w_5 - w_1 w_4)(w_7 p_2 w_5 - C w_3) + C w_1 w_4}{w_5 (w_7 p_2 w_5 - C w_3) + w_1 w_4 (w_1 p_1 w_6 - C w_2)}$$
$$y_2^* = \frac{C w_5 - (w_5 - w_1 w_4)(w_1 p_1 w_6 - C w_2)}{w_5 (w_7 p_2 w_5 - C w_3) + w_1 w_4 (w_1 p_1 w_6 - C w_2)}$$

The positive non-zero biological equilibrium $E_6 = (y_1^*, y_2^*, x^*)$ exists provided the conditions (5) are satisfied.

It may further be observed that conditions (3) and (4) imply (5), that is if $E_4 = \left(y_1^*, 0, x^*\right)$ and $E_5 = \left(0, y_2^*, x^*\right)$ exists then E_6 will also exists. However, the existence of E_6 is possible irrespective of E_4 and E_5 provided the condition (5) is satisfied.

IV. STABILITY ANALYSIS

The variational matrix about the point E_0 is given by

$$J_0 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & w_4 & 0 \\ 0 & 0 & -C \end{bmatrix}$$

From the above variational matrix, it is seen that there are two unstable manifolds along both X , Y axis and one stable manifold along Z axis. Therefore the point E_0 is a saddle point, that is, at very small densities of species the effort decreases and tends to zero, while for small efforts the densities of harvesting species will start increasing,

The variational matrices about the axial point $E_1 = (1,0,0)$ and $E_2 = (0,1,0)$ are given by

$$J_1 = \begin{bmatrix} -1 & 0 & -1/(1+w_2) \\ 0 & w_4 & 0 \\ 0 & 0 & \dfrac{w_1 p_1 w_6}{1+w_2} - C \end{bmatrix} \text{ and } J_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -w_4 & -w_5/(1+w_3) \\ 0 & 0 & (\dfrac{w_5 p_2 w_7}{1+w_3} - C) \end{bmatrix} \text{ respectively.}$$

From the matrix J_1, it is seen that there exists a stable manifold along X axis and an unstable manifolds along Z axis. Stable manifold along Y axis exists provided $w_1 p_1 w_6 - C(1+w_2) < 0$. Observe that this condition violates the existence of $E_4 = \left(y_1^*, 0, x^*\right)$. The point E_1 is a saddle point.

Similarly, from the matrix J_2, it is seen that there exists a stable manifold along Y axis and an unstable manifolds along X axis. Stable manifold along Z axis exists provided $w_1 p_1 w_6 - C(1+w_2) < 0$. This condition excludes the existence of equilibrium point $E_5 = \left(0, y_2^*, x^*\right)$.The point E_2 is a saddle point.

The variational matrix about the point $E_3 = (1,1,0)$ is given by

$$J_3 = \begin{bmatrix} -1 & 0 & -w_1/(1+w_2+w_3) \\ 0 & -w_4 & -w_5/(1+w_2+w_3) \\ a_{31} & a_{32} & \dfrac{w_1 p_1 w_6 + w_5 p_2 w_7}{(1+w_2+w_3)} - C \end{bmatrix}$$

Thus, the equilibrium point $E_3 = (1,1,0)$ is stable provided the following condition is satisfied:

$$\frac{w_1 p_1 w_6 + w_5 p_2 w_7}{(1+w_2+w_3)} - C < 0 \tag{6}$$

Theorem 4.1: The equilibrium point $E_4 = \left(y_1^*, 0, x^*\right)$ is locally asymptotically stable provided

$$(w_2 - 1)/2w_2 < y_1^* < (w_1 - w_5)/w_5 < 1 \tag{7}$$

Proof. The variational matrix about the point $E_4 = \left(y_1^*, 0, x^*\right)$ is given by

$$J_4 = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{23} \end{bmatrix}$$

$$a_{11} = y_1^*\left[-1 + \frac{w_1 w_2 x^*}{(1+w_2 y_1^*)^2}\right], \quad a_{12} = \frac{w_1 w_3 y_1^* x^*}{(1+w_2 y_1^*)^2}, \quad a_{13} = -\frac{w_1 y_1^*}{(1+w_2 y_1^*)};$$

$$a_{22} = \left[w_4 - \frac{w_5 x^*}{(1+w_2 y_1^*)}\right], \qquad a_{21} = a_{23} = a_{33} = 0$$

$$a_{31} = \frac{x^* w_1 w_6 p_1}{(1+w_2 y_1^*)^2}; \qquad a_{32} = \frac{x^*\left(w_7 w_5 p_2 + y_1^*\left(w_2 w_7 w_5 p_2 - w_3 w_1 w_6 p_1\right)\right)}{(1+w_2 y_1^*)^2};$$

The equilibrium point E_4 is locally stable if the following conditions are satisfied:

$$w_1 x^* > w_2(1 - y_1^*)^2; \text{ and } \quad y_1^* < (w_1 - w_5)/w_5;$$

Substitution for x^* and simplification yields the stability conditions as

$$\left((w_2 - 1)/2w_2\right) < y_1^* < \left((w_4 w_1 - w_5)/w_1 w_4\right) < 1$$

The equilibrium E_4 is unstable when the condition (10) is violated.

Similarly, the stability conditions for the equilibrium E_5 are stated in the theorem 4.2. Its proof is omitted.

Theorem 4.2: The equilibrium point $E_5 = \left(0, y_2^*, x^*\right)$ is locally asymptotically stable provided

$$(w_3 - 1)/2w_3 < y_1^* < (w_1 - w_5)/w_5 < 1 \tag{8}$$

The equilibrium E_5 is unstable when the condition (11) is violated.

The following theorem gives the conditions for the locally stability of the nonzero positive equilibrium point $E_6 = \left(y_1^*, y_2^*, x^*\right)$.

Theorem 4.3: The positive non-zero biological feasible equilibrium $E_6 = \left(y_1^*, y_2^*, x^*\right)$ is locally asymptotically stable if the following conditions are satisfied:

$$x^* > w_2(1 - y_1^*)^2; \tag{9}$$

$$w_4 w_1^2 x^* > w_3 w_5 (1 - y_1^*)^2 \tag{10}$$

$$w_1^2 x^* y_1^* > (w_1 w_2 y_1^* + w_3 w_5 y_2^*)(1 - y_1^*)^2 \tag{11}$$

$$w_4 w_1^2 x^* y_2^* > (w_1 w_2 y_1^* + w_3 w_5 y_2^*)(1 - y_1^*)^2 \tag{12}$$

Proof: Proof is given in [6].

Thus, the positive non-zero biological feasible equilibrium E_6 is locally asymptotically stable if the conditions given by (9-12) are satisfied.

The following theorem gives the conditions for the global stability of the nonzero positive equilibrium point.

Theorem 4.4 Let the local stability conditions given by (9-12) hold. The positive non-zero biological feasible equilibrium $E_6 = \left(y_1^*, y_2^*, x^*\right)$ is global stable if the following condition is satisfied:

$$(w_3 - \alpha w_2 w_4)^2 < 4\alpha w_4 \left((1+w_2 y_1^* + w_3 y_2^*) - w_2\right)\left((1+w_2 y_1^* + w_3 y_2^*) - w_3\right)$$

$$\alpha = \frac{w_1(w_7 w_5 p_2 - w_3 C)}{w_5(w_1 w_6 p_1 - w_2 C)} > 0 \tag{13}$$

Proof: Proof is given in [6];

V. THE OPTIMUM HARVEST POLICY

The economic rent (net revenue) at any time is given by

$$p(x, y_1, y_2) = x\left(\frac{p_1 w_1 w_6 y_1 + p_2 w_5 w_7 y_2}{1 + w_2 y_1 + w_3 y_2} - C\right)$$

The present value J of a continuous time stream of revenues is given by the expression

$$J = \int_0^\infty p(x, y_1, y_2) e^{-\delta t} dt = \int_0^\infty x\left(\frac{p_1 w_1 w_6 y_1 + p_2 w_5 w_7 y_2}{1 + w_2 y_1 + w_3 y_2} - C\right) e^{-\delta t} dt \tag{14}$$

Consider the integrand of the present value

$$G(y_1, y_2, x, t) = x\left(\frac{p_1 w_1 w_6 y_1 + p_2 w_5 w_7 y_2}{1 + w_2 y_1 + w_3 y_2} - C\right) e^{-\delta t}$$

$$= \left[\{y_1(1-y_1) - \dot{y}_1\} p_1 w_6 + \{w_4 y_2 (1-y_2) - \dot{y}_2\} p_2 w_5 - Cx\right] e^{-\delta t}.$$

Here δ is the instantaneous annual rate of discount.
Using classical Euler necessary conditions to maximize the positive nonzero equilibrium such that

$$\frac{\partial G}{\partial y_1} - \frac{d}{dt}\left(\frac{\partial G}{\partial \dot{y}_1}\right) = 0; \quad \frac{\partial G}{\partial y_2} - \frac{d}{dt}\left(\frac{\partial G}{\partial \dot{y}_2}\right) = 0; \quad \frac{\partial G}{\partial x} - \frac{d}{dt}\left(\frac{\partial G}{\partial \dot{x}}\right) = 0.$$

On solving the first equation, we get

$$\frac{\partial G}{\partial y_1} - \frac{d}{dt}\left(\frac{\partial G}{\partial \dot{y}_1}\right) = 0 \quad \Rightarrow \{1 - 2y_1 - \delta\} p_1 w_6 e^{-\delta t} = 0; \text{ or}$$

$$\{1 - 2y_1 - \delta\} e^{-\delta t} = 0 \ \ \& \ \ p_1 w_6 \neq 0 \tag{15}$$

Similarly other conditions yield

$$\{w_4(1 - 2y_2) - \delta\} e^{-\delta t} = 0 \ \ \& \ \ p_2 w_5 \neq 0 \tag{16}$$

$$C e^{-\delta t} = 0 \tag{17}$$

Case (1): Let $e^{-\delta t} \neq 0$ for $\delta t \neq \infty$, then

$$1 - 2y_1 = \delta. \tag{18}$$

$$w_4(1 - 2y_2) = \delta \tag{19}$$

$$C = 0 \tag{20}$$

Combining (22) and (23) gives

$$y_1 - w_4 y_2 = (1 - w_4)/2. \tag{21}$$

Case (2): Let $e^{-\delta t} = 0$ for $\delta t = \infty$ but $\delta \neq 0$ then

$$1 - 2y_1 - \delta = c_1. \tag{22}$$

$$w_4(1 - 2y_2) - \delta = c_2 \tag{23}$$

$$C \neq 0 \tag{24}$$

Eliminating δ, c_1, c_2 using (26) and (27) we get

$$y_1 - w_4 y_2 = (1 - w_4)/2. \tag{25}$$

Hence the optimum line (21) and (25) are same in the both cases. Therefore the optimum equilibrium will be the intersection of line (25) and first isocline of (2).

VI. EXISTENCE OF HOPF BIFURCATION

The characteristic equation of the above variational matrix about E_6 is obtained as [6]. By the Routh-Hurwitz criterion, the positive nonzero equilibrium point is locally asymptotically stable under the conditions (9-12).

The Hopf bifurcation occurs at $C = C^*$ where the value of $a_0 a_1 - a_2$ becomes zero. Substituting $a_0 a_1 = a_2$ into (16), we get,

$$\lambda^3 + a_0\lambda^2 + a_1\lambda + a_0 a_1 = 0 \Rightarrow (\lambda^2 + a_1)(\lambda + a_0) = 0.$$

This gives purely imaginary roots and one real root:

$$\lambda_1 = -a_0 \quad ; \quad \lambda_{2,3} = \pm i\sqrt{a_1} \tag{26}$$

Transversality condition: - Let the characteristic equation be such that it contains a pair of purely imaginary roots $\lambda_1 = \lambda_1' + i\lambda_2'$ and one real root, say c_1:

$$\begin{aligned} &(\lambda - \lambda_1)((\lambda - \bar{\lambda}_1)(\lambda - c_1) = 0. \\ &or \quad \lambda^3 - (2\lambda_1' + c_1)\lambda^2 + (\left|\lambda_1\right|^2 + 2\lambda_1' c_1) - \left|\lambda_1\right|^2 c_1 = 0\cdot \end{aligned} \tag{27}$$

Comparing the coefficients of (31) and (16) we get

$$a_1(-a_0 - 2\lambda_1') = -a_2 + 2\lambda_1'(2\lambda_1' + a_0)^2 \tag{28}$$

Differentiating (30) with respect to bifurcation parameter C, setting $C = C^*$, and rescaling that $\lambda_1'(C^*) = 0$ .we get [18],

$$\left.\frac{\partial \lambda_1'}{\partial C}\right|_{C=C^*} = -\frac{\left(a_0 \dfrac{\partial a_1}{\partial C} + a_1 \dfrac{\partial a_0}{\partial C} - \dfrac{\partial a_2}{\partial C}\right)}{2(a_0^2 + a_1)} \neq 0 \tag{29}$$

Thus the transversality condition is satisfied. So there exists a family of periodic solutions bifurcating from E_6 in the neighborhood of C^* that is, $C \in (C^* - \delta, C^* + \delta)$.

VII. NUMERICAL SIMULATION

As the solution of the system is bounded, the long time behavior of the solution is obtained as limit cycle, limit point attractor, quasi-periodic. For global dynamic behavior, numerical simulations of the underlying non-linear system are carried out. Consider the biological feasible set of parameters as

$$w_1 = 2.0, w_2 = 1.2, w_3 = 1.6, w_4 = 1.12, w_5 = 2.5, \; w_6 = 1.3, w_7 = 1.7, p_1 = 0.15, p_2 = 0.12, \tag{30}$$

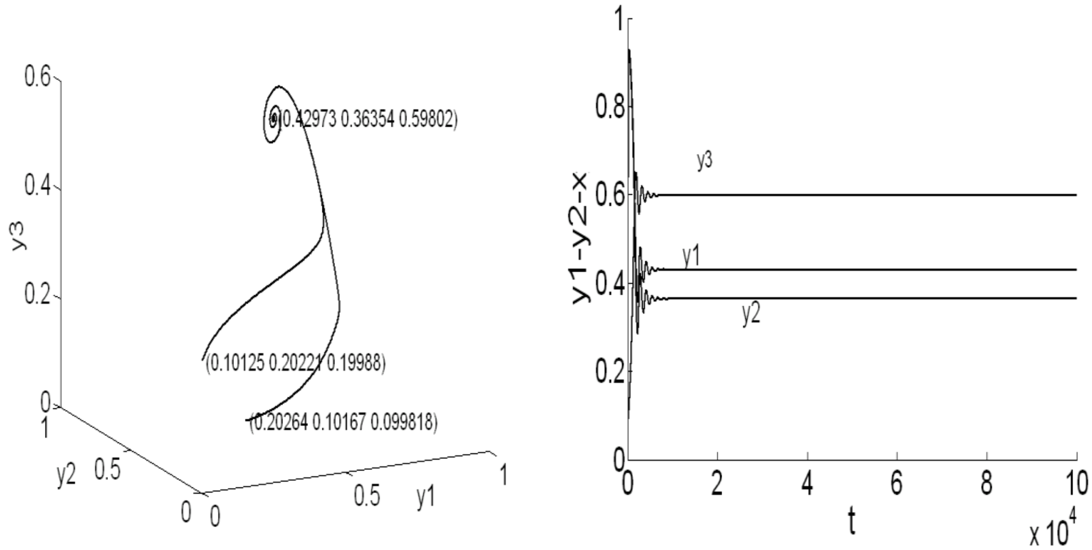
The sign change from positive to negative for the expression $a_0 a_1 - a_2$ is observed as the values of parameter C are varied. The existence of hopf bifurcation is observed in the neighborhood of $C = 0.14$. The variational matrix at the hopf bifurcation point $C = 0.14$ is given by

$$J_1 = \begin{bmatrix} -0.1725 & 0.1967 & -0.3617 \\ 0.1390 & -0.0848 & -0.3406 \\ 0.0755 & 0.0972 & 0 \end{bmatrix}$$

The eigenvalues of the matrix J_1 are obtained as $-0.2989,\ 0.0208 + 0.2443i,\ 0.0208 - 0.2443i$. From the eigenvalues the Hopf bifurcation is evident is evident in the neighborhood of $C = 0.14$.

The value of the expression $a_0a_1 - a_2$ is $0.0030 > 0$ at C =0.156. Fig. 1 (a) shows the phase space trajectories converging to the point $(0.42973,\ 0.36954,\ 0.59802)$ for the data set (30) at $C = 0.156$, starting with two different initial conditions. In other words, the nontrivial equilibrium point $(0.42973,\ 0.36954,\ 0.59802)$ is stable giving the persistence of the system for the given set of parameters. Fig 1(b) shows the time series for the data set (30) at $C = 0.156$.

Figure 2 (a) shows the phase space trajectories and their time series for the data set (30) at $C = 0.14$. Figure 2 (b) shows the long time behavior in phase space trajectories and their time series for the same data set (30) at $C = 0.14$. The solution is quasi periodic. The value of the expression $a_0a_1 - a_2$ is $-0.0057 < 0,$ at $C = 0.14$



(a) **(b)**

Fig. 1: For the data set (34); (a) 3D behavior (b) time series

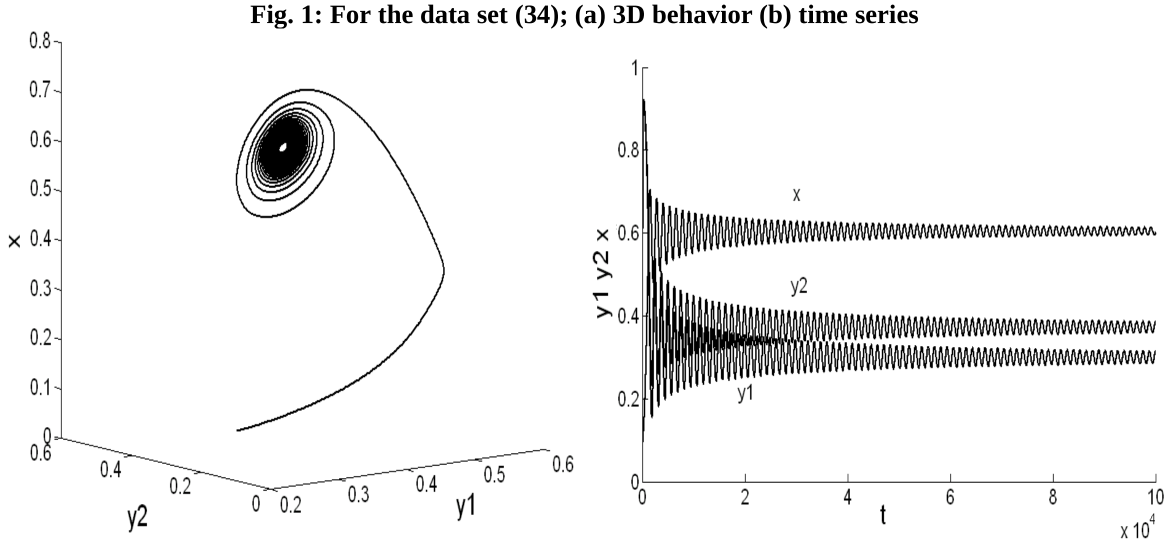


Fig. 2 (a).

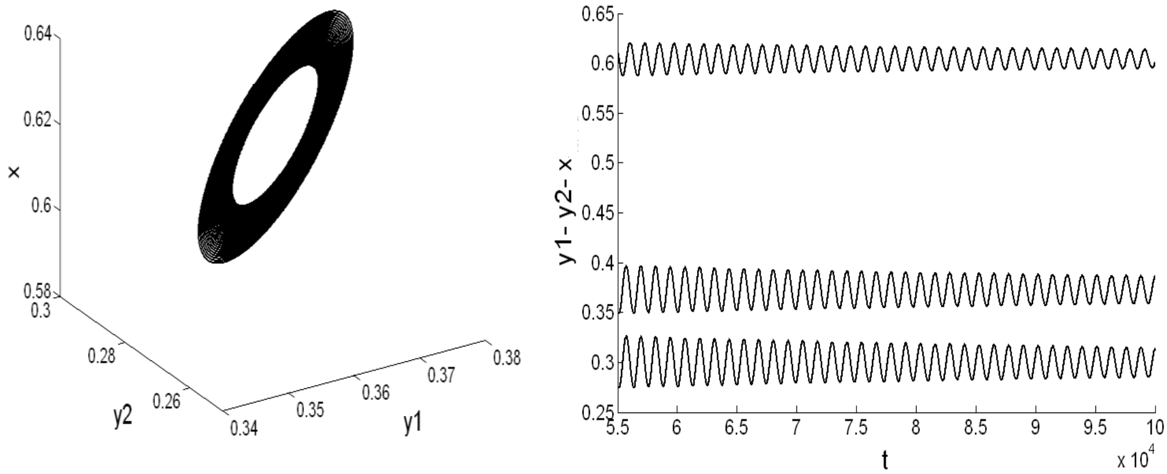


Fig. 2 (b). Quasiperiodic behavior

However, for $C = 0.11$, there exists a limit cycle for the same set of data. As is evident from the Fig 3. The value of the expression $a_0 a_1 - a_2$ is $-0.0077 < 0$, at $C = 0.11$.

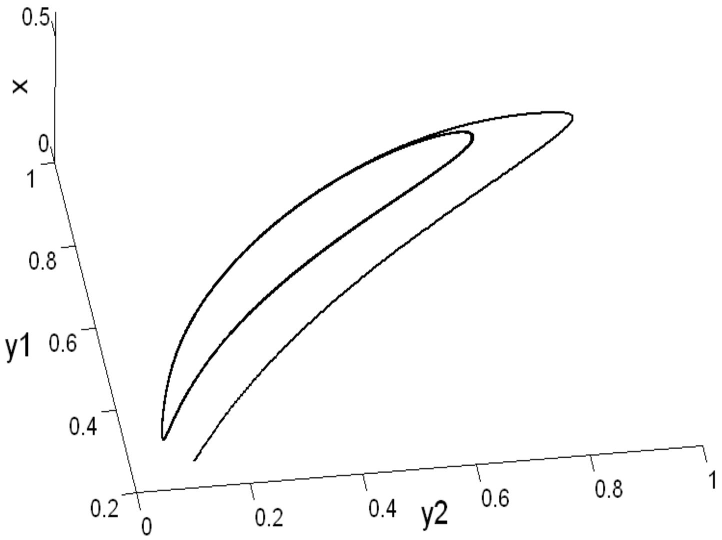


Fig. 3. Limit cycle

VIII. CONCLUSIONS

In this model, separate dynamics of harvesting effort is considered. The local and global persistence of the harvested preys has been analyzed. The optimum population level is investigated. The existence of hopf bifurcation with respect to total cost of fishing is observed.

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