

## Effects of Heat, Mass Transfer and Viscous Dissipation on Unsteady Flow past an Oscillating Infinite Vertical Plate with Variable Temperature through Porous Medium

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**ABSTRACT:** An analysis is performed to study the effect of viscous dissipation on unsteady flow past an oscillating infinite vertical plate through porous medium with variable temperature, heat and mass transfer. The non-linear, coupled partial differential equations together with boundary condition are reduced to dimensionless form. The governing equations are solved numerically using implicit finite difference scheme of Crank-Nicolson. The results are obtained for Velocity, Temperature, and Concentration profiles for different physical parameters like Phase angle, Thermal Grashof number, Modified Grashof number, Permeability parameter, Eckert number, Prandtl number, Schmidt number and time. It is observed that heat and mass transfer, viscous dissipation and porous medium affect the flow pattern significantly.

**Keywords:** Heat Transfer, Mass Transfer, Oscillating plate, viscous dissipation and Porous Medium.

### I. INTRODUCTION

Simultaneous heat and mass transfer from different geometrics embedded in porous media has many engineering and geophysical application such as geothermal reservoirs, drying of porous solids, thermal insulation, enhanced oil recovery, packed – bed catalytic reactors, cooling of nuclear reactors, and underground energy transport. Convection in porous media has applications in geothermal energy recovery, oil extraction, thermal energy storage and flow through filtering devices. Convective heat transfer in porous media has received considerable attention in recent years owing to its importance in various technological applications such as fiber and granular insulation, electronic system cooling, cool combustors, and porous material regenerative heat exchangers. The phenomena of mass transfer are also very common in theory of stellar structure. Viscous mechanical dissipation effects are important in geophysical flows and also in certain industrial operations and are usually characterized by the Eckert number.

Soundalgekar V.M. et al [1], was studied on flow past oscillating plate with variable temperature. Soundalgekar V.M. et al [2], was discussed free convection effects on the flow past a vertical oscillating plate. Vajravelu K., et al [3], presented a solution for Heat transfer in a viscous fluid over a stretching sheet with viscous dissipation and internal heat generation. Pop I, et al [4], was analyzed viscous dissipation effects on unsteady free convection flow past an infinite plate with constant suction and heat source. Tania S. Khaleque, et al [5], was studied Effects of radiation, heat generation and viscous dissipation on MHD free convection flow along a stretching sheet. Mansour M.A., et al [6], was investigated radiative and free convection effects on the oscillatory flow past a vertical plate. Soundalgekar V.M, et al [7], presented a solution for Effects of free convection currents and mass transfer on the flow past a vertical oscillating plate. Soundalgekar V.M, et al [8], discussed Effects of mass transfer on the flow past an oscillating infinite vertical plate with constant heat flux. Bala siddulu malga , et al [9], was analyzed viscous dissipation effect on unsteady free convection and mass transfer flow past an accelerated vertical porous plate with suction. Hitesh Kumar [10], was investigated radiative Heat Transfer with Hydro magnetic flow and viscous dissipation over a stretching surface in the presence of variable heat flux. Israel – cookey, C., Ogulu, A., Omubo – Pepple, V.M.,[11], was studied The influence of viscous dissipation and radiation on unsteady MHD free convection flow past an infinite heated vertical plate in a porous medium with time dependent suction. Mahajan, R.L., Gebhart, B.B.[12] ,presented a solution for viscous dissipation effects in Buoyancy- Induced flows. Zueco Jordan, J., [13], was discussed Network Simulation Method Applied to Radiation and Dissipation Effects on MHD Unsteady Free Convection over Vertical Porous Plate. Anjana Bhattacharya et al, [14] were analyzed Theoretical study of Chemical Reaction Effects on Vertical Oscillating Plate Immersed in a Stably Stratified Fluid. Saraswat Amit and Srivastava R.K.,[15] was discussed Heat and Mass Transfer Effects on Flow past an Oscillating Infinite vertical

plate with Variable Temperature through Porous Media. P.M. Kishore, et al,[16] was studied Effects of Heat transfer and viscous dissipation on MHD free convection flow past an exponentially accelerated vertical plate with variable temperature. V.Rajesh, [17] was investigated Heat Source and Mass Transfer Effects on MHD Flow an Elastic-Viscous Fluid through a porous medium.

The objective of the present paper is to study the unsteady flow of an incompressible viscous fluid past an oscillating vertical plate on taking into account of viscous dissipation, heat and mass in presence of variable temperature. The dimensionless governing equations are solved by using an implicit finite difference method of Crank – Nicolson’s type.

## II. FORMULATION OF THE PROBLEM

Consider the unsteady flow of an incompressible viscous fluid which is initially at rest past an infinite vertical plate with variable temperature through a porous medium. The flow is assumed to be in x-direction, which is taken along the vertical plate in the upward direction. The y-axis is taken be normal to the plate. Initially the plate and the fluid are at same Temperature  $T'_{\infty}$  with same concentration  $C'_{\infty}$  level at all points. At time  $t' > 0$  the plate starts oscillating in its own plane with a velocity  $u = u_0 \cos \omega t'$ . The plate Temperature is raised to  $T'_w$  and the levels of concentration near the plate are raised to  $C'_w$  linearly with time t. The viscosity is taken into account with constant permeability of porous medium. Then by usual Boussinesq’s approximation the unsteady flow is governed by the following equation.

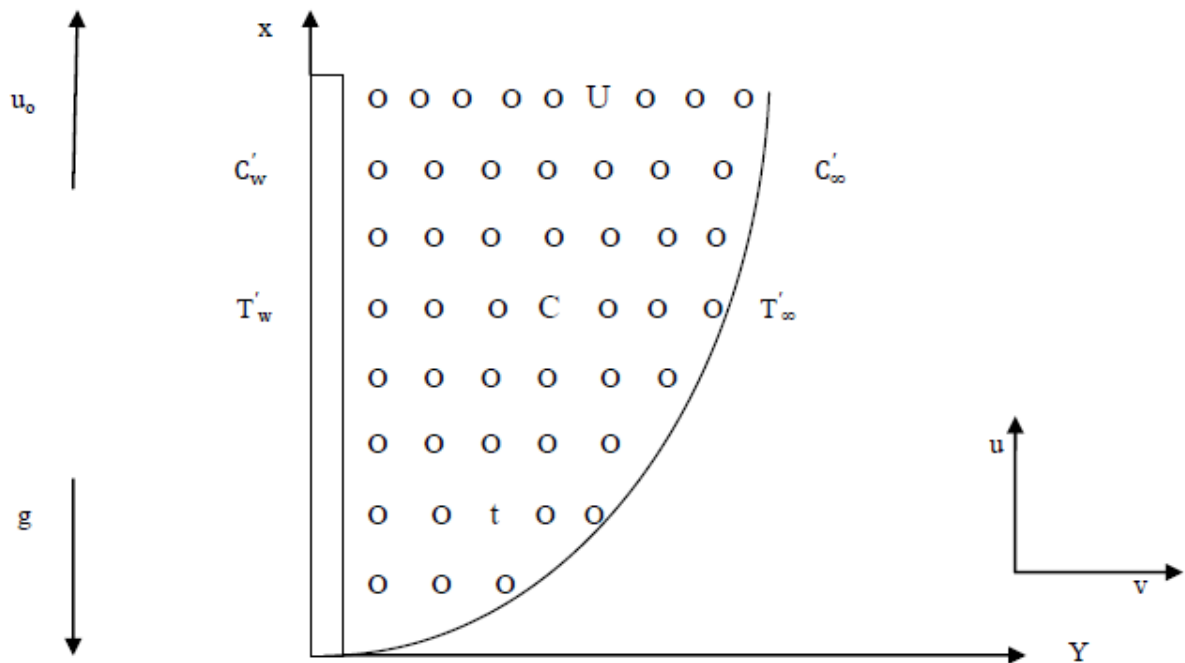


Figure: 1 Physical model of the problem

$$\frac{\partial u}{\partial t'} = g \beta ( T' - T'_{\infty} ) + g \beta^* ( C' - C'_{\infty} ) + \nu \frac{\partial^2 u}{\partial y^2} - v \left( \frac{u}{k'} \right) \quad (1)$$

$$\rho c_p \frac{\partial T'}{\partial t'} = \kappa \frac{\partial^2 T'}{\partial y^2} + \frac{\nu}{c_p} \left( \frac{\partial u}{\partial y} \right)^2 \quad (2)$$

$$\frac{\partial C'}{\partial t'} = D \frac{\partial^2 C'}{\partial y^2} \quad (3)$$

with the following initial and boundary conditions

$$t' \leq 0, \quad u = 0, \quad T' = T'_{\infty}, \quad C' = C'_{\infty} \quad \text{for all } y.$$

$$t' > 0, \quad u = u_0 \cos \omega t', \quad T' = T'_{\infty} + ( T'_w - T'_{\infty} ) A t',$$

$$C' = C'_{\infty} + ( C'_w - C'_{\infty} ) A t', \quad \text{at } y = 0.$$

$$u = 0, \quad T' \rightarrow T'_{\infty}, \quad C' \rightarrow C'_{\infty} \quad \text{as } y \rightarrow \infty, \quad t' > 0. \quad (4)$$

where  $A = \left( \frac{u_0^2}{\nu} \right)$

On introducing the following dimensionless quantities

$$U = \left( \frac{u}{u_0} \right), \quad t = \left( \frac{t' u_0^2}{\nu} \right), \quad Y = \left( \frac{y u_0}{\nu} \right), \quad \Theta = \left[ \frac{T' - T'_{\infty}}{T'_w - T'_{\infty}} \right], \quad \omega = \left( \frac{\nu \omega'}{u_0^2} \right)$$

$$S_c = \left( \frac{\nu}{D} \right), \quad P_r = \left( \frac{\mu c_p}{k} \right), \quad G_r = \left[ \frac{g \beta \nu (T'_w - T'_{\infty})}{u_0^3} \right], \quad C = \left[ \frac{C' - C'_{\infty}}{C'_w - C'_{\infty}} \right]$$

$$G_c = \left[ \frac{g \beta^* \nu (C'_w - C'_{\infty})}{u_0^3} \right], \quad \frac{1}{K} = \left( \frac{u_0^2 k'}{\nu^2} \right), \quad E_c = \frac{u_0^2}{c_p (T'_w - T'_{\infty})} \quad (5)$$

in equations (1) –(3) leads to

$$\frac{\partial U}{\partial t} = Gr \Theta + Gc C + \frac{\partial^2 U}{\partial Y^2} - K U \quad (6)$$

$$\frac{\partial \Theta}{\partial t} = \frac{1}{Pr} \frac{\partial^2 \Theta}{\partial Y^2} + E_c \left( \frac{\partial U}{\partial Y} \right)^2 \quad (7)$$

$$\frac{\partial C}{\partial t} = \frac{1}{Sc} \frac{\partial^2 C}{\partial Y^2} \quad (8)$$

Initial and boundary conditions in non dimensional form are

$$\begin{aligned} U = 0, \quad \Theta = 0, \quad C = 0, \quad \text{for all } Y \leq 0, \quad t \leq 0 \\ U = \cos \omega t, \quad \Theta = t, \quad C = t, \quad \text{at } Y = 0, \quad t > 0 \\ U = 0, \quad \Theta \rightarrow 0, \quad C \rightarrow 0, \quad \text{as } Y \rightarrow \infty, \quad t > 0 \end{aligned} \quad (9)$$

### III. METHOD OF SOLUTION

Numerical technique: Equations (6) – (8) are coupled non-linear partial differential equations and are to be solved under the initial and boundary conditions of equations (9). However exact or approximate solutions are not possible for this set of equations and hence we use implicit finite difference technique of Crank-Nicolson method. The finite difference equations corresponding to equations (6) – (8) are as follows

$$\begin{aligned} \left( \frac{u_{i,j+1} - u_{i,j}}{\Delta t} \right) = \frac{1}{2} \left( \frac{u_{i-1,j} - 2u_{i,j} + u_{i+1,j} + u_{i+1,j+1} - 2u_{i,j+1} + u_{i-1,j+1}}{2 (\Delta Y)^2} \right) \\ + G_r \left( \frac{\theta_{i,j+1} + \theta_{i,j}}{2} \right) + G_c \left( \frac{C_{i,j+1} + C_{i,j}}{2} \right) - K \left( \frac{u_{i,j+1} + u_{i,j}}{2} \right) \end{aligned} \quad (10)$$

$$\begin{aligned} \left( \frac{\theta_{i,j+1} - \theta_{i,j}}{\Delta t} \right) = \frac{1}{2 Pr} \left( \frac{\theta_{i+1,j} - 2\theta_{i,j} + \theta_{i-1,j} + \theta_{i+1,j+1} - 2\theta_{i,j+1} + \theta_{i-1,j+1}}{2 (\Delta Y)^2} \right) \\ + E_c \left( \frac{u_{i+1,j} - u_{i,j}}{\Delta Y} \right)^2 \end{aligned} \quad (11)$$

$$\left( \frac{C_{i,j+1} - C_{i,j}}{\Delta t} \right) = \frac{1}{2 S_c} \left( \frac{C_{i+1,j} - 2C_{i,j} + C_{i-1,j} + C_{i+1,j+1} - 2C_{i,j+1} + C_{i-1,j+1}}{2 (\Delta Y)^2} \right) \quad (12)$$

Initial and boundary conditions are in the following form

$$\begin{aligned}
 u_{i,0} &= 0, & \theta_{i,0} &= 0, & C_{i,0} &= 0 \\
 u_{0,j} &= \cos\omega t, & \theta_{0,j} &= j\Delta t, & C_{0,j} &= j\Delta t \\
 u_{L,j} &\rightarrow 0, & \theta_{L,j} &\rightarrow 0, & C_{L,j} &\rightarrow 0
 \end{aligned}
 \tag{13}$$

Here the suffix  $i$  corresponds to  $Y$ ,  $j$  corresponds to  $t$  and  $L \rightarrow \infty$ . Also  $\Delta t = t_{j+1} - t_j$  and  $\Delta Y = Y_{i+1} - Y_i$  knowing the values of  $C, \Theta, u$ , at a time  $t$ , we can calculate the values at a time  $t + \Delta t$  as follows we substitute  $i = 1, 2, 3, \dots, N-1$  in equation (12) which results in a tridiagonal system of equations in unknown values of  $C$ . using initial and boundary conditions the system can be solved by Thomas algorithm. Thus  $C$  is known at all values of  $Y$  at a time  $t + \Delta t$ . Then the knowing the values of  $C$  and applying the same procedure and using boundary conditions we calculate  $\Theta$  from equation (11) the knowing the values of  $\Theta$  and applying the same procedure and using boundary conditions we calculate  $u$  from equation (10). This procedure is continued to obtain the solution till desired time  $t$ .

#### IV. RESULT AND DISCUSSION

The velocity, Temperature and concentration profiles have been computed by using Crank- Nicolson finite difference method. The numerical calculations are carried out for the effect of the flow parameters such as Phase angle ( $\omega t = \pi/4$ ), Thermal Grashof Number ( $Gr = 2$ ), Modified Grashof Number ( $Gc = 5$ ), Prandtl Number ( $Pr = 0.71$ (air)  $Pr = 7$ (water)), Schmidt Number ( $Sc = 0.6$ ), Eckert Number ( $Ec = 0.5$ ), Permeability parameter ( $K = 0.5$ ), time ( $t = 0.2$ ). The velocity, temperature and concentration profiles has been studied and presented in fig 2 to 9.

- ❖ **Effect of Phase angle ( $\omega t$ ) :** The velocity profile for different values of phase angle ( $\omega t = 0, \pi/6, \pi/4, \pi/2$ ) is presented in Figure 2. It is observed from the figure that the velocity decreases with increasing phase angle ( $\omega t$ )
- ❖ **Effect of Thermal Grashof Number ( $Gr$ ) :** The velocity profile for different values of thermal Grashof Number ( $Gr = -10, -5, 5, 10$ ) is presented in Figure 3. It is observed from the figure that the velocity increases with increasing Thermal Grashof Number ( $Gr$ ).
- ❖ **Effect of Modified Grashof Number ( $Gc$ ) :** The velocity profile for different values of modified Grashof Number ( $Gc = -5, -3, 3, 5$ ) is presented in Figure 4. It is observed from the figure that The velocity increases with increasing modified Grashof Number ( $Gc$ ).
- ❖ **Effect of permeability parameter ( $K$ ) :** For the different values of permeability ( $K = 5, 10, 15$ ) the velocity profile is drawn and is presented in Figure 5. It is observed from the figure that the velocity decreases with increasing permeability parameter ( $K$ ).
- ❖ **Effect of time ( $t$ ) :** The velocity profile for different values of time ( $t = 0.2, 0.4, 0.6, 0.8$ ) is shown in Figure 6. It is clear that the velocity profile decreases with increasing time ( $t$ ).
- ❖ **Effect of Eckert Number ( $Ec$ ) :** The Temperature profile for different values of Eckert Number ( $Ec = 0.5, 2, 4$ ) is represented in Figure 7 and it is shown that the Temperature profile increases with increasing Eckert Number ( $Ec$ ).
- ❖ **Effect of prandtl Number ( $Pr$ ) :** The Temperature profile for different values of Prandtl Number ( $Pr = 0.71, 7.0, 11.4$ ) is shown in Figure 8 and it is observed that the Temperature decreases with increasing Prandtl Number ( $Pr$ ).
- ❖ **Effect of Schmidt Number ( $Sc$ ) :** The Concentration profile for different values of Schmidt Number ( $Sc = 0.22, 0.6, 0.78, 0.96$ ) is presented in Figure 9. It is observed from the figure that the concentration decreases with increasing Schmidt Number ( $Sc$ ).

### V. CONCLUSIONS

The effect of different parameters like, phase angle( $\omega t$ ), Thermal Grashof Number( $Gr$ ), modified Grashof Number( $Gc$ ), Permeability parameter( $k$ ), Eckert Number( $Ec$ ), Prandlt Number( $Pr$ ), Schmidt Number( $Sc$ ), and time( $t$ ) are shown through graphs.

- It is clearly observed the the velocity profile increases when Thermal Grashof Number ( $Gr$ ) and modified Grashof Number ( $Gc$ ) increases. But velocity profile decreases when Phase angle ( $\omega t$ ), Permeability parameter ( $k$ ), time ( $t$ ) increases.
- It is clearly observed the Temperature profile increases when Eckert Number ( $Ec$ ) increases. But Temperature profile decreases when Prandlt Number ( $Pr$ ) increases.
- It is clearly observed the concentration profile decreases when Schmidt Number ( $Sc$ ) increases.

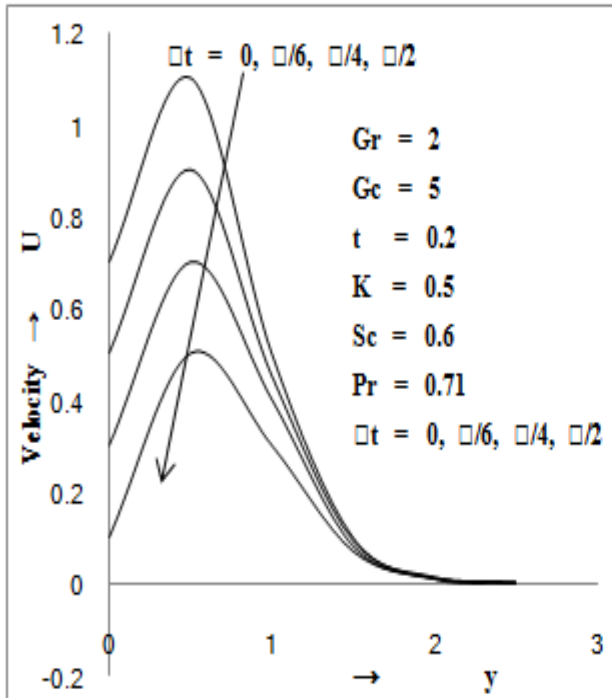


Fig 2: Velocity profile for different values of ( $\omega t$ )

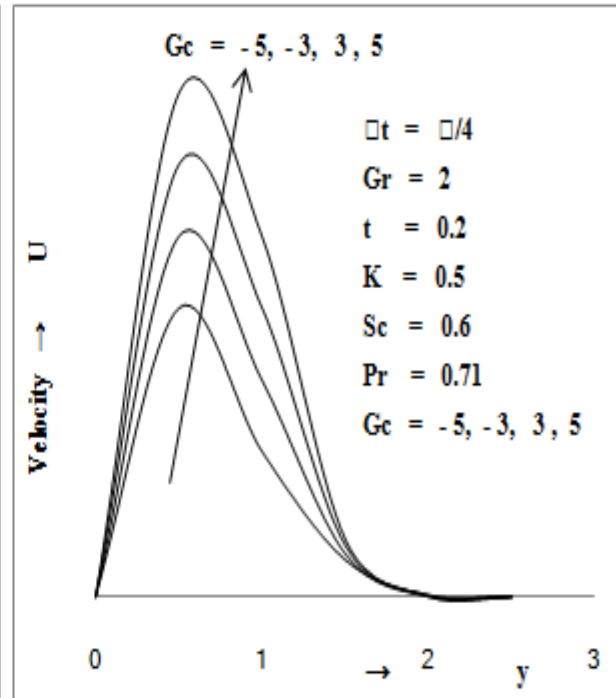


Fig 4: Velocity profile for different values of  $Gc$

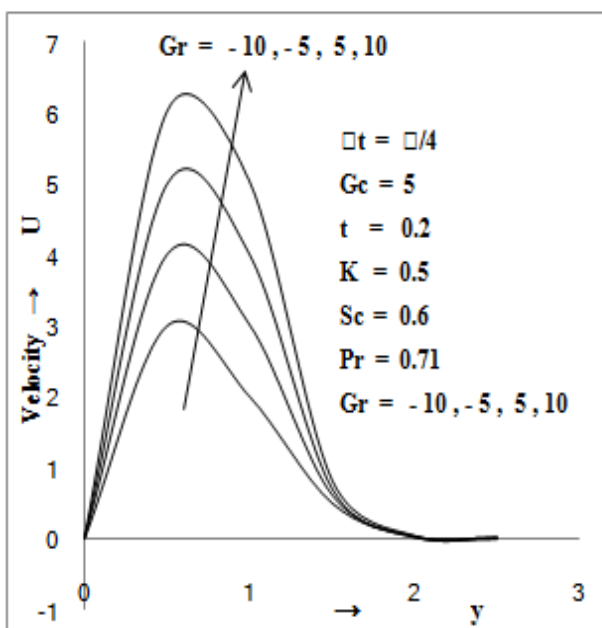


Fig 3: Velocity profile for different values of  $Gr$

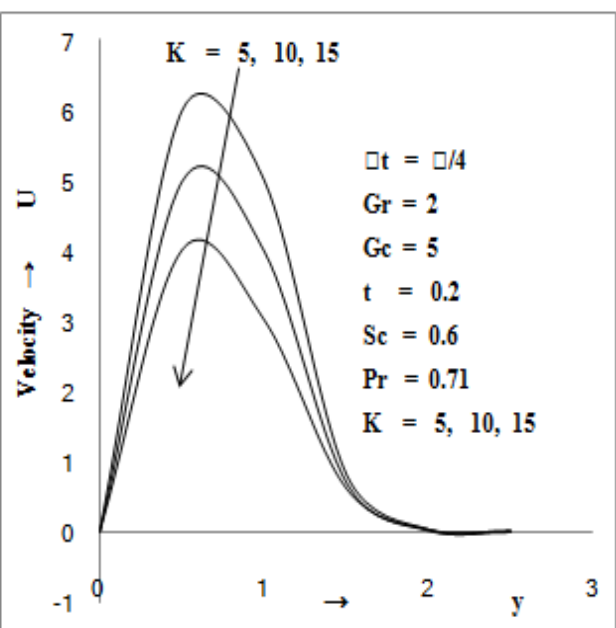


Fig 5: Velocity profile for different values of  $K$

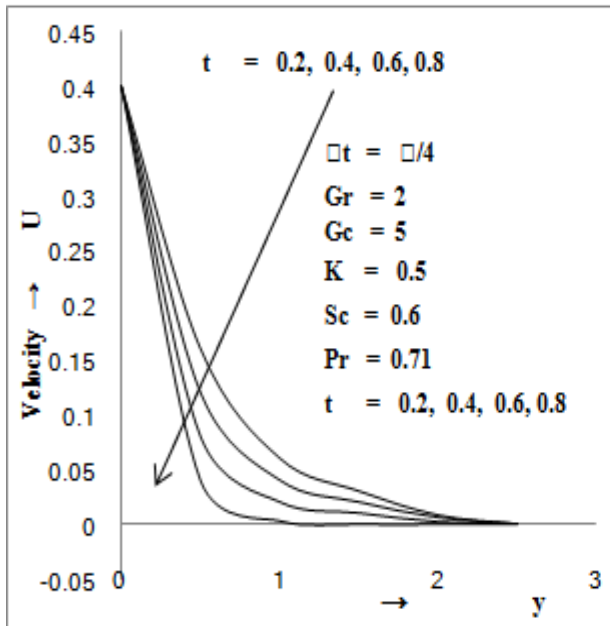


Fig-6: Velocity profile for different values of  $t$

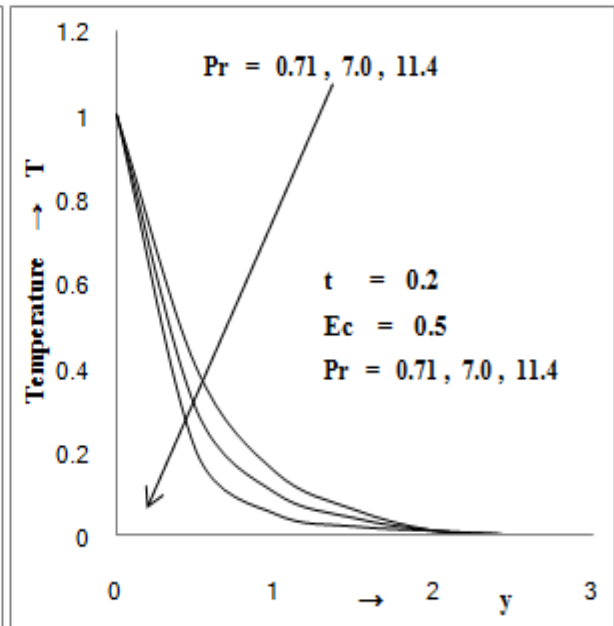


Fig 8: Temperature profile for different values of  $Pr$

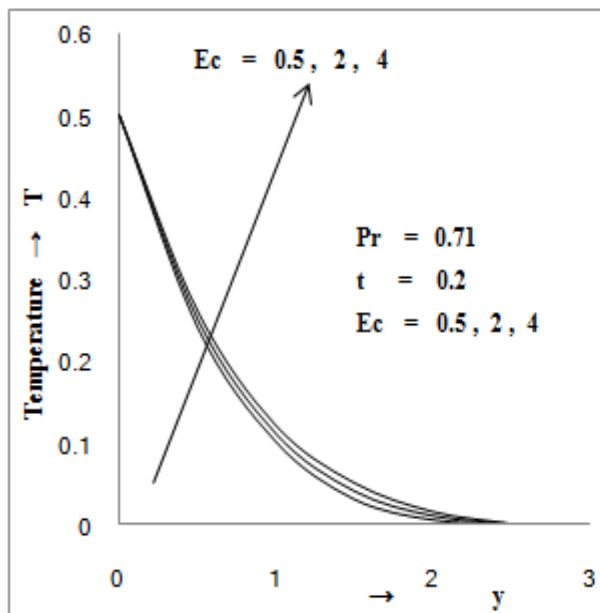


Fig 7: Temperature profile for different values of  $Ec$

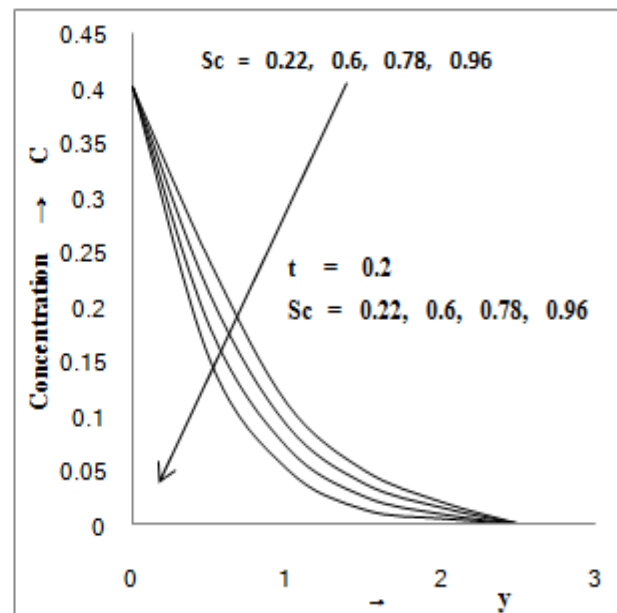


Fig 9: Concentration profile for different values of  $Sc$

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