Theoretical Development of Ellipse Generating Mechanism

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ABSTRACT: This paper presents the development of ellipse generating mechanism. The various components of mechanism are explained and relative motions of these components are described. Design of string and spring on the basis of their dimensions and deformation has been performed. Velocity analysis of various components of the mechanism has been performed. Also analysis for selection of electric motor on the basis of stall torque and speed has been carried out.

Keywords: Ellipse, Kinematic analysis, Pointer.

I. INTRODUCTION

In our day to day life elliptical shapes are used in tables, kitchens, logos, showcases etc. for their better aesthetic look. Drawing a perfect ellipse is a difficult task thus for drawing perfect ellipse, specific mechanism like elliptical trammel which is inversion of double slider crank mechanism is used [1].But in elliptical trammel manual input is needed. To make elliptical trammel automated efforts are being made using rack and pinion but it adds to mechanism cost and manufacturability [2]. In this paper theoretical development of ellipse generating mechanism is given which works on property of ellipse that sum of focal distances of a point on ellipse is constant and is equal to length of major axis. According to our requirements of dimensions of ellipse, various ellipses can be generated for this mechanism. Input to this mechanism is given by electric motor.

In this way chapter II explains ellipse and its property. Chapter III explains system description in which relative positions and relative motions of various components is explained. Chapter IV includes design of different system components.

II. ELLIPSE

An ellipse is locus of a point in a plane which moves so that its distance from a fixed point bears a constant ratio e (less than 1) to its distance from fixed line called as directrix. The fixed point is called the focus and is denoted by S; the constant ratio is called as eccentricity and is denoted by e.



The standard equation of an ellipse is given by equation (1), where a is semi major axis, b is semi minor axis and a is greater than b,

$$\frac{x^2}{x^2} + \frac{y^2}{y^2} = 1.....(1)$$

The sum of the focal distances of a point for all points on the ellipse remains constant and is equal to length of major axis,

 $S_1P + S_2P = 2a = Constant...(2)$

If the two ends of an inelastic string having length equal to major axis are held at the focus, the ellipse can be traced with the pointer by passing the string through the pointer and gently following the path, provided the string should not contain any slack and whirl.

III. SYSTEM DESCRIPTION



Figure 2. Schematic diagram of ellipse generating mechanism

1. StepperMotor

Stepper motor has rotor movement in discrete steps. The angular rotation is determined by the number of pulses fed into the control circuit. Each input pulse initiates the drive circuit which produces one step of angular movement. The angle by which the rotor of the stepper motor moves when one pulse is applied to the input is called step angle. To achieve a smoother movement of rotor, we have to increase the resolution of motor by reducing the step angle. Higher the resolution, greater the accuracy of the positioning of objects by the motor. In stepper motor step angle can be achieved up to 0.36° , it will have 1000 steps in one revolution and greater resolution [4].

2.Crank

The link which makes complete revolution is called crank. Input for rotation of crank is given by stepper motor.

3. Extension Spring

Extension spring absorbs and stores energy as well as creates resistance to a pulling force. Free Length is overall length of spring in unloaded condition or position and is denoted by *l*.Extended Lengthis the length at full rated extension and is denoted by $l + \delta$.

4. Pointer

Pointer will trace an ellipse. For drawing ellipse pointer can be replaced by pen or chalk. For cutting operations, cutting tool can be used in place of pointer.

5. String

String used in this mechanism is inelastic. While tracing an ellipse string should not be slack or whirl. According to the dimensions of ellipse, length of the string will have to be varied.

6. Base

Whole mechanism is mounted on the base. Scales along with sliders are mounted in opposite direction in diametrically opposite directions at the origin on the base as shown in Fig 3. With the help of locking screw, sliders can be fixed at a particular distance along the scale. The ends of string are attached to respective sliders at the points which serve the purpose of foci of ellipse.



Figure 3. Arrangement of scales, string and pointer

Electric motor is fixed to the base such that its axis coincides with the origin. Crank is attached to the electric motor. One end of the spring is attached to the crank and other end to pointer. The string is passed through the pointer and its ends fixed to slider which is fixed with locking screw at foci of ellipse to be drawn. The entire assembly is shown in Fig. 2.

As the crank rotates, its end traces the circle and pointer traces an ellipse. By giving the pulses to the stepper motor corresponding to the subscribed angle and setting the crank at proper position some part of the ellipse can also be drawn using this mechanism.

IV. COMPONENT DESIGN

1. Slider

As the sliders serve the purpose of the foci of the ellipse sliders are fixed at the foci $S_1(-ae, 0)$ and $S_2(ae, 0)$ of the ellipse which is to be traced.

2. String

Length of the string required to the draw a particular ellipse is equal to 2a as given by equation (2).

3. Spring

 P_1 is the vertex of ellipse on abscissa and P_2 is the vertex of ellipse on ordinate. As the crank rotates, deformation of spring varies. At point P_1 deformation of spring is minimum, assuming it as zero. Thus $\delta_1 = 0$. Hence, length of spring at P_1 is free length. At point P_2 , deformation of spring is maximum $\delta_2 = a - b$ Length of spring at point P_2 is extended length.

Spring of this mechanism should be selected such that its maximum possible deformation is greater than δ_2 .

4. Stepper motor

Stepper motor is selected according to our requirement of stall torque and speed of revolution. 4.1 Stall Torque

Stall torque is the torque produced by mechanical device whose output rotational speed is zero.

Assumptions for Torque calculation:

- Mass of cutting tool assembly is more than mass of spring.
- Centre of gravity of cutting tool assembly lies on point of cutting.
- Weight distribution of spring and crank is uniform along its length.

The required torque of the motor at a particular crank angle while tracing the ellipse is given by equation (3) $T = m c \bar{u}$

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T = m. g. \bar{x}....(3)
Where, m = Mass of whole assembly of cutting tool, spring and crank
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 \bar{x} =Combined centre of gravity of whole assembly



Figure 4. Mass distribution of cutting tool assembly, spring and crank

Distance of combined center of gravity of whole assembly,

$$\bar{x} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}$$

Where, $m_1 = Mass$ of pointer

 m_2 = Mass of spring

 $m_3 = Mass of crank$

 x_1 =Distance of centre of gravity of cutting tool from origin

 x_2 = Distance of centre of gravity of spring from origin = $r - \frac{(l+\delta)}{2}$

 x_3 =Distance of centre of gravity of crank from origin = $\frac{r}{2}$

The value of \bar{x} changes with the values of x_1 and x_2 . As m_1 is more than m_2 , dependence of \bar{x} is more on x_1 than x_2 . Maximum value of x_1 is at point P₁ thus $x_1 = a$. At point P₁, $\delta = 0$, thus $x_2 = r - \frac{l}{2}$

The stall torque of stepper motor should be greater than or equal to the maximum value of torque obtained by equation (4).

4.2 Speed of rotation

Maximum feedrate of depends on tool material and material to be cut. The velocity of feed of cutting tool should not exceed the maximum feed rate at any point while cutting.



Figure. 5 Velocity analysis

Velocity of feed of cutting tool is given by equation (5)

Where, ω = Angular velocity of crank

Maximum feed rate of cutting tool occurs when x_1 is maximum and at point P₁, x_1 is maximum.

$$v_{p_{max}} = \omega a$$

Taking maximum value of feed of cutting tool equal to maximum feed rate,

$$\frac{2\pi Na}{60} = v_f$$
$$N = \frac{60v_f}{2\pi a}$$

Speed of motor in revolution per minute should not exceed the value obtained in the above equation.

V. CONCLUSION

In this way this paper discusses about development of a mechanism for drawing and cutting of ellipse of various dimensions which are used in industries and furniture works. This mechanism uses electric motor as an input instead of manual input. This mechanism uses string and spring as chief components which has to be replaced after prolonged use because of permanent deformation which may lead to error. This paper talks about kinematic analysis of various components of mechanism. Few suggestions for future work include analysis of various components on strength basis.

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