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Some New Operations on Fuzzy Soft Sets

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Abstract: *Fuzzy soft set is one of the recent topics developed for dealing with the uncertainties present in most of our real life situations. The parameterization tool of soft set theory enhances the flexibility of its application. In this paper, we have studied membership grade, power set, α -cut set , strong fuzzy α -cut set ,some standard operation fuzzy soft set, degree of subset hood and proposed some results with examples.*
Keywords: *Fuzzy set, Soft sets, fuzzy soft class, standard union fuzzy soft set, standard intersection fuzzy soft set*

I. Introduction

In many complicated problems arising in the fields of engineering, social science, economics, environment, medical science etc. have various uncertainties. Fuzzy soft sets are very useful structures arising in many areas of mathematics and computer science. The theory soft sets was initiated by Molodtsov[4] in 1999 for modeling uncertainty present in real life. Roughly speaking a soft set is a parameterized classification of the objects of the universe. He has shown several applications of soft sets in different areas like integration, decision making etc. In 2003, Maji, Biswas and Ray [5] studied the theory of soft sets initiated by Molodtsov. They defined equality of two soft sets, subset and super set of a set, complement of a soft set , null soft set and absolute soft set with examples. Soft binary operations like AND, OR and also the operations of union, intersection were also defined. In 2005, Pie and Miao [10] and Chen et al [2] improved the work of Maji et al [5]. In 2012, Borah, Neog and Sut [9] studied the theory of soft sets initiated by Molodtsov [4] and put forwarded some more propositions regarding fuzzy soft set. In this paper, an analysis has been made to discuss about some properties related to fuzzy soft sets along with examples and proofs of certain results.

II. Preliminaries

Definition 2.1[1]

Let U be an initial universal set and E be a set of parameters. Let P (U) denote the power set of U. A pair (F, A) is called a soft set over U if F is a mapping given by F: A→ $\tilde{P}$ (U) ,Where A $\tilde{\subset}$ E

Definition 2.2[6]

Let U be a universe and E a set of attributes. Then the pair (U, E) denotes the collection of all fuzzy soft sets on U with attributes from E and is called a fuzzy soft class.

Definition 2.3[6]

For two fuzzy soft sets (F, A) and (G,B) over a common universe U , we say that (F,A) is a fuzzy soft subset of (G,B) if

i) A $\tilde{\subset}$ B

ii) For all ε Є A, F(ε) is a sub set G(ε) and is written as F(A) $\tilde{\subset}$ (G,B)

Definition 2.4[6]

For two fuzzy soft sets (F, A) and (G, B) over a common universe U are said to be soft set equal if (F,A) is a soft subset of (G,B) and (G,B) is a soft subset of (F,A)

Definition 2.6[6]

Intersection of two fuzzy soft sets (F,A) and (G,B) over a common universe U is a fuzzy soft set (H,C), Where C= A ∩ B and ε Є C, H(ε)= F(ε) ∩ G(ε) and is written as (F,A)∩ (G,B) = (H,C)

III. Main Result

Definition3.1

let X={x} be a collection of objects denoted by x than a fuzzy soft set Ă in X is a set of ordered pairs Ă = {x, $\mu_{Ă}(x)$:x ∈ X}, where $\mu_{Ă}(x)$ is called the membership grade of x in Ă.

Example 3.1

Let X= {p, q ,r, s} and A:X $\rightarrow$I defined A(p)=o.1 A(q)=0.2 ,A(r)= 0.3 A(s)=0.4. Then fuzzy soft set A can be written as A= {(p, 0.1),(q,0.2),(r,0.3)(s,0.4)}

Definition3.2

Let X be a domain. Then set of all fuzzy soft subsets of X is called the fuzzy soft power set.

Example 3.2 Let X= {a, b} be a fuzzy soft set Then fuzzy soft power set is { ϕ ,[a],[b],[a, b]}

Definition 3.3

A fuzzy soft set a is defined on X .The α -cut set ${}^{\alpha}A$ is made up of members X. Whose members grade is not less than α ,therefore ${}^{\alpha}A$ = { x$\in$x, A(x) $\geq$ α } Where α $\in$[0,1]

Example3.3 Let x={p, q, r, s} and fuzzy soft set A= $\frac{0.9}{p}+\frac{1.0}{q}+\frac{0.2}{r}+\frac{0.1}{s}$ and α -cut set for α =0.1,0.2,0.9,1.0 than ${}^{1.0}A$={q}, ${}^{0.9}A$=[p, q}, ${}^{0.2}A$={p, q, r}, ${}^{0.1}A$={p, q, r, s}=X

Definition 3.4

Let A be a fuzzy soft set on X. Then strong fuzzy soft α -cut set ${}^{\alpha+}A$ is made up of member of X. whose membership grade is greater than α i. e ${}^{\alpha+}A$={x$\in$X, A(x)$>\alpha$ } where α $\in$[0, 1]

Example 3.4 In Examle3.3 the strong α -cut set is ${}^{1.0+}A$={q], ${}^{0.9+}A$={q], ${}^{0.2+}A$=[p,q}, ${}^{0.1+}A$={p,q,r}

Definition3.5

The set of all levels α $\in$[0,1] that represents distinct α -cut set of a given fuzzy soft set A is called a fuzzy soft level set of A then Λ(A) ={ α : A(x)= α for all x$\in$X}

Example3.5 Let X= {a, b, c, d} and fuzzy soft set A = $\frac{0.1}{a}+\frac{0.4}{b}+\frac{0.3}{c}+\frac{0.5}{d}+\frac{0.3}{e}$ then fuzzy soft level set of A= {0.1, 0.3, 0.4, 0.5}

Definition3.6

The support of a fuzzy soft set A over the universal set X is the crisp set that contains all the elements that have non-zero membership grades in A and is denoted by Supp(A) i.e. Supp(A)={ x$\in$X, A(x) >0}

Definition3.7

Let A be a fuzzy soft set over universal set X. than the set of all those elements of X whose membership grade is 1 called core of fuzzy soft set A and is denoted by core(A) i. e core(A) ={ x$\in$X, A(x) =1}

Definition3.8

Let A be a fuzzy soft set over a universal set X. Then the height of A is the largest membership grade obtained by any element in that set and is denoted by h(A) i.e h(A) =$\text{supp}_{x\in X}$A(x)

Example3.6 If A= {0,0.2,0.8} than h(A)=0.8

Definition3.9

A fuzzy soft set A of a classical set X is called fuzzy soft normal if there exists an x$\in$X such that A(x) =1

Example3.7 If a= {0, 0.1, 0.2, 1.0} than A(x) =1

IV. Standard Operation Of Fuzzy Soft Set

Definition4.1

Let A be a fuzzy soft subset of a nonempty set X. Then the complement of A is denoted by $\bar{A}$ and is denoted by $\bar{A}$= 1-A(x) $\forall$ x$\in$X

Definition4.2

Let A and B are two fuzzy soft subset over nonempty set X. Then the standard union of A and B are denoted by $(A\tilde{\bigcup}B)(x)=\max[A(x),B(x)]\forall x\in X$

Definition4.3

Let A and B are two fuzzy soft subset over nonempty set X. Then the standard intersection of A and B are denoted by $(A\tilde{\bigcap}B)(x)=\min[A(x),B(x)]\forall x\in X$

Proposition 4.1 Let A, B be fuzzy soft sets over a universal set X then $|A|+|B|=|A\bigcup B|+|A\bigcap B|$

$$\Pr oof : \left|A \bigcup B\right| = \sum_{x \in X} \max[A(x), B(x)]$$

$$\left|A \bigcap B\right| = \sum_{x \in X} \min[A(x), B(x)]$$

$$\left|A \bigcup B\right| + \left|A \bigcap B\right| = \sum_{x \in X} \max[A(x), B(x)] + \sum_{x \in X} \min[A(x), B(x)]$$

$$= \sum_{x \in X} [A(x) + B(x)] \text{ or } \sum_{x \in X} [B(x) + A(x)]$$

$$= \sum_{x \in X} A(x) + \sum_{x \in X} B(x)$$

$$= |A| + |B|$$

Definition4.4

Let A be a fuzzy soft subset over a finite set X then scalar cardinality is denoted by $|A| = \sum_{x \in X} A(x)$

Example4.4 If A= {0.1, 0.2, 0.3} be a fuzzy soft set then $|A| =$ 0.1+0.2+0.3+0.6

Definition4.5

Let A be a fuzzy soft set over a finite set X is obtained by computing the magnitude of fuzzy soft set A with the universal set X and is denoted by $\|A\| = \frac{|A|}{|X|}$

Example4.4 If A= {0.1, 0.2, 0.8, 0.3} be a fuzzy soft set then $|A| =$ 0.1+0.2+0.8+0.3=1.4

$|x| = 4$, therefore $\|A\| = \frac{|A|}{|X|} = \frac{1.4}{4} = .35$

V. Degree Of Subset Hood

Definition5.1

The support of a fuzzy soft set A over a finite universal set X is the crisp set that contains all elements of X that have non zero membership grades in A i.e. supp(A) ={ x∈X, A(x) >0}

Definition5.2

The height of a fuzzy soft set A is the largest membership grades obtained by any element in that set and is denoted by h(A)=sup(A)

Definition5.3

Let A and B are two fuzzy soft sets of universal set X. Then degree of subset hood denoted by S(A,B) is defined as $S(A,B) = \frac{1}{|A|}[|A| - \sum_{x \in X} \max[0, A(x) - B(x)]$

Preposition 5.13 Let A and B are two fuzzy soft sets of universal set X . Then degree of subset hood denoted by S(A,B) is defined as $S(A,B) = \frac{1}{|A|}[|A| - \sum_{x \in X} \max[0, A(x) - B(x)]$ or $S(A,B) = \frac{|A \cap B|}{|A|}$

Proof: $S(A,B) = \frac{1}{|A|}[|A| - \sum_{x \in X} \max[0, A(x) - B(x)]$ ……………………………(1)

We have $[|A| - \sum_{x \in X} \max[0, A(x) - B(x)]]$

$$=[\sum_{x \in X} A(x) - \sum_{x \in X} \max[0, A(x) - B(x)]]$$

$$= \sum_{x \in X} [A(x) - \max[0, A(x) - B(x)]]$$

$$= \sum_{x \in X} \min[A(x) - 0, A(x) - \{A(x) - B(x)\}]$$

$= \sum_{x \in X} \min[A(x), B(x)]$

$= |A \cap B|$ ……………………………………………………………………(2)

From (1) and (2), we get $S(A,B) = \dfrac{|A \cap B|}{|A|}$

VI. Conclusion

In this paper, we have given definitions of power soft set, core of fuzzy soft set, strong fuzzy α -cut set, standard operations of fuzzy soft set and illustrate with some examples. We have introduced the concept fuzzy soft set, degree of subset hood. It is hoped that our findings will help enhancing this study on fuzzy soft sets for the researchers.

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