Investigating Waste Water Treatment in a Closed Environment

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Abstract: *This paper focuses on the investigation of the claim of 'The Sound and Environmental Water and Effluent Re-cycling company' that water can be completely purified if a population of at least 10 organisms per cubic centimeter can be maintained in its cleaning tank for a period of at least 8 days. We investigated the growth patterns of each of these organisms and make recommendations about which one will provide the most efficient water treatment and how many organisms should be introduced initially.*

Keyword: *Environment, Waste, Micro-organisms and Water Treatment,*

I. INTRODUCTION

The importance of these kinds of investigations as well as the complimentary experimental works, say in large-scale water treatment and redistribution networks, thus makes them of great relevance [1-6]. It should however be remarked that to date, the literature on the waste water treatment is still quite sparse. This then forms the crux of the current investigation [8]. Untreated water supply causes a serious socio-ecological hazard. If the problem is not carefully controlled and monitored, large communities can be exposed to extensive health risks. It then requires taking immediate corrective measure to redress and mitigate against its impact in the society [9].

The use of micro organisms to clean waste water and make it safe for re-use is good idea for water treatment. The micro-organisms feed on harmful bacteria, thus removing them from the water. The efficiency of this method depends upon choosing the correct species of micro-organism and ensuring that it is able to thrive in the tank of waste water successfully for long enough to consume all the bacteria [7]. However, it is important that the population declines after completing its function, since the ecological balance could be upset by discharge number of micro-organism into the environment.

II. MODEL EQUATION

The growth of micro-organism in a closed environment (such as the cleaning tank) is determined by three principle factors.

- The reproduction rate
- The effect of increased crowding
- The build-up of toxins produced as waste products by the organisms.

This can be modeled by the equation [9]

$$\frac{du}{dt} = au - bu^2 - cu\int_0^t u(\tau)d\tau \qquad (1)$$

Where $u(t)$ the number of organisms per cubic centimeter of water is, a is the reproduction ratio, b is the crowding coefficient, and c is a measure of the sensitivity of the organisms to toxins. The integral term is necessary since the toxins in the environment are built up gradually over a period of time at a rate proportional to the number of organisms present.

The model equation in (1) which is in Integra-differential equation is reduced to differential equations of the form

Let $x(t) = \int_0^t u(\tau)d\tau$

$$\frac{dx}{dt} = \int_0^t \frac{d}{dt}u(\tau)d\tau + u(t)$$

$$\frac{dx}{dt} = u(t)$$

$$x' = u \qquad (2)$$

Then (1) becomes

$$\frac{du}{dt} = au - bu^2 - cux$$

$$u' = au - bu^2 - cux \qquad (3)$$

III. NUMERICAL SOLUTION

The claim of SEWER scientists that water can be completely purified if a population of at least 10 organisms per cubic centimeter can be maintained in its cleaning tank for a period of at least 8 days. We investigated by a MATLAB code for equations (2) and (3) that give the plots below on which the investigation is made.

The maximum permissible population within water discharged into the environment is 0.05 per cubic centimeter. We considered the micro-organism (multiplicanda numerosa, solo anti-socials and reproductio sensitova) whose growth is determined by the three principle factors a, b and c defined above. Alternatively, they could use the combination of these three. These can be categorized into 7 cases for the organism coefficients.

Case 1
Multiplicanda numerosa

$$a = 20, b = 0.01, c = 0.003$$

Case 2
Solo anti-socialis

$$a = 5, b = 0.5, c = 0.01$$

Case 3
Reproductio sensitova

$$a = 0.5, b = 0.001, c = 0.08$$

Case 4
Multiplicanda numerosa and Solo anti-socialis

$$a = 25, b = 0.51, c = 0.013$$

Case 5
Multiplicanda numerosa and Reproductio sensitova

$$a = 20.5, b = 0.011, c = 0.083$$

Case 6
Solo anti-socialis and Reproductio sensitova

$$a = 5.5, b = 0.501, c = 0.09$$

Case 7
Multiplicanda numerosa, Solo anti-socialis and Reproductio sensitova

$$a = 25.5, b = 0.511, c = 0.093$$

In summary the following table shows the different cases.

Case	Organism	Reproduction rate	Crowding effect	Toxin Build-up
1	mn	20	0.01	0.03
2	sa	5.0	0.5	0.01
3	rs	0.5	0.001	0.08
4	$mn + sa$	25	0.51	0.013
5	$mn + rs$	20.5	0.011	0.083
6	$sa + rs$	5.5	0.501	0.09
7	$mn + sa + rs$	25.5	0.511	0.093

Where
mn = Multiplicanda numerosa
sa = Solo anti-socialis
rs = Reproductio sensitova

Plot for cases 1 -7 are given in figures 1 – 7 below for 50 days in which the analysis of the results are based. The number of organisms per cubic centimetre chosen by experiments as initial values is 10, 20 and 30. The plots are given together for comparison from which the following inferences are drawn.

Case 1

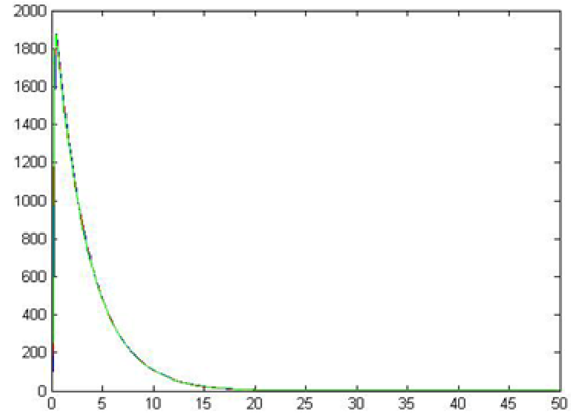

Figure 1: a = 20; b = 0:01; c = 0:03

Case 2

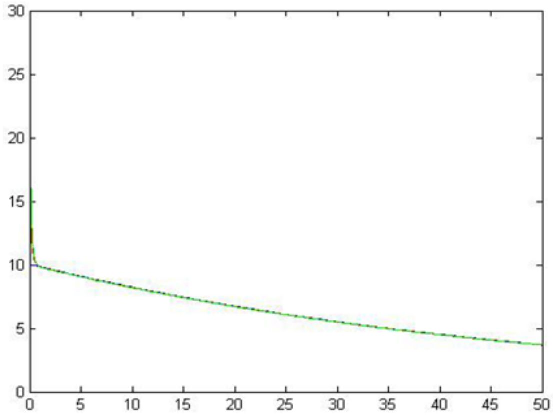

Figure 2: a = 5; b = 0:5; c = 0:01

Case 3

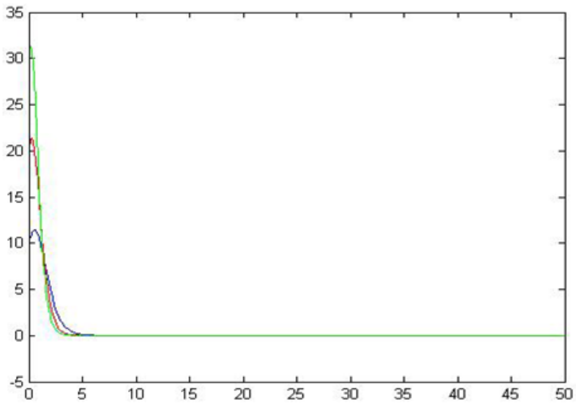

Figure 3: a = 0:5; b = 0:001; c = 0:08

Case 4

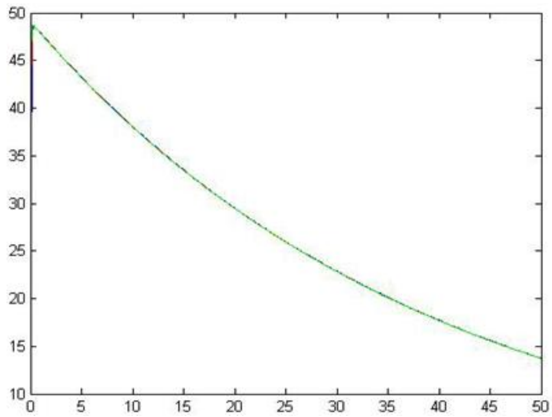

Figure 4: a = 25; b = 0:51; c = 0:013

Case 5

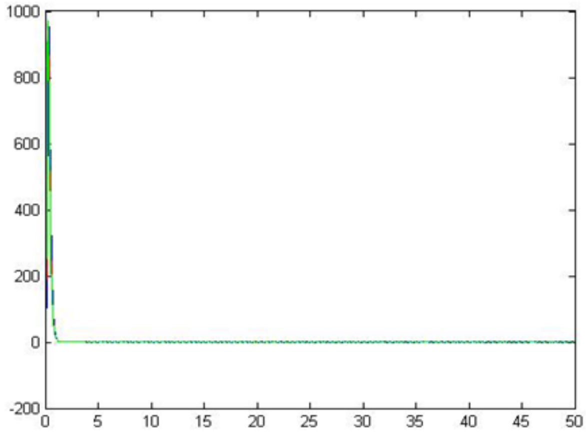

Figure 5: a = 20:5; b = 0:011; c = 0:083

Case 6

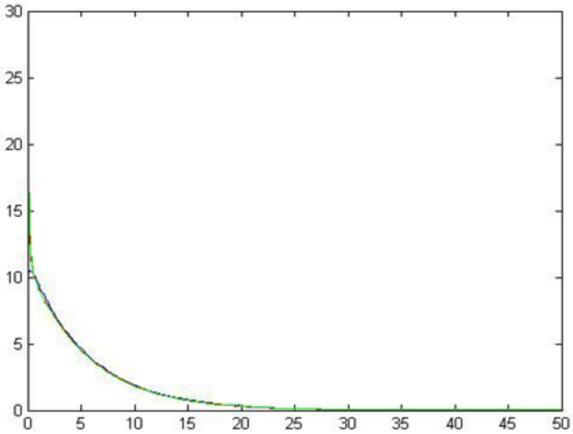

Figure 6: a = 5:5; b = 0:501; c = 0:09

Case 7

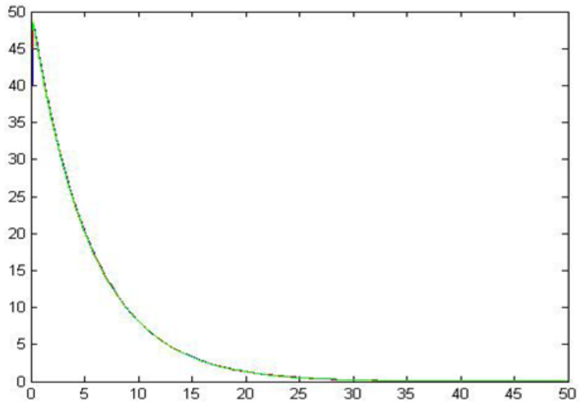

Figure 7: a = 25:5; b = 0:511; c = 0:093

IV. DISCUSSION OF RESULTS

It is observed from figures 1, 4, and 7 that number of organisms is more than 10 so growth of organisms is limited due to crowding effects. Toxin build-up only dominating in the long run, hence the population is insensitive to toxins. They are all good methods of treatment.

Also for figures 2, 3, 5 and 6, it is observed that the number of organisms is less than 10. Growth is immediate due to lack of crowding effects and checked by accumulation of toxins. The populations are sensitive to toxins. They are not good methods of waste water treatment.

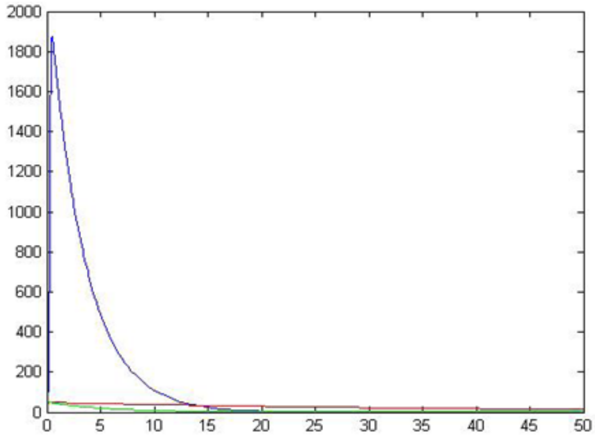

Figure 8a

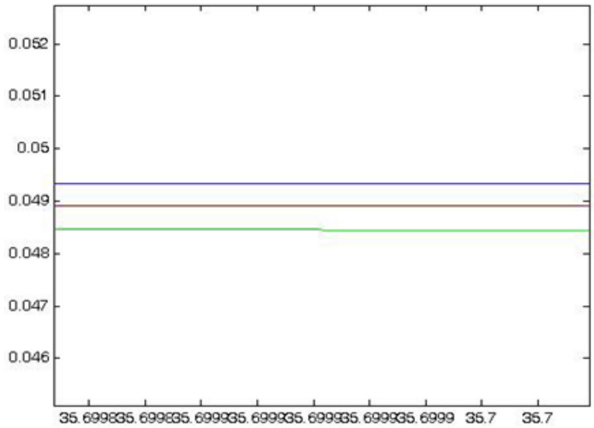

Figure 8b

V. CONCLUSION

Cases 1, 4 and 7 provide more efficient water treatment as evident in the observation compared to cases 2, 3, 5 and 6. From the figures, it is obvious that case 1 (i.e. Multiplicanda numerosa) with the largest amplitude curves brings the logistic curve with slow decay will provide the most efficient water treatment because it can successfully thrive in the tank of waste water to consume all the bacteria in fewer days and has the maximum permissible population below 0.05 per cubic centimeter (see figure 8b) within water discharge into the environment. This is evident in figure 8a which shows the comparison between 1, 4 and 7. So from figure 8b, 10 organisms should be introduced initially and this confirms the claim of the SEWER scientists.

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