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Effect of rotation on the onset of Rayleigh-Bénard convection in a layer of Ferrofluid

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ABSTRACT: *The effect of rotation on the Rayleigh-Bénard convection in a horizontal layer of ferrofluid is investigated by using Galerkin weighted residuals method. Linear stability theory based upon normal mode analysis and perturbation method is used to find expressions for Rayleigh number for free-free boundary layer of fluid. It is observed that the system is more stable in the rotating fluid than non-rotating fluid layer. 'Principle of exchange of stabilities' not valid and the oscillatory convection is possible only for certain conditions. The effect of rotation and magnetic parameters on the stationary convection is investigated analytically.*

KEY WORDS: *Ferro fluid, Convection, rotation, Galerkin method, Prandtl number.*

Nomenclature

a	wave number
B	magnetic induction
g	acceleration due to gravity
H	magnetic field Intensity
k	thermal conductivity
K_1	pyomagnetic coefficient
M	magnetization
M_1	buoyancy magnetization
M_3	magnetic parameter
N	magnetic thermal Rayleigh number
n	growth rate of disturbances
p	pressure (Pa)
P_r	Prandtl number
q	fluid velocity
R	Rayleigh number
t	time
T	temperature
Ta	average temperature
T_A	Taylor number
u, v, w	fluid velocity components
(X, y, z)	space co-ordinates

Greek symbols

α	thermal expansion coefficient
β	uniform temperature gradient
μ_o	magnetic permeability
μ	viscosity
ρ	density of the fluid
(ρ c)	heat capacity of fluid
κ	thermal diffusivity
Φ'	perturbed magnetic potential
ω	dimensional frequency
χ	magnetic susceptibility

Superscripts

'	non dimensional variables
' '	perturbed quantity

Subscripts

0	lower boundary
1	upper boundary
H	horizontal plane

I. INTRODUCTION

Ferromagnetic fluid has wide ranges of applications in instrumentation, lubrication, printing, vacuum technology, vibration damping, metals recovery, acoustics and medicine, its commercial usage includes vacuum feed through for semiconductor manufacturing in liquid-cooled loudspeakers and computer disk drives etc. Owing the applications of the ferrofluid its study is important to the researchers. A detailed account on the stability of ferrofluid has been given by Rosensweig (1985) in his monograph. This monograph reviews several applications of heat transfer through ferrofluid. One such phenomenon is enhanced convective cooling having a temperature-dependent magnetic moment due to magnetization of the fluid. This magnetization, in general, is a function of the magnetic field, temperature, salinity and density of the fluid.

In our analysis, we assume that the magnetization is aligned with the magnetic field. Convective instability of a ferromagnetic fluid for a fluid layer heated from below in the presence of uniform vertical magnetic field has been considered by Finlayson (1970). He explained the concept of thermo-mechanical interaction in ferromagnetic fluids.

Thermo convective stability of ferromagnetic fluids without considering buoyancy effects has been investigated by Lalas and Carmi (1971). Linear and nonlinear convective instability of a ferromagnetic fluid for a fluid layer heated from below under various assumptions is studied by many authors Shliomis (2002), Blennerhassett et.al.(1991), Gupta and Gupta (1979), Stiles and Kagan (1990), Sunil et.al. (2005, 2006), Sunil, Mahajan (2008), Venkatasubramanian and Kaloni (1994), Zebib (1996), Mahajan (2010). Rotation also play important role in the thermal instability of fluid layer and has applications in rotating machineries such as nuclear reactors, petroleum industry bio mechanics etc. Owing to the various applications of ferrofluid an attempt has been made to investigate the thermal instability of a ferromagnetic fluid in the presence of rotation in a fluid layer heated from below using Galerkin weighted residuals method.

II. MATHEMATICAL FORMULATION OF THE PROBLEM

Consider an infinite, horizontal layer of an electrically non-conducting incompressible ferromagnetic fluid of thickness ‘d’, bounded by plane z = 0 and z = d. Fluid layer is acted upon by gravity force **g** (0, 0, -g) and a uniform magnetic field $\mathbf{H} = H_0^{ext}\hat{k}$ acts outside the fluid layer. The layer is heated from below such that a uniform temperature gradient $\beta\left(=\left|\frac{dT}{dz}\right|\right)$ is to be maintained. The temperature T at z = 0 taken to be T_0 and T_1 at z = d, ($T_0 > T_1$ as shown in Fig.1.

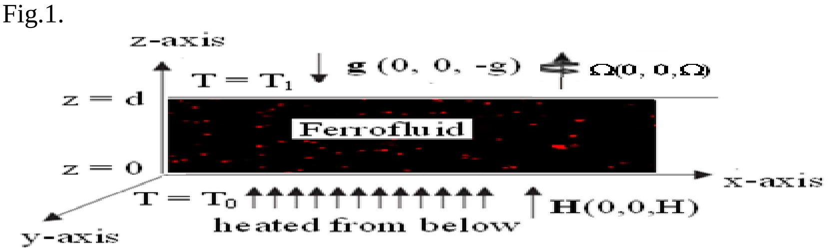

Fig.1 Geometrical configuration of the problem

The governing equations under Boussinesq approximation for the above model (Finlayson (1970), Resenweig (1997), and Mahajan (2010) are:

$$\nabla.\mathbf{q} = 0, \tag{1}$$

$$\rho_0 \frac{d\mathbf{q}}{dt} = -\nabla p + \rho_0 \mathbf{g} + \mu\nabla^2\mathbf{q} + \mu_0(\mathbf{M}.\nabla)\mathbf{H} + 2\rho_0(\mathbf{q}\times\Omega), \tag{2}$$

$$(\rho_\circ C_0)_f \frac{dT}{dt} + (\rho_\circ C_0)_f\, q.\nabla T = k\nabla^2 T, \tag{3}$$

Maxwell’s equations, in magnetostatic limit:

$$\nabla.\mathbf{B} = 0,\ \nabla\times\mathbf{H} = 0,\quad B = \mu_0(\mathbf{H}+\mathbf{M}). \tag{4}$$

The magnetization has the relationship

$$\mathbf{M} = \frac{\mathbf{H}}{H}\left[M_0 + \chi(H - H_0) - K_1(T - T_a)\right]. \tag{5}$$

The density equation of state is taken as

$$\rho = \rho_\circ\left[1 - \alpha(T - T_a)\right]. \tag{6}$$

Here ρ, ρ_0, **q**, t, p, μ, μ_0, **H**, **B**, C_0, T, M, K_1, and α are the fluid density, reference density, velocity, time, pressure, dynamic viscosity (constant), magnetic permeability, magnetic field, magnetic induction, specific heat at constant pressure, temperature, magnetization, thermal conductivity and thermal expansion coefficient, T_a is the average temperature given by,

$T_a = \left(\frac{T_0 + T_1}{2}\right)$ $H = |\mathbf{H}|$, $M = |\mathbf{M}|$ and $M_0 = M(H_0, T_a)$. The magnetic susceptibility and pyomagnetic coefficient are defined by $\chi = \left(\frac{\partial M}{\partial H}\right)_{H_\circ, T_a}$ and $K_1 = \left(\frac{\partial M}{\partial T}\right)_{H_\circ, T_a}$ respectively.

Since the fluid under consideration is confined between two horizontal planes z = 0 and z = d, on these two planes certain boundary conditions must be satisfied. We take case of free-free surface and assume the temperature and volumetric fraction of the nanoparticles are constant and the boundary conditions [Chandrasekhar (1961)] are

$$w = 0, \quad T = T_0 \quad \text{at} \quad z = 0, \text{ and } \quad w = 0, \quad T = T_1 \quad \text{at} \quad z = d. \tag{7}$$

II.1 Basic Solutions

The basic state is assumed to be a quiescent state and is given by

$$q(u, v, w) = q_b(u, v, w) = 0 \;, \; p = p_b(z) \;, \; T = T_b(z) = -\beta z + T_a, \; H_b = \left[H_\circ + \frac{K_1(T_b - T_a)}{1+\chi}\right]\hat{k},$$

$$M_b = \left[M_\circ - \frac{K_2(T_b - T_a)}{1+\chi}\right]\hat{k} \quad], \; H_\circ + M_\circ = H_\circ ext \; . \tag{8}$$

II.2 The Perturbation Equations

We shall analyze the stability of the basic state by introducing the following perturbations:

$$q = q_b + q', \; p = p_b(z) + \delta p, T = T_b(z) + \theta \;, \; H = H_b(z) + H' \; M = M_b(z) + M'. \tag{9}$$

where $q'(u,v,w)$, δp, θ, $H'(H'_1,H'_2,H'_3)$ and $M'(M'_1,M'_2,M'_3)$ are perturbations in velocity, pressure, temperature, magnetic field and magnetization. These perturbations are assumed to be small and then the linearized perturbation equations are

$$\nabla.\mathbf{q}' = 0, \tag{10}$$

$$\rho_\circ \frac{\partial \mathbf{q}'}{\partial t} = -\nabla\delta p + \mu\nabla^2\mathbf{q}' + \rho_0 \alpha g \theta \hat{k} - \frac{\mu_\circ K_1 \beta}{1+\chi}\left((1+\chi)\frac{\partial \varphi_1'}{\partial z}\hat{k} - K_1\theta\hat{k}\right) + 2\rho_0(\mathbf{q} \times \Omega), \tag{11}$$

$$\frac{\partial \theta}{\partial t} = \kappa \nabla^2 \theta + \beta w, \tag{12}$$

$$\left(1 + \frac{M_0}{H_0}\right)\nabla^2\phi_1' - \left(\frac{M_0}{H_0} - \chi\right)\frac{\partial^2 \phi_1'}{\partial z^2} = K_1 \frac{\partial \theta}{\partial z}. \tag{13}$$

Where $\mathbf{H}' = \nabla\varphi_1'$ and φ' is the perturbed magnetic potential and $\kappa = \frac{k}{(\rho_0 c_0)_f}$ is thermal diffusivity of the fluid.

And boundary conditions are

$$w = 0, \quad T = T_0, D\varphi = 0 \quad \text{at} \quad z = 0 \text{ and } \quad w = 0, \quad T = T_1, D\varphi = 0 \; \text{ at } \; z = d. \tag{14}$$

We introduce non-dimensional variables as

$$(x'', y'', z'') = \left(\frac{x', y', z'}{d}\right), \mathbf{q}'' = \mathbf{q}'\frac{d}{\kappa}, \; t' = \frac{\kappa}{d^2}t, \; \delta p' = \frac{d^2}{\mu\kappa}\delta p, \; \theta' = \frac{\theta}{\beta d}, \; \varphi_1'' = \frac{(1+\chi)}{K_1 \beta d^2}\varphi_1'.$$

There after dropping the dashes (") for simplicity.

Equations (10) - (14), in non dimensional form can be written as

$$\nabla.\mathbf{q} = 0, \tag{15}$$

$$\frac{1}{Pr}\frac{\partial \mathbf{q}}{\partial t} = -\nabla\delta p + \nabla^2\mathbf{q} + R(1+M_1)\theta\hat{k} - RM_1\frac{\partial \varphi_1}{\partial z}\hat{k} + \sqrt{T_A}(ve_x - ue_y), \tag{16}$$

$$\frac{\partial \theta}{\partial t} = \nabla^2\theta + w, \tag{17}$$

$$M_3\nabla^2\phi_1 - (M_3 - 1)\frac{\partial^2 \phi_1}{\partial z^2} = \frac{\partial \theta}{\partial z}. \tag{18}$$

Where non-dimensional parameters are:

$P_r = \frac{\mu}{\rho\kappa}$ is Prandtl number; $R = \frac{\rho_0 g\alpha\beta d^4}{\mu\kappa}$ is Rayleigh number; $M_1 = \frac{\mu_0 K_1^2 \beta}{\alpha\rho_0 g(1+\chi)}$ measure the ratio of magnetic to gravitational forces, $T_A = \left(\frac{2\Omega d^2}{\nu}\right)^2$ is the Taylor number, $N = RM_1 = \frac{\mu_0 K_1^2 \beta^2 d^4}{\mu\kappa(1+\chi)}$ is magnetic thermal Rayleigh number; $M_3 = \frac{\left(1+\frac{M_0}{H_0}\right)}{(1+\chi)}$ measure the departure of linearity in the magnetic equation of state and values from one $(M_0 = \chi H_0)$ higher values are possible for the usual equation of state.

The dimensionless boundary conditions are

$$w = 0, \quad T = 1, D\varphi = 0 \text{ at } z = 0 \text{ and } w = 0, \quad T = 0, D\varphi = 0 \text{ at } z = 1. \tag{19}$$

Eliminating p from equation (16) we get

$$\left(\nabla^2 - \frac{1}{Pr}\frac{\partial}{\partial t}\right)\nabla^2 w + R(1+M_1)\nabla_H^2\theta - RM_1\nabla_H^2 D\varphi_1 + \sqrt{T_A}\frac{\partial\xi}{\partial z} = 0, \tag{20}$$

and

$$\left(\nabla^2 - \frac{1}{Pr}\frac{\partial}{\partial t}\right)\xi = -\sqrt{T_A}\frac{\partial w}{\partial z}. \tag{21}$$

Where stands for z- component of the vorticity ∇_H^2, is two-dimensional Laplacian operator on horizontal plane.

Eliminating ξ from equations (20) and (21), we get

$$\left(\nabla^2 - \frac{1}{Pr}\frac{\partial}{\partial t}\right)^2 \nabla^2 w + T_A D^2 w + R(1+M_1)\nabla_H^2\left(\nabla^2 - \frac{1}{Pr}\frac{\partial}{\partial t}\right)\theta - RM_1\nabla_H^2\left(\nabla^2 - \frac{1}{Pr}\frac{\partial}{\partial t}\right)D\varphi_1 = 0. \tag{22}$$

III. NORMAL MODE ANALYSIS

Analyzing the disturbances of normal modes and assume that the perturbation quantities are of the form

$$[w, \theta, \varphi_1] = [W(z), \Theta(z), \Phi(z)]\exp(ik_x x + ik_y y + nt), \tag{23}$$

where, k_x, k_y are wave numbers in x- and y- direction and n is growth rate of disturbances.

Using equation (21), equations (20) and (17) - (18) becomes

$$\left(\left(D^2 - a^2 - \frac{n}{Pr}\right)^2 (D^2 - a^2) + T_A D^2\right)W - a^2 R(1+M_1)\left(D^2 - a^2 - \frac{n}{Pr}\right)\Theta + a^2 RM_1\left(D^2 - a^2 - \frac{n}{Pr}\right)D\Phi = 0, \tag{24}$$

$$W + (D^2 - a^2 - n)\Theta = 0, \tag{25}$$

$$D\Theta - (D^2 - a^2 M_3)\Phi = 0. \tag{26}$$

where $D = \frac{d}{dz}$ and $a^2 = k_x^2 + k_y^2$ is dimensionless the resultant wave number.

The boundary conditions of the problem in view of normal mode analysis are

$$W = 0, D^2W = 0, \ \Theta = 0, \ D\Phi = 0 \text{ at } z = 0,1. \tag{27}$$

IV. METHOD OF SOLUTION

The Galerkin weighted residuals method is used to obtain an approximate solute on to the system of equations (24) – (26) with the corresponding boundary conditions (27). In this method, the test functions are the same as the base (trial) functions. Accordingly W, Θ and Φ are taken as

$$W = \sum_{p=1}^{n} A_p W_p, \ \Theta = \sum_{p=1}^{n} B_p \Theta_p, \ D\Phi = \sum_{p=1}^{n} C_p D\Phi_p. \tag{28}$$

Where A_p, B_p and C_p are unknown coefficients, p =1, 2, 3...N and the base functions W_p, Θ_p and $D\Phi_p$ are assumed in the following form for free-free boundaries are:

$$W_p = \text{Cos}\,p\pi z, \ \Theta_p = \text{Cos}\,p\pi z, \ D\Phi_p = \text{Cos}\,p\pi z, \tag{29}$$

Such that W_p, Θ_p and Φ_p satisfy the corresponding boundary conditions. Using expression for W, Θ and DΦ in equations (24) – (26) and multiplying first equation by W_p second equation by Θ_p and third by $D\Phi_p$ and integrating in the limits from zero to unity, we obtain a set of 3N linear homogeneous equations in 3N unknown A_p, B_p and C_p; p =1,2,3,...N. For existing of non trivial solution, the vanishing of the determinant of coefficients produces the characteristics equation of the system in term of Rayleigh number R.

V. LINEAR STABILITY ANALYSIS

We confined our analysis to the one term Galerkin approximation; for one term Galerkin approximation, we take N=1, we get the expression for Rayleigh number R as:

$$R = \frac{\left(\pi^2 + a^2M_3\right)\left(\pi^2 + a^2\right)\left(\pi^2 + a^2 + \frac{n}{Pr}\right)^2 + T_A\pi^2}{a^2\left(\pi^2 + a^2M_3 + a^2M_1M_3\right)\left(\pi^2 + a^2 + \frac{n}{Pr}\right)}. \tag{30}$$

For neutral stability, the real part of n is zero. Hence we put n = iω, in equation (30), where ω is real and is dimensionless frequency, we get

$$R = \frac{\left(\pi^2 + a^2M_3\right)\left(\pi^2 + a^2\right)\left(\pi^2 + a^2 + \frac{i\omega}{Pr}\right)^2 + T_A\pi^2}{a^2\left(\pi^2 + a^2M_3 + a^2M_1M_3\right)\left(\pi^2 + a^2 + \frac{i\omega}{Pr}\right)}. \tag{31}$$

Equating real and imaginary parts of equation (31), we get

$$\frac{a^2R\left(\pi^2 + a^2M_3 + a^2M_1M_3\right)\left(\pi^2 + a^2\right)}{\left(\pi^2 + a^2M_3\right)} = \left(\pi^2 + a^2\right)^4 + T_A\pi^2 - \omega^2\left(\frac{J^2}{Pr^2} + \frac{2J^2}{Pr}\right), \tag{32}$$

and

$$\omega\left(\frac{a^2R\left(\pi^2 + a^2M_3 + a^2M_1M_3\right)}{Pr\left(\pi^2 + a^2M_3\right)} - \left(\left(\pi^2 + a^2\right)^2 + \frac{2\left(\pi^2 + a^2\right)^2}{Pr} - \frac{\omega^2}{Pr^2}\right)\right) = 0. \tag{33}$$

From equation (36) it follows that ω is real it is necessary that

$$\frac{a^2R\left(\pi^2 + a^2M_3 + a^2M_1M_3\right)\left(\pi^2 + a^2\right)}{\left(\pi^2 + a^2M_3\right)} \le \left(\pi^2 + a^2\right)^4 + T_A\pi^2. \tag{34}$$

Hence equation (32) gives the oscillatory stability provided that inequalities (34) hold.

(a) Stationary Convection

Consider the case of stationary convection i.e., ω = 0, from equation (30), we have

$$R = \frac{\left(\pi^2 + a^2M_3\right)\left(\pi^2 + a^2\right)^3 + T_A\pi^2}{a^2\left(\pi^2 + a^2M_3 + a^2M_1M_3\right)}. \tag{35}$$

From equation (35) we have

$$\frac{\partial R}{\partial T_A} > 0, \quad \frac{\partial R}{\partial M_3} < 0 \text{ and } \frac{\partial R}{\partial M_1} < 0.$$

Thus rotation has stabilizing effect while magnetization parameters have destabilizing effect on fluid layer.
In the absence of rotation T_A=0, the Rayleigh number R for steady onset is given by

$$R = \frac{\left(\pi^2 + a^2M_3\right)\left(\pi^2 + a^2\right)^3}{a^2\left(\pi^2 + a^2M_3 + a^2M_1M_3\right)}.$$

This is the good agreement of the result as obtained by Finlayson (1970).
In the absence of rotation T_A=0 and magnetic parameters M_1=M_3=0, the Rayleigh number R for steady onset is given by

$$R = \frac{\left(\pi^2 + a^2\right)^3}{a^2}.$$

Consequently critical Rayleigh number is given by $Rc = \frac{27\pi^2}{4}$.

This is exactly the same the result as obtained by Chandrasekhar (1961) in the classical Bénard problem.

VI. CONCLUSIONS

A linear stability analysis of thermal instability for ferrofluid in the presence of rotation is investigated for free-free boundary layer. Galerkin-type weighted residuals method is used for the stability analysis. The behavior of rotation and magnetization on the onset of convection analyzed.

The main conclusions are as follows:

a. Rotation stabilizes the fluid layer while magnetization parameters destabilize the fluid in case of stationary convection.
b. The ‘principle of exchange of stabilities’ is not valid for the problem.
c. The oscillatory convection is possible if

$$\frac{a^2R\left(\pi^2 + a^2M_3 + a^2M_1M_3\right)\left(\pi^2 + a^2\right)}{\left(\pi^2 + a^2M_3\right)} \leq \left(\pi^2 + a^2\right)^4 + T_A\pi^2 .$$

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