On Topological $\tilde{g}\alpha$--WG Quotient Mappings

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Abstract: *The aim of this paper is to introduce $\tilde{g}_\alpha$wg-quotient map using $\tilde{g}_\alpha$wg-closed sets and study their basic properties. We also study the relation between weak and strong form of $\tilde{g}_\alpha$wg-quotient maps. We also derive the relation between $\tilde{g}_\alpha$wg-quotient maps and $\tilde{g}_\alpha$-quotient maps and also derive the relation between the $\tilde{g}_\alpha$wg-continuous map and $\tilde{g}_\alpha$wg-quotient maps. Examples are given to illustrate the results.*

Keywords: *$\tilde{g}_\alpha$-closed sets, $\tilde{g}_\alpha$-open sets, $\tilde{g}_\alpha$wg-closed sets, $\tilde{g}_\alpha$wg-open sets.*

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I. Introduction

Njastad [12] introduced the concept of ~~an~~ α-sets and Mashhour et al [9] introduced α-continuous mappings in topological spaces. The topological notions of semi-open sets and semi-continuity, and preopen sets and precontinuity were introduced by Levine [5] and Mashhour et al [10] respectively. After advent of these notions, Reilly [14] and Lellis Thivagar [1] obtained many interesting and important results on α-continuity and α-irresolute mappings in topological spaces. Lellis Thivagar [1] introduced the concepts of α-quotient mappings and α*-quotient mappings in topological spaces. Jafari et al.[15] have introduced $\tilde{g}_\alpha$-closed set in topological spaces. The author [6] introduced $\tilde{g}_\alpha$-WG closed set using $\tilde{g}_\alpha$-closed set. In this paper we have introduced $\tilde{g}_\alpha$-WG quotient mappings.

II. Preliminaries

Throughout this paper (X, τ), (Y, σ) and (Z,η) (or X, Y and Z) represent non empty topological spaces on which no separation axiom is defined unless otherwise mentioned. For a subset A of a space the closure of A, interior of A and complement of A are denoted by cl(A), int(A) and A^c respectively.
We recall the following definitions which are useful in the sequel.

Definition 2.1: A subset A of a space (X, t) is called:
(i)semi-open set [5] if A⊆cl(int(A));
(ii) a α-open set [12] if A⊆ int(cl(int(A))).

The complement of semi-open set(resp.α-open set) is said to be semi closed (resp.α-closed)

Definition 2.2: A subset A of a topological space(X, τ)is called
(i)w-closed set [13] if cl(A) ⊆ U, whenever A ⊆ U and U is semi-open in (X, τ).
(ii) * g-closed set [16] if cl(A) ⊆ U, whenever A ⊆ U and U is w-open in (X, τ).
(iii) a # g-semi closed set(# gs-closed)[17] if scl(A) ⊆ U, whenever A ⊆ U and U is * g -open in (X, τ).
(iv) a $\tilde{g}_\alpha$-closed [15] if αcl(A) ⊆ U, whenever A ⊆ U and U is # gs-open in (X, τ).
(v) a $\tilde{g}_\alpha$-Weakly generalized closed set($\tilde{g}_\alpha$wg-closed) [6] if Cl(Int(A)) ⊆ U, whenever A ⊆ U,U is $\tilde{g}_\alpha$-open in (X, τ).

The complements of the above sets are called their respective open sets.

Definition 2.3: A function $f:(X,\tau)\to(Y,\sigma)$ is called
(i) a α-continuous [1] if f(V) is a α -closed set of (X, τ) for each closed set V of (Y, σ).
(ii)a α -irresolute [1] if f-1(V) is an α -open in (X,τ) for each α -open set V of (Y, σ).
(iii) a $\tilde{g}_\alpha$-continuous [3] if f^{-1}(V) is a $\tilde{g}_\alpha$_-closed set of (X, τ) for each closed set V of (Y, σ),
(iv)a $\tilde{g}_\alpha$_-irresolute [3] if f□1(V) is $\tilde{g}_\alpha$-open in (X, τ) for each $\tilde{g}_\alpha$-open set V of (Y, σ),
(v)$\tilde{g}_\alpha$wg - continuous [7] if f^{-1}(V) is $\tilde{g}_\alpha$wg-closed in (X, τ) for every closed set V of (Y, σ).
(vi) $\tilde{g}_\alpha$wg - irresolute [7] if f^{-1}(v) is $\tilde{g}_\alpha$wg-closed in (X, τ) for every $\tilde{g}_\alpha$wg-closed set V in (Y, σ)

Definition 2.4: A space(X, τ) is called $T_{\tilde{g}_\alpha wg}$-space [6] if every $\tilde{g}_\alpha$wg-closed set is closed.

Definition 2.5: A function $f:(X,\tau)\to(Y,\sigma)$ is said to be
(i)a $\tilde{g}_\alpha$wg -open [8]if the image of each open set in (X,τ) is $\tilde{g}_\alpha$wg -open set in (Y,σ).
(ii) a strongly $\tilde{g}_\alpha$wg -open or $((\tilde{g}_\alpha wg)^*$-open)[8] if the image of each $\tilde{g}_\alpha$wg -open set in (X,τ) is a $\tilde{g}_\alpha$wg -open in (Y,σ).

Definition 2.6: A surjective map $f:(X,\tau)\rightarrow(Y,\sigma)$ is said to be

(i) a quotient map [4] provided a subset U of (Y,σ) is open in (Y, σ) if and only if f^{-1}(U) is open in (X, τ),
(ii) An α-quotient map [1] if f is α -continuous and f^{-1}(U) is open in (X, τ) implies U is an α -open in (Y, σ)
(iii) An α^* -quotient map [1] if f is α -irresolute and f^{-1}(U) is an α -open set in (X, τ) implies U is an open set in (Y, σ).
(iv) a $\tilde{g}_\alpha$-quotient map[2] if f is $\tilde{g}_\alpha$-continuous and f^{-1}(U) is open in (X,τ) implies U is a $\tilde{g}_\alpha$-open set in (Y,σ).
(v) a strongly $\tilde{g}_\alpha$-quotient map[2], provided a set U of (Y,σ) is open in Y if and only if f^{-1}(U) is a $\tilde{g}_\alpha$-open set in (X,τ).
(vi) a $\tilde{g}_\alpha{}^*$-quotient map[2]if f is $\tilde{g}_\alpha$ -irresolute and f^{-1}(U) is an $\tilde{g}_\alpha$ -open set in (X, τ) implies U is an open set in (Y, σ).

Remark 2.7: The collection of all $\tilde{g}_\alpha$wg-closed ($\tilde{g}_\alpha$wg-open sets) are denoted by $\tilde{G}_\alpha$WG_Cl(X) and ($\tilde{G}_\alpha$WG-O(X)), respectively.

III. $\tilde{g}_\alpha$ - Weakly Generalized quotient maps.

Definition 3.1: A surjective map $f:(X,\tau)\rightarrow(Y,\sigma)$ is said to be $\tilde{g}_\alpha$wg-quotient map if f is $\tilde{g}_\alpha$wg-continuous and f^{-1} (V) is open in (X,τ) implies V is $\tilde{g}_\alpha$wg-open set in (Y,σ).

Example3.2:Let$X=\{a,b,c,d\},\tau=\{\emptyset,X,\{a\},\{a,c\},\{a,b,d\}\},Y=\{1,2,3\},\sigma=\{\emptyset,Y,\{1\},\{1,2\},\{1,3\}\}$,
$\tilde{g}_\alpha$wg$O(X)$={∅,X,{a},{a,c},{a,b},{a,d},{a,b,c},{a,b,d},{a,c,d}}, $\tilde{g}_\alpha$wgO(Y) = {∅,Y,{1},{1,2},{1,3}}
The map $f:(X,\tau)\rightarrow(Y,\sigma)$ is defined as f(a)=1, f(b)=2=f(d), f(c)=3. The map f is $\tilde{g}_\alpha$wg-quotient map.

Proposition 3.3: If a map $f:(X,\tau)\rightarrow(Y,\sigma)$ is $\tilde{g}_\alpha$wg-continuous and $\tilde{g}_\alpha$wg-open then f is $\tilde{g}_\alpha$wg-quotient map.
Proof: We only need to prove f^{-1}(V) is open in (X,τ) implies V is $\tilde{g}_\alpha$wg-open in (Y,σ). Let f^{-1}(V) is open in (X,τ). Then f(f^{-1}(V)) is $\tilde{g}_\alpha$wg-open in(Y,σ). Since f is $\tilde{g}_\alpha$wg-open. Hence V is $\tilde{g}_\alpha$wg-open in (X,τ).

IV. Strong form of $\tilde{g}_\alpha$ - Weakly Generalized quotient maps.

Definition 4.1: Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be a surjective map. Then f is called strongly $\tilde{g}_\alpha$wg-quotient map provided a set U of (Y,σ) is open in Y if and only if f^{-1}(U) is $\tilde{g}_\alpha$wg-open set in (X,τ)

Example 4.2: Let $X=\{a,b,c,d\},\tau=\{\emptyset,X,\{a\},\{b,c\},\{a,b,c\}\},Y=\{1,2,3\},\sigma=\{\emptyset,Y,\{1\},\{2\},\{1,2\}\}$
$\tilde{g}_\alpha$wg$O(X)$={∅,X,{a},{b},{c},{a,c},{a,b},{b,c},{a,b,c},{a,b,d},{a,c,d}}, $\tilde{g}_\alpha$wgO(Y) = {∅,Y,{1},{2},{1,2}}
The map $f:(X,\tau)\rightarrow(Y,\sigma)$ is defined as f(a)=1, f(b)=2=f(c), f(d)=3.The map f is strongly $\tilde{g}_\alpha$wg-quotient map.

Theorem 4.3: Every strongly $\tilde{g}_\alpha$wg-quotient map is $\tilde{g}_\alpha$wg-open map.
Proof: Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be a strongly $\tilde{g}_\alpha$wg $-$ *quotient map*. Let V be any open set in (X,τ). Since every open set is $\tilde{g}_\alpha$wg-open by theorem 3.2[6]. Hence V is $\tilde{g}_\alpha$wg-open in (X,τ). That is f^{-1}(f(V)) is $\tilde{g}_\alpha$wg-open in(X,τ). Since f is strongly $\tilde{g}_\alpha$wg-quotient, then f(V) is open in (Y,σ) and hence f(V) is $\tilde{g}_\alpha$wg-open in (Y,σ). This shows that f is $\tilde{g}_\alpha$wg-open map.

Remark 4.4: Converse need not be true

Example 4.5: Let $X=\{a,b,c,d\},\tau=\{\emptyset,X,\{a\},\{b\},\{a,b\}\},Y=\{1,2,3\},\sigma=\{\emptyset,Y,\{1\},\{2\},\{1,2\},\{1,3\}\}$,
$\tilde{g}_\alpha$wg$O(X)$={∅,X,{a},{b},{a,b},{a,b,c},{a,b,d}}, $\tilde{g}_\alpha$wgO(Y) = {∅,Y,{1},{2},{1,2},{1,3}}. $f:(X,\tau)\rightarrow(Y,\sigma)$ is defined by f(a)=1=f(b), f(c)=3, f(d)=2 is a $\tilde{g}_\alpha$wg-open map but not a strongly $\tilde{g}_\alpha$wg-quotient map. Since the set {2} is open in (Y,σ) but f^{-1}({2})={d} is not $\tilde{g}_\alpha$wg-open in (X,τ).

Theorem 4.6: Every strongly $\tilde{g}_\alpha$wg-quotient map is strongly $\tilde{g}_\alpha$wg-open.
Proof: Let $f:(X,\tau)\rightarrow(Y,\sigma)$be a strongly $\tilde{g}_\alpha$wg-quotient map. Let V be a $\tilde{g}_\alpha$wg-open in (X,τ). That is f^{-1}(f(V)) is $\tilde{g}_\alpha$wg-open in(X,τ). Since f is strongly $\tilde{g}_\alpha$wg-quotient, then f(V) is open in (Y,σ) and hence f(V) is $\tilde{g}_\alpha$wg-open in (Y,σ). This shows that f is $\tilde{g}_\alpha$wg-open map.

Example4.7: Let $X=\{a,b,c\},\tau=\{\emptyset,X,\{a\},\{c\},\{a,c\}\},Y=\{1,2,3\},\sigma=\{\emptyset,Y,\{1,3\}\}$, $\tilde{g}_\alpha$wg$O(X)$={S∅,X,{a},{c},{a,c}}, $\tilde{g}_\alpha$wgO(Y) = {∅,Y,{1},{3},{1,2},{1,3},{2,3}}. The map $f:(X,\tau)\rightarrow(Y,\sigma)$ is defined as identity map. Then f is strongly $\tilde{g}_\alpha$wg-open but not strongly $\tilde{g}_\alpha$wg-quotient map. Since
f^{-1}{3}={c} is $\tilde{g}_\alpha$wg-open in (X,τ) but {3} is not open in (Y,σ).

Definition 4.8: Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be a surjective map. Then f is called a $(\tilde{g}_\alpha wg)^*$ -quotient map if f is $\tilde{g}_\alpha$wg-irresolute and f^{-1}(V) is a $\tilde{g}_\alpha$wg-open set in (X,τ) implies V is open in (Y,σ).

Example 4.9: X={a,b,c,d}, $\tau=\{\emptyset,X,\{a\},\{b\},\{a,b\}\}$, Y={1,2,3}, $\sigma=\{\emptyset,Y,\{1\},\{3\},\{1,3\}\}$
$\tilde{g}_\alpha$wg$O(X)$={∅,X,{a},{b},{a,b},{a,b,c},{a,b,d}}, $\tilde{g}_\alpha$wgO(Y) = {∅,Y,{1},{3},{1,3}}. The map $f:(X,\tau)\rightarrow(Y,\sigma)$ defined by f(a)=1, f(b)=3, f(c)=2=f(d). The map f is $(\tilde{g}_\alpha wg)^*$-quotient map.

Theorem 4.10: Every $(\tilde{g}_\alpha wg)^*$-quotient map is $\tilde{g}_\alpha$wg-irresolute.
Proof: Obviously true from the definition.

Remark 4.11: Converse need not be true.

Example 4.12: X={a,b,c,d}, $\tau=\{\emptyset,X,\{a\},\{a,c\},\{a,b,d\}\},Y=\{1,2,3\},\sigma=\{\emptyset,Y,\{1\},\{1,3\}\}$
$\tilde{g}_\alpha$wg$O(X)$={∅,X,{a},{a,c},{a,b},{a,d},{a,b,c},{a,b,d},{a,c,d}}, $\tilde{g}_\alpha$wgO(Y) = {∅,Y,{1},{1,2},{1,3}} Let $f:(X,\tau)\rightarrow(Y,\sigma)$ is defined by f({a})={1}, f({b})={2}=f({d}),f({c})={3}. The map f is $\tilde{g}_\alpha$wg-irresolute but not $(\tilde{g}_\alpha wg)^*$-quotient map Since f^{-1}{1,2}={a,b,d} is $\tilde{g}_\alpha$wg-open in (X,τ) but the set {1,2} is not open in (Y,σ).

Theorem 4.13: Every $(\tilde{g}_\alpha wg)^*$-quotient map is strongly $\tilde{g}_\alpha$wg-open map.

Proof: Let $f:(X,\tau)\to(Y,\sigma)$ be a $(\tilde{g}_\alpha wg)^*$-quotient map. Let V be $\tilde{g}_\alpha$wg-open set in (X,τ). Then f^{-1}(f(V)) is $\tilde{g}_\alpha$wg-open in (X,τ). Since f is $(\tilde{g}_\alpha wg)^*$-quotient this implies that f(V) is open in (Y,σ) and thus $\tilde{g}_\alpha$wg-open in (Y,σ) and thus f(V) is $\tilde{g}_\alpha$wg-open in (Y,σ). Hence f is strongly $\tilde{g}_\alpha$wg-open.

Remark 4.14: Converse need not be true

Example 4.15: Let X={a,b,c}, $\tau=\{\emptyset,X,\{c\},\{a,c\},\{b,c\}\}$, Y={1,2,3}, $\sigma=\{\emptyset,Y,\{1\},\{2,3\}\}$,
$\tilde{g}_\alpha$wg$O(X)$={∅,X,{c}.{a,c},{b,c}}, $\tilde{g}_\alpha$wgO(Y) = {∅,Y,{1},{2},{3},{1,2},{2,3},{1,3}}
Let $f:(X,\tau)\to(Y,\sigma)$ be a identity map, f is strongly $\tilde{g}_\alpha$wg –open map but not $(\tilde{g}_\alpha wg)^*$-quotient map. since F-1{1}={a} is $\tilde{g}_\alpha$wg-open in (Y,σ) but the set{1} is not $\tilde{g}_\alpha$wg-open set in (X,τ).

Proposition 4.16: Every $\tilde{g}_\alpha$-irresolute (α-irresolute) map is $\tilde{g}_\alpha$wg-irresolute.
Proof: Let U be $\tilde{g}_\alpha$-closed (α-closed) set in(Y,σ). Since every $\tilde{g}_\alpha$-closed (α-closed) set is $\tilde{g}_\alpha$wg-closed by theorem 3.7(by theorem 3.11)[6]. Then U is $\tilde{g}_\alpha$wg-closed in (Y,σ). Since f is $\tilde{g}_\alpha$-irresolute (α-irresolute), f^{-1}(U) is $\tilde{g}_\alpha$-closed (α-closed) in (X,τ) which is $\tilde{g}_\alpha$wg-closed in (X,τ). Hence f is $\tilde{g}_\alpha$wg-irresolute.

V. Comparisons

Proposition 5.1:
(i) Every quotient is $\tilde{g}_\alpha$wg-quotient map.
(ii) Every α-quotient map is $\tilde{g}_\alpha$wg-quotient map.
Proof: Since every continuous and α-continuous map is $\tilde{g}_\alpha$wg-continuous by theorem 2.3 1nd 2.7[7] and every open set and α-open set is $\tilde{g}_\alpha$wg-open by theorem 3.11[6]. The proof follows from the definition.

Remark 5.2: Converse of the above proposition need not be true.

Example 5.3: Let X={a,b,c,d}, $\tau=\{\emptyset,X,\{a\},\{b\},\{a,b\}\}, Y=\{1,2,3\}, \sigma=\{\emptyset,Y,\{1,2\}\}$
$\tilde{g}_\alpha$wg$O(X)$={∅,X,{a},{b}, {a,b}, {a,b,c},{a,b,d}}, $\tilde{g}_\alpha$wgO(Y) = {∅,Y,{1},{1,2},{1,3},{2,3}}. Let $f:(X,\tau)\to(Y,\sigma)$ is defined by f({a})={1}, f({b})={2},f({c})={3}=f({d}).The function f is $\tilde{g}_\alpha$wg-quotient but not a quotient map. Since f^{-1}{1}={a} is open in(X,τ) but the set {1} is not open in (Y, σ).

Example 5.4: Let X={a,b,c,d}, $\tau=\{\emptyset,X,\{a\},\{b\},\{a,b\}\}, Y=\{1,2,3\}, \sigma=\{\emptyset,Y,\{1,2\}\}$
$\tilde{g}_\alpha$wg$O(X)$={∅,X,{a},{b}, {a,b}, {a,b,c},{a,b,d}},$\alpha O(X)$= {∅,X,{a},{b}, {a,b}, {a,b,c},{a,b,d}}, $\tilde{g}_\alpha$wgO(Y) = {∅,Y,{1},{1,2},{1,3},{2,3}}, $\alpha O(Y)$= {∅,Y,{1,2}}. Let $f:(X,\tau)\to(Y,\sigma)$ is defined by f({a})={1}, f({b})={2},f({c})={3}=f({d}).The function f is $\tilde{g}_\alpha$wg-quotient but not α-quotient map. Since
f^{-1}{1}={a} is open in(X,τ) but the set {1} is not α- open in (Y, σ).

Theorem 5.5: Every $\tilde{g}_\alpha$-quotient map is $\tilde{g}_\alpha$wg-quotient map.
Proof: Let $f:(X,\tau)\to(Y,\sigma)$ be a $\tilde{g}_\alpha$-quotient map. Then f is $\tilde{g}_\alpha$-continuous function. By theorem 2.5[7] every $\tilde{g}_\alpha$-continuous function is $\tilde{g}_\alpha$wg-continuous function, then f is $\tilde{g}_\alpha$wg-continuous map. Let f^{-1}(V) is open in (Y,σ). Since every $\tilde{g}_\alpha$-open set is $\tilde{g}_\alpha$wg-open . Then V is $\tilde{g}_\alpha$wg-open in (Y,σ). Hence V is $\tilde{g}_\alpha$wg-open in (Y,σ). Hence f is $\tilde{g}_\alpha$wg-quotient map.

Remark 5.6: Converse need not be true:

Example 5.7: Let X={a,b,c,d}, $\tau=\{\emptyset,X,\{d\},\{a,c\},\{a,c,d\}\}$, $Y=\{1,2,3\}, \sigma=\{\emptyset,Y,\{2\},\{3\},\{1,3\},\{2,3\}\}$
$\tilde{g}_\alpha$wg$O(X)$={∅,X,{a},{c},{d},{a,c},{a,d},{c,d},{a,b,d},{a,c,d},{b,c,d}}, $\tilde{g}_\alpha$O(X) = {∅,Y,{a},{c},{d},{a,c},{a,d},{c,d},{a,c,d}}. Let $f:(X,\tau)\to(Y,\sigma)$ is defined by f(a)=2, f(b)=1,f(c)=3=f(d). f is $\tilde{g}_\alpha$wg-quotient map but not $\tilde{g}_\alpha$-quotient map Since f^{-1}{1,3}={b,c,d} is $\tilde{g}_\alpha$wg-open in(X,τ) but not $\tilde{g}_\alpha$- open in (Y, σ).

Theorem 5.8: Every strongly $\tilde{g}_\alpha$wg-quotient map is $\tilde{g}_\alpha$wg-quotient.

Proof: Let V be any open set in(Y,σ). Since f is strongly $\tilde{g}_\alpha$wg-quotient, f^{-1}(V) is $\tilde{g}_\alpha$wg-open set in (X,τ). Hence f is $\tilde{g}_\alpha$wg continuous. Let f^{-1}(V) be open in (X,τ). Then f^{-1}(V) is $\tilde{g}_\alpha$wg-open in (X,τ). Since f is strongly $\tilde{g}_\alpha$wg-quotient, V is open in (Y,σ). Hence f is a $\tilde{g}_\alpha$wg quotient map.

Remark 5.9: Converse need not be true

Example 5.10: Let X= {a, b, c, d}, $\tau=\{\emptyset,X,\{a\},\{a,c\},\{a,b,d\}\}, Y=\{1,2,3\}, \sigma=\{\emptyset,Y,\{1\}\}$
$\tilde{g}_\alpha$wg$O(X)$={∅,X,{a},{a,c},{a,b},{a,d},{a,b,c},{a,b,d},{a,c,d}}, $\tilde{g}_\alpha$wgO(Y) = {∅,Y,{1},{1,2},{1,3}} Let $f:(X,\tau)\to(Y,\sigma)$ is defined by f({a})={1}, f({b})={3},f({c})=2=f({d}).The function f is $\tilde{g}_\alpha$wg-quotient but not strongly $\tilde{g}_\alpha$wg-quotient. Since f^{-1}{1,3}={a,b} is $\tilde{g}_\alpha$wg-open in(X,τ) but {1,3} is not open in (Y, σ).

Theorem 5.11: Every strongly $\tilde{g}_\alpha$-quotient map is strongly $\tilde{g}_\alpha wg$-quotient map.
Proof: Let $f:(X,\tau)\to(Y,\sigma)$ strongly $\tilde{g}_\alpha$-quotient map. Let V be any open set in (Y,σ). Since f is strongly $\tilde{g}_\alpha$-quotient, f^{-1}(V) is $\tilde{g}_\alpha$-open in (X,τ). Since every $\tilde{g}_\alpha$-open set is $\tilde{g}_\alpha wg$-open by theorem. Then f^{-1}(v) is $\tilde{g}_\alpha$wg-open in(X,τ). Hence f is strongly $\tilde{g}_\alpha$wg-quotient map.

Remark 5.12: Converse need not be true.

Example 5.13: Let X={a,b,c,d}, $\tau=\{\emptyset,X,\{d\},\{a,c\},\{a,c,d\}\}$, $Y=\{1,2,3\}, \sigma=\{\emptyset,Y,\{b\},\{c\},\{b,c\},\{a,c\}\}$
$\tilde{g}_\alpha$wg$O(X)$={∅,X,{a},{c},{d},{a,c},{a,d},{c,d},{a,b,d},{a,c,d},{b,c,d}}, $\tilde{g}_\alpha$O(X) = {∅,Y,{a},{c},{d},{a,c},{a,d},{c,d},{a,c,d}}. Let $f:(X,\tau)\to(Y,\sigma)$ is defined by f({a})={b}, f({b})={a},f({c})=c=f({d}). Since f^{-1}{a,c}={b,c,d} is $\tilde{g}_\alpha$wg-open in(X,τ) but not $\tilde{g}_\alpha$- open in (Y, σ).

Theorem 5.14: Every strongly $\tilde{g}_\alpha$wg quotient map is $\tilde{g}_\alpha$wg quotient

Proof: Let V be any open set in (Y,σ). Since f is strongly $\tilde{g}_\alpha$wg quotient, f^{-1}(V) is $\tilde{g}_\alpha$wg-open set in(X,τ). Hence f is $\tilde{g}_\alpha$wg-continuous. Let f^{-1}(V) be open in (X,τ). Then f^{-1}(V) is $\tilde{g}_\alpha$wg open in (X,τ) . Since f is strongly $\tilde{g}_\alpha$wg-quotient V is open in (Y,σ). Hence f is a $\tilde{g}_\alpha$wg-quotient map.

Remark 5.15: Converse need not be true

Example 5.16: Let X={a,b,c,d}, $\tau = \{\emptyset, X, \{a\}, \{a,c\}, \{a,b,d\}\}, Y = \{1,2,3\}, \sigma = \{\emptyset, Y, \{1\}\}$
$\tilde{g}_\alpha$wg$O(X)$={∅,X,{a},{a,c},{a,b},{a,d},{a,b,c},{a,b,d},{a,c,d}}, $\tilde{g}_\alpha$wgO(Y) = {∅,Y,{1},{1,2},{1,3}} Let $f: (X,\tau) \to (Y,\sigma)$ is defined by f({a})={1}, f({b})={3},f({c})=2=f({d}).The function f is $\tilde{g}_\alpha$wg-quotient but not strongly $\tilde{g}_\alpha$wg-quotient. Since f^{-1}{1,3}={a,b} is $\tilde{g}_\alpha$wg-open in(X,τ) but {1,3} is not open in (Y,σ).

Theorem 5.17: Every $(\tilde{g}_\alpha wg)^*$-quotient map is $\tilde{g}_\alpha$wg-quotient map.

Proof: Let f be $(\tilde{g}_\alpha wg)^*$-quotient map. Then f is $\tilde{g}_\alpha$wg-irresolute, by theorem 3.2[7] f is $\tilde{g}_\alpha$wg-continuous. Let
f^{-1}(V) be an open set in (X,τ). Then f^{-1}(V) is a $\tilde{g}_\alpha$wg-open in(X,τ). Since f is $(\tilde{g}_\alpha wg)^*$-quotient, V is open in (Y,σ). It means V is $\tilde{g}_\alpha$wg-open in (Y,σ). Therefore f is a $\tilde{g}_\alpha$wg-quotient map.

Remark 5.18: Converse need not be true

Example 5.19: Let X={a,b,c,d}, $\tau = \{\emptyset, X, \{a\}, \{a,c\}, \{a,b,d\}\}, Y = \{1,2,3\}, \sigma = \{\emptyset, Y, \{1\}\}$
$\tilde{g}_\alpha$wg$O(X)$={∅,X,{a},{a,c},{a,b},{a,d},{a,b,c},{a,b,d},{a,c,d}}, $\tilde{g}_\alpha$wgO(Y) = {∅,Y,{1},{1,2},{1,3}} Let $f: (X,\tau) \to (Y,\sigma)$ is defined by f({a})={1}, f({b})={3},f({c})=2=f({d}).The function f is $\tilde{g}_\alpha$wg-quotient but not ($\tilde{g}_\alpha$wg)*-quotient. Since f^{-1}{1,2}={a,c,d} is $\tilde{g}_\alpha$wg-open in(X,τ) but {1,2} is not open in (Y,σ).

Theorem 5.20: Every α^*-quotient map is $(\tilde{g}_\alpha wg)^*$-quotient map.

Proof: Let f be α^*-quotient map. Then f is surjective, α-irresolute and f^{-1}(U) is α-open in (X,τ) implies U is open set in(Y,σ). Then U is $\tilde{g}_\alpha wg$-open in (Y,σ). Since every α-irresolute map is $\tilde{g}_\alpha$wg-irresolute by proposition 4.16 and every α-irresolute map is α-continuous. Then f^{-1}(U) is α-open which is $\tilde{g}_\alpha wg$-open in(X,τ). Since f is α^*-quotient map, U is open in (Y,σ). Hence f is $(\tilde{g}_\alpha wg)^*$-quotient map.

Remark 5.21: Converse need not be true

Example 5.22: X={a,b,c,d}, $\tau = \{\emptyset, X, \{d\}, \{a,c\}, \{a,c,d\}\}, Y = \{1,2,3\}, \sigma = \{\emptyset, Y, \{2\}, \{3\}, \{2,3\}, \{1,3\}\}$
$\tilde{g}_\alpha$wg$O(X)$={∅,X,{a},{c},{d},{a,c},{a,d},{c,d},{a,b,d},{a,c,d},{b,c,d}}, $\tilde{g}_\alpha$wgO(Y) = {∅,Y,{2},{3},{2,3},{1,3}}, $\alpha O(X)$={{d},{a,c},{a,c,d}}, αO(Y)={{2},{3},{2,3},{1,3}}. The map $f: (X,\tau) \to (Y,\sigma)$ is defined by f(a)=2, f(b)=1,f(c)=3=f(d).The function f is ($\tilde{g}_\alpha$wg)*-quotient but not $(\alpha)^*$-quotient map. Since the set {2} is α-open in (Y,σ) but f^{-1}{2}={a} is not α -open in(X,τ).

Theorem 5.23: Every $(\tilde{g}_\alpha)^*$ quotient map is $(\tilde{g}_\alpha wg)^*$-quotient map:

Proof: Let f be $(\tilde{g}_\alpha)^*$-quotient map. Then f is surjective, $\tilde{g}_\alpha$-irresolute and f(U) is $\tilde{g}_\alpha$-open in (X,τ) implies U is open in(Y,σ). Then U is $\tilde{g}_\alpha$wg-open in(Y,σ). Since every $\tilde{g}_\alpha$-irresolute map is $\tilde{g}_\alpha$wg-irresolute and every $\tilde{g}_\alpha$-irresolute map is $\tilde{g}_\alpha$-continuous. Then f^{-1}(U) is $\tilde{g}_\alpha$-open set which is $\tilde{g}_\alpha$wg-open set in(X,τ). Since f is a $(\tilde{g}_\alpha)^*$-quotient map, U is open in (Y,σ). Hence f is a $(\tilde{g}_\alpha wg)^*$-quotient map.

Remark 5.24: Converse need not be true

Example 5.25: Let X={a,b,c,d}, $\tau = \{\emptyset, X, \{d\}, \{a,c\}, \{a,c,d\}\}, Y = \{1,2,3\}, \sigma = \{\emptyset, Y, \{2\}, \{3\}, \{2,3\}, \{1,3\}\}$
$\tilde{g}_\alpha$wg$O(X)$={∅,X,{a},{c},{d},{a,c},{a,d},{c,d},{a,b,d},{a,c,d},{b,c,d}}, $\tilde{g}_\alpha$wgO(Y) = {∅,Y,{2},{3},{2,3},{1,3}}, $\tilde{g}_\alpha O(X)$={{a},{d},{c},{a,c},{a,d},{c,d},{a,b,d},{a,c,d},{b,c,d}}. $\tilde{g}_\alpha$O(Y)={{2},{3},{2,3},{1,3}}. The map $f: (X,\tau) \to (Y,\sigma)$ is defined by f({a})={2}, f({b})={1},f({c})=3=f({d}).The function f is ($\tilde{g}_\alpha$wg)*-quotient but not $(\tilde{g}_\alpha)^*$-quotient map. Since the set {1,3} is $\tilde{g}_\alpha$-open in (Y,σ) but f^{-1}{1,3}={b,c,d} is not $\tilde{g}_\alpha$-open in(X,τ).

Theorem 5.26: Every $(\tilde{g}_\alpha wg)^*$-quotient map is strongly $\tilde{g}_\alpha$wg-quotient.

Proof: Let V be an open set in (Y,σ). Then it is $\tilde{g}_\alpha$wg-open in(Y,σ) since f is $(\tilde{g}_\alpha wg)^*$-quotient map. f^{-1}(V) is $\tilde{g}_\alpha$wg-open in (X,τ).(Since f is $\tilde{g}_\alpha$wg-irresolute). Also V is open in(Y,σ) implies f^{-1}(V) is $\tilde{g}_\alpha$wg-open in (X,τ). since f is $(\tilde{g}_\alpha wg)^*$-open V is open in(Y,σ). Hence f is strongly $\tilde{g}_\alpha$wg-quotient map.

Remark 5.27: Converse need not be true

Example 5.28: Let X={a,b,c,d}, $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a,b\}, \{a,b,c\}\}$, $Y = \{1,2,3\}, \sigma = \{\emptyset, Y, \{1\}, \{1,2\}\}$
$\tilde{g}_\alpha$wg$O(X)$={∅,X,{a},{b},{a,b},{a,b,c},{a,b,d}}, $\tilde{g}_\alpha$wgO(Y) = {∅,Y,{1},{1,2},{1,3}} Let $f: (X,\tau) \to (Y,\sigma)$ is defined by f({a})={1}, f({b})={2}=f({c}),f({d})={3}.The function f is strongly $\tilde{g}_\alpha$wg-quotient but not a $(\tilde{g}_\alpha wg)^*$-quotient. Since the set {1,3} is $\tilde{g}_\alpha$wg-open in(Y,σ) but f^{-1}{1,3}={a,d} is not open in (X,τ).

Remark 5.29: Quotient map and α^*-quotient map are independent

Example 5.30: Let X={a,b,c,d}, $\tau = \{\emptyset, X, \{a\}, \{a,b\}\}$, $Y = \{1,2,3\}, \sigma = \{\emptyset, Y, \{1\}, \{1,2\}\}$
αO (X)={∅,X,{a},{a,b},{a,c},{a,d},{a,b,c},{a,b,d},{a,c,d}}, αO(Y) = {∅,Y,{1},{1,2},{1,3}} Let $f: (X,\tau) \to (Y,\sigma)$ is defined by f(a)=1, f(b)={2},f(c)=3=f(d).The function f is quotient but not a $(\alpha)^*$-quotient. Since
f^{-1}{1,3}={a,c,d} is α-open in(X,τ) but the set {1,3} is not open in (Y,σ).

Example 5.31: Let X={a,b,c,d}, $\tau = \{\emptyset, X, \{a\}, \{a,c\}, \{a,b,c\}\}$, $Y = \{1,2,3\}, \sigma = \{\emptyset, Y, \{1\}, \{1,2\}, \{1,3\}\}$

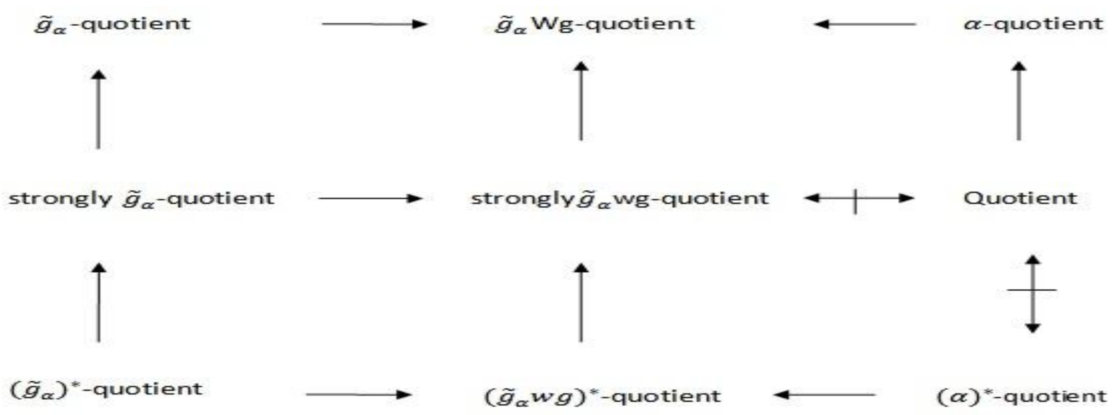
$\alpha O(X)$={∅,X,{a},{a,b},{a,c},{a,d},{a,b,c},{a,b,d},{a,c,d}}, αO(Y) = {∅,Y,{1},{1,2},{1,3}} Let $f:(X,\tau)\rightarrow(Y,\sigma)$ is defined by f(a)=1, f(b)={2},f(c)=3=f(d).The function f is α^{*} quotient but not a quotient map. Since f^{-1}{1,3}={a,c,d} is α-open in(X,τ) but the set {1,3} is not open in (Y,σ)

Remark 5.32: Quotient map and strongly $\tilde{g}_{\alpha}$wg-quotient map are independent.

Example 5.33: X={a,b,c,d}, $\tau=\{\emptyset,X,\{d\},\{a,c\},\{a,c,d\}\}, Y=\{1,2,3\}, \sigma=\{\emptyset,Y,\{2\},\{3\},\{2,3\},\{1,3\}\}$ $\tilde{g}_{\alpha}$wg$O(X)$={∅,X,{a},{c},{d},{a,c},{a,d},{c,d},{a,b,d},{a,c,d},{b,c,d}}, $\tilde{g}_{\alpha}$wgO(Y) = {∅,Y,{2},{3},{2,3},{1,3}}, $\alpha O(X)$={{d},{a,c},{a,c,d}}, αO(Y)={{2},{3},{2,3},{1,3}}. The map $f:(X,\tau)\rightarrow(Y,\sigma)$ is defined by f(a)=2, f(b)=1,f(c)=3=f(d).The function f is strongly $\tilde{g}_{\alpha}$wg-quotient but not quotient map. Since the set {2} is open in (Y,σ) but f^{-1}{2}={a} is not open in(X,τ).

Example 5.34: X={a,b,c,d}, $\tau=\{\emptyset,X,\{a\},\{a,b\}\}, Y=\{1,2,3\}, \sigma=\{\emptyset,Y,\{1\},\{1,2\}\}$ $\tilde{g}_{\alpha}$wg$O(X)$={∅,X,{a},{a,b},{a,c},{a,d},{a,b,c},{a,b,d},{a,c,d}}, $\tilde{g}_{\alpha}$wgO(Y) = {∅,Y,{1},{1,2},{1,3}}, The map $f:(X,\tau)\rightarrow(Y,\sigma)$ is defined by f(a)=1, f(b)=2,f(c)=3=f(d).The function f is quotient but not strongly $\tilde{g}_{\alpha}$wg-quotient map. Since f^{-1}({1,3} is $\tilde{g}_{\alpha}$wg -open in (X,τ) but the set {1,3} is not open in (Y,σ).

Remark 5.35: From the above results we have the following diagram where A ⟶ B represent A implies B but not conversely, A ⟷ B represents A and B are independent each other.



VI. Applications

Theorem 6.1: The composition of two $(\tilde{g}_{\alpha}wg)^{*}$-quotient maps is $(\tilde{g}_{\alpha}wg)^{*}$-quotient.

Proof: Let $f:(X,\tau)\rightarrow(Y,\sigma)$and $g:(Y,\sigma)\rightarrow(Z,\eta)$ be two $(\tilde{g}_{\alpha}wg)^{*}$-quotient maps. Let V be any $\tilde{g}_{\alpha}$wg-open set in (Z,η). Since g is $(\tilde{g}_{\alpha}wg)^{*}$-quotient, g^{-1}(V) is $\tilde{g}_{\alpha}$wg-open in(Y,σ) and since f is $(\tilde{g}_{\alpha}wg)^{*}$-quotient then ($f^{-1}$($g^{-1}$(V)))= $(g\circ f)^{-1}$(V) is $\tilde{g}_{\alpha}$wg-open in(X,τ). Hence g∘f is $\tilde{g}_{\alpha}$wg-irresolute.

Let $(g\circ f)^{-1}$(V) is $\tilde{g}_{\alpha}$wg-open in(X,τ). Then f^{-1}(g^{-1}(V)) is $\tilde{g}_{\alpha}$wg-open in(X,τ). Since f is $(\tilde{g}_{\alpha}wg)^{*}$-quotient, g^{-1}(V) is open in (Y,σ). Since g is $(\tilde{g}_{\alpha}wg)^{*}$-quotient, V is open in (Z,η). Hence g∘f is $(\tilde{g}_{\alpha}wg)^{*}$-quotient.

Proposition 6.2: If $h:(X,\tau)\rightarrow(Y,\sigma)$ is $\tilde{g}_{\alpha}$wg-quotient map and $g:(X,\tau)\rightarrow(Z,\eta)$ is a continuous map that is constant on each set h^{-1}(y) for y∈Y, then g induces a $\tilde{g}_{\alpha}$wg-continuous map $f:(Y,\sigma)\rightarrow(Z,\eta)$ such that $f\circ h=g$.

Proof: g is a constant on h^{-1}(y) for each y∈Y, the set g(h^{-1}(y)) is a one point set in (Z,η). If f(y) denote this point, then it is clear that f is well defined and for each $x\in X, f(h(x))=g(x)$. We claim that f is $\tilde{g}_{\alpha}$wg-continuous. For if we let V be any open set in (Z,η), then g^{-1}(V) is an open set in (X,τ) as g is continuous. But g^{-1}(V)=h^{-1}(f^{-1}(V)) is open in (X,τ). Since h is $\tilde{g}_{\alpha}$wg-quotient map, f^{-1}(V) is a $\tilde{g}_{\alpha}$wg-open in(Y,σ). Hence f is $\tilde{g}_{\alpha}$wg-continuous.

Proposition 6.3: If a map $f:(X,\tau)\rightarrow(Y,\sigma)$ is surjective, $\tilde{g}_{\alpha}$wg-continuous and $\tilde{g}_{\alpha}$wg-open then f is a $\tilde{g}_{\alpha}$wg-quotient map.

Proof: To prove f is $\tilde{g}_{\alpha}$wg-quotient map. We only need to prove f^{-1}(V) is open in (X,τ) implies V is $\tilde{g}_{\alpha}$wg-open in (Y,σ). Since f is $\tilde{g}_{\alpha}$wg-continuous. Let f^{-1}(V) is open in (X,τ). Then f(f^{-1}(V)) is $\tilde{g}_{\alpha}$wg-open set, since f is $\tilde{g}_{\alpha}$wg-open. Hence V is $\tilde{g}_{\alpha}$wg-open set. Hence V is $\tilde{g}_{\alpha}$wg-open set, as f is surjective, f(f^{-1}(V))=V. Thus f is a $\tilde{g}_{\alpha}$wg-quotient map.

Proposition 6.4: If $f:(X,\tau)\rightarrow(Y,\sigma)$be open surjective, $\tilde{g}_{\alpha}$wg-irresolute map, and $g:(Y,\sigma)\rightarrow(Z,\eta)$is a $\tilde{g}_{\alpha}$wg-quotient map then $g\circ f:(X,\tau)\rightarrow(Z,\eta)$ is $\tilde{g}_{\alpha}$wg-quotient map.

Proof: Let V be any open set in (Z, η). Since g is $\tilde{g}_{\alpha}$wg-quotient map then g^{-1}(V) is $\tilde{g}_{\alpha}$wg-open in (Y,σ). Since f is $\tilde{g}_{\alpha}$wg-irresolute, f^{-1}(g^{-1}(V)) =$(g\circ f)^{-1}$(V) is $\tilde{g}_{\alpha}$wg-open in (X,τ). This implies $(g\circ f)^{-1}$(V) is $\tilde{g}_{\alpha}$wg-open set in (X,τ). This shows that g∘f is $\tilde{g}_{\alpha}$wg-continuous map. Also assume that $(g\circ f)^{-1}(V)$ is open in (X,τ). For $V\subseteq Z$, that is $(f^{-1}(g^{-1}(V))$ is open in (Y,σ). It follows that g^{-1}(V) is open in (Y,σ). Since f is surjective and g is $\tilde{g}_{\alpha}$wg-quotient map. V is $\tilde{g}_{\alpha}$wg-open set in(Z,η).

Proposition 6.5: Let $f:(X,\tau)\rightarrow(Y,\sigma)$be strongly $\tilde{g}_{\alpha}$wg-open surjective and $\tilde{g}_{\alpha}$wg-irresolute map and $g:(Y,\sigma)\rightarrow(Z,\eta)$ be strongly $\tilde{g}_{\alpha}$wg-quotient map then $g\circ f$ is a strongly $\tilde{g}_{\alpha}$wg-quotient map.

Proof: Let V be an open set in(Z,η). Since g is strongly $\tilde{g}_\alpha$wg-quotient, $g^{-1}(V)$ is a $\tilde{g}_\alpha$wg-open in(Y,σ). Since f is $\tilde{g}_\alpha$wg-irresolute, $f^{-1}(g^{-1}(V))$ is a $\tilde{g}_\alpha$wg-open in (X,τ) implies $(g\circ f)^{-1}(V)$ is a $\tilde{g}_\alpha$wg-open in (X,τ). Assume $(g\circ f)^{-1}(V)$ is $\tilde{g}_\alpha$wg-open in(X,τ). Then $(f^{-1}\left(g^{-1}(V)\right)$ is $\tilde{g}_\alpha$wg-open in (X,τ). Since f is strongly $\tilde{g}_\alpha$wg-open, then $f(f^{-1}(g^{-1}(V)))$ is $\tilde{g}_\alpha$wg-open in (Y,σ) which implies $g^{-1}(V)$ is $\tilde{g}_\alpha$wg-open in(Y,σ). Since g is strongly $\tilde{g}_\alpha$wg-quotient map, V is open in (Z,η). Thus g∘f is strongly $\tilde{g}_\alpha$wg-quotient map.

Theorem 6.6: Let p:(X,τ) $\rightarrow (Y,\sigma)$be $\tilde{g}_\alpha$wg-quotient map and the spaces (X,τ),(Y,σ) are $T_{\tilde{g}_\alpha wg}$-spaces. f:(Y,σ)→(Z,η) is a strongly $\tilde{g}_\alpha$wg-continuous iff f∘ $p:(X,\tau)\rightarrow(Z,\eta)$is a strongly $\tilde{g}_\alpha$wg-continuous.
Proof: Let f be strongly $\tilde{g}_\alpha$wg-continuous and U be any $\tilde{g}_\alpha$wg-open set in (Z,η). Then $f^{-1}(U)$ is open in (Y,σ). Since p is $\tilde{g}_\alpha$wg-quotient map, then $p^{-1}(f^{-1}(U))=(f\circ p)^{-1}(U)$ is $\tilde{g}_\alpha$wg-open in(X,τ). Since(X,τ) is $T_{\tilde{g}_\alpha wg}$ -space, $p^{-1}(f^{-1}(U))$ is open in (X,τ). Thus f∘p is strongly $\tilde{g}_\alpha$wg-continuous.
Conversely, Let the composite function f∘p is strongly $\tilde{g}_\alpha$wg-continuous. Let U be any $\tilde{g}_\alpha$wg-open set in (Z,η), $p^{-1}(f^{-1}(U))$ is open in (X,τ). Since p is a $\tilde{g}_\alpha$wg-quotient map, it implies that $f^{-1}(U)$ is $\tilde{g}_\alpha$wg-open in (Y,σ). Since (Y,σ) is a $T_{\tilde{g}_\alpha wg}$ -space, $f^{-1}(U)$ is open in (Y,σ). Hence f is strongly is $\tilde{g}_\alpha$wg-continuous.

Theorem 6.7: Let $f:(X,\tau)\rightarrow(Y,\sigma)$ be a surjective, strongly $\tilde{g}_\alpha$wg-open and $\tilde{g}_\alpha$wg-irresolute map and $g:(Y,\sigma)\rightarrow(Z,\eta)$ be a $(\tilde{g}_\alpha wg)^*$-quotient map, then g∘f is a $(\tilde{g}_\alpha wg)^*$-quotient map.
Proof: Let V be $\tilde{g}_\alpha$wg-open set in (Z,η). Since g is a $(\tilde{g}_\alpha wg)^*$-quotient map, $g^{-1}(V)$ is a $\tilde{g}_\alpha$wg-open set in(Y,σ), since f is $\tilde{g}_\alpha$wg-irresolute, $f^{-1}(g^{-1}(V))$ is $\tilde{g}_\alpha$wg-open in (X,τ). Then $(g\circ f)^{-1}(V)$ is $\tilde{g}_\alpha$wg-open in (X,τ). Since f is strongly $\tilde{g}_\alpha$wg-open, $f(f^{-1}(g^{-1}(V)))$ is $\tilde{g}_\alpha$wg-open in (Y,σ) which implies $g^{-1}(V)$ is a $\tilde{g}_\alpha$wg-open in (Y,σ). Since g is $(\tilde{g}_\alpha wg)^*$-quotient map then V is open in (Z,η).
Hence g∘f is a $(\tilde{g}_\alpha wg)^*$-quotient map.

Theorem 6.8: Let $f:(X,\tau)\rightarrow(Y,\sigma)$be a strongly $\tilde{g}_\alpha$wg-quotient map and $g:(Y,\sigma)\rightarrow(Z,\eta)$be a $(\tilde{g}_\alpha wg)^*$-quotient map and f is $\tilde{g}_\alpha$wg-irresolute then g∘f is $(\tilde{g}_\alpha wg)^*$-quotient map.
Proof: Let V be a $\tilde{g}_\alpha$wg-open in (Z,η). Then $g^{-1}(V)$ is $\tilde{g}_\alpha$wg-open in (Y,σ) since g is $(\tilde{g}_\alpha wg)^*$-quotient map. Since f is $\tilde{g}_\alpha$wg-irresolute map, then $f^{-1}(g^{-1}(V))$ is $\tilde{g}_\alpha$wg-open in (X,τ). ie, $(g\circ f)^{-1}(V)$ is $\tilde{g}_\alpha$wg-open in (X,τ) which shows that g∘f is $\tilde{g}_\alpha$wg –irresolute. Let $f^{-1}(g^{-1}(V))$ is $\tilde{g}_\alpha$wg-open in (X,τ). Since f is strongly $\tilde{g}_\alpha$wg-quotient map then $g^{-1}(V)$ is open in (Y,σ) which implies $g^{-1}(V)$ is $\tilde{g}_\alpha$wg-open in(Y,σ). Since g is $(\tilde{g}_\alpha wg)^*$-quotient map, V is open in (Z,η). Hence g∘f is $(\tilde{g}_\alpha wg)^*$-quotient map.

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