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3-Chromatic Cubic Graphs with Complementary Connected Domination Number Three

Selvam Avadayappan, [1] S. Kalaimathy, [2] G. Mahadevan[3]
[1, 2] *Department of Mathematics, VHNSN College, Virudhunagar - 626001, India.*
[3]*Department of Mathematics, Gandhi gram Rural University, Gandhi gram - 624302, India.*

Abstract: *Let G (V, E) be a graph. A subset S of V is called a dominating set of G if every vertex in V-S is adjacent to at least one vertex in S. The domination number γ (G) is the minimum cardinality taken over all such dominating sets in G. A subset S of V is said to be a complementary connected dominating set (ccd-set) if S is a dominating set and < V-S > is connected. The chromatic number χ is the minimum number of colours required to colour all the vertices such that no two adjacent vertices receive the same colour. In this paper, we characterize the r - regular graphs with $\gamma_{cc} = \chi = 2$ and the 3 - regular graphs with $\gamma_{cc} = \chi = 3$.*

Keywords: *Dominating set, domination number, complementary connected domination number, chromatic number, regular graphs and cubic graphs.*

AMS Subject Classification Code: 05C (primary).

I. INTRODUCTION

Throughout this paper, by a graph we mean a finite, simple, connected and undirected graph G (V, E). The number of vertices in G is denoted by n. We denote a cycle on n vertices by C_n. Δ denotes the maximum degree in G. If S is a subset of V, then < S > denotes the induced subgraph of G induced by S. For any two subsets V_1 and V_2 of V, let E (V_1, V_2) denote the set of all edges with one end in V_1 and the other in V_2. The girth of G is the length of a shortest cycle in G if exists, otherwise it is infinite. The terms which are not defined here can be found in [3].

A subset S of V is called a dominating set of G if every vertex in V-S is adjacent to at least one vertex in S. The domination number γ(G) of G is the minimum number of vertices in a dominating set G. Various types of dominating set and domination number have been introduced and studied by several authors [1], [4], [5], [6], [7], [8], [9], [10], [11], [12], [13] and [16]. Many graph theoretic parameters such as χ, α, β etc. have been compared and combined with γ (G). Paulraj Joseph and Arumugam discussed the relationship between domination number and connectivity in [14], as well as domination number and chromatic number in [15].

T. Tamizhchelvam and B. Jayaprasad [17] have introduced the complementary connected domination number. A subset S of V is said to be a complementary connected dominating set (ccd-set) of G if S is a dominating set and < V-S > is connected. The minimum cardinality of S is called the complementary connected domination number and is denoted by γ_{cc}.

For example, the graph H_1 shown in Figure 1, has γ_{cc} = 3. Here S = {v_2, v_4, v_7} is a γ_{cc} - set of H_1. For the graph H_2 shown in Figure 1, γ_{cc} = 4 and S = {v_1, v_2, v_3, v_6} is a γ_{cc} - set of H_2.

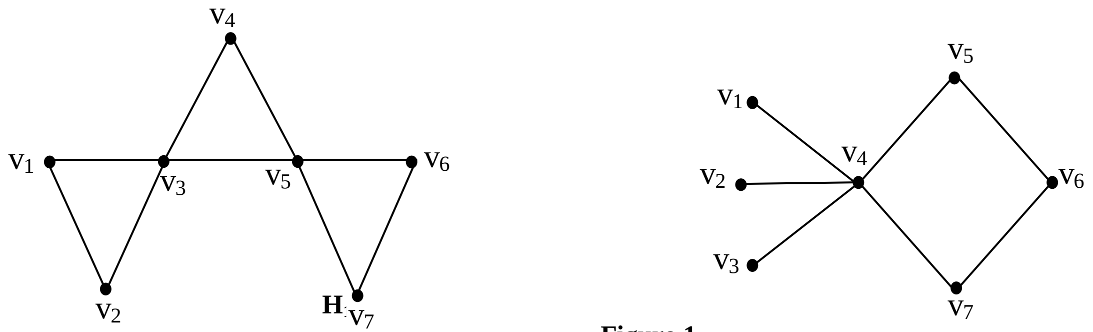
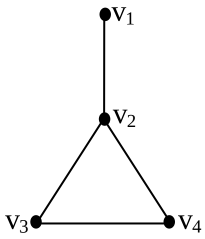


Figure 1

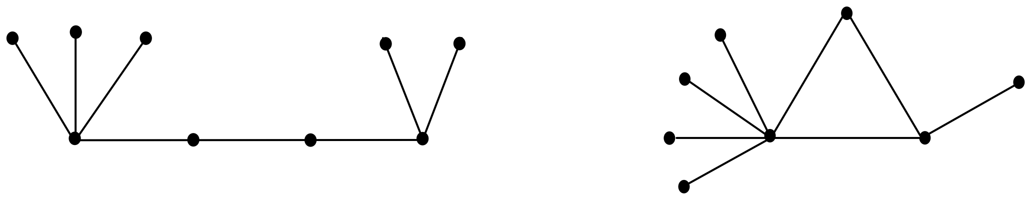
Note that a graph may have more than one γ_{cc} - set. For example, S_1 = {v_1, v_4, v_6} is also a γ_{cc} - set of H_1.
In [17], the following results have been obtained:
1. For n > 3, γ_{cc} (P_n) = n - 2.
2. For n $\geq$ 3, γ_{cc} (C_n) = n - 2.
3. For n $\geq$ 1, γ_{cc} ($K_{1, m}$) = m.
4. For any graph G, γ (G) $\leq$ γ_{cc} (G) and the inequality is strict.
For example, consider the graph G shown in Figure 2. Here γ (G) = 1 where as γ_{cc} (G) = 2.



G
Figure 2

In fact, we can easily note that for any integer $i \geq 0$, there exists a graph G with $\gamma_{cc}(G) - \gamma(G) = i$.
5. If P_k $(k \geq 4)$ is a subgraph of the graph G, then $\gamma_{cc}(G) \leq n - 2$.
6. Let G be a graph with $n \geq 3$. Then there exists a γ_{cc} - set S of G which contains all pendant vertices of G.
7. If $\gamma_{cc}(G) \leq n - 2$, then any γ_{cc} - set S of G contains all pendant vertices of G.
8. For any connected graph G with n vertices and m edges, $\left\lceil \frac{n}{\Delta+1} \right\rceil \leq \gamma_{cc} \leq 2m - n + 1$.

Let $P_k(m_1, m_2)$ where $k \geq 2$ and $m_1, m_2 \geq 1$ be the graph obtained by identifying the centers of the stars K_{1,m_1} and K_{1,m_2} at the ends of a path P_k respectively. The graph $C_3(m_1, m_2, 0)$ is obtained from C_3 by identifying the centers of the stars K_{1,m_1} and K_{1,m_2} at any two vertices of C_3 respectively.
For example, the graphs $P_4(3, 2)$ and $C_3(4, 1, 0)$ are shown in Figure 3.



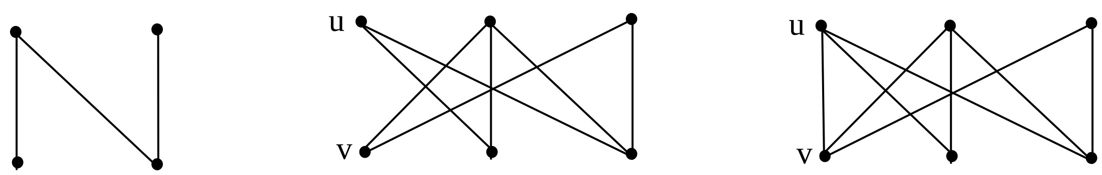
$P_4(3, 2)$ **$C_3(4, 1, 0)$**
Figure 3

In [2], S. Avadayappan and C.S. Senthilkumar have characterized the graphs with $\gamma_{cc} = n - 2$. More precisely, they have proved that for any graph G, $\gamma_{cc}(G) = n - 2$ if and only if G is isomorphic to any one of the following graphs: (i) $P_k(m_1, m_2)$ where $k \geq 2$, $m_1, m_2 \geq 1$, (ii) C_n, $n \geq 3$, and (iii) $C_3(m_1, m_2, 0)$ where $m_1, m_2 \geq 0$.

II. Complementary Connected Domination Number

In this section, we characterize the graphs with $\gamma_{cc} = \chi = 2$ and hence the r - regular graphs with $\gamma_{cc} = \chi = 2$.
Let G be a bipartite graph with bipartition (X, Y) where $X = \{u_1, u_2, \ldots, u_m\}$ and $Y = \{v_1, v_2, \ldots, v_n\}$. Then $G_{u,v}$ is the graph obtained from G by adding two new vertices u and v and the new edges uv_i, $1 \leq i \leq n$; vu_j, $1 \leq j \leq m$. Take $G^*_{u,v} = G_{u,v} + uv$. It is clear that $G_{u,v}$ and $G^*_{u,v}$ are bipartite graphs.

For example, the graph G and the corresponding graphs $G_{u,v}$ and $G^*_{u,v}$ are shown in Figure 4.



G **$G_{u,v}$** **$G^*_{u,v}$**
Figure 4

It is easy to note that for the graphs $G_{u,v}$ and $G^*_{u,v}$, $\gamma_{cc} = 2$. In fact, the converse is also true which is proved in the next theorem.

Theorem 1 Let G be a graph. Then $\gamma_{cc} = \chi = 2$ if and only if $G \cong H_{u,v}$ or $H^*_{u,v}$ for some connected bipartite graph H.

Proof Suppose $\gamma_{cc} = \chi = 2$. Then G is a bipartite graph with bipartition, say, (X, Y). Let S = {u, v} be a γ_{cc} - set. Then clearly both u and v cannot be in the same partition. Now, consider the bipartite graph H = G – S. If u and v are not adjacent in G, then $G \cong H_{u,v}$. Otherwise, $G \cong H^*_{u,v}$. The converse part is obvious. Hence the theorem. ■

The next theorem shows that there are only two r - regular graphs for which $\gamma_{cc} = \chi = 2$.

Theorem 2 Let G be an r - regular graph. Then $\gamma_{cc} = \chi = 2$ if and only if $G \cong K_{r,r}$ or $K_{r+1,r+1}$-F.

Proof Let G be an r - regular graph on n vertices with $\gamma_{cc} = \chi = 2$.

Then $2 = \gamma_{cc} \geq \left\lceil \frac{n}{\Delta+1} \right\rceil = \left\lceil \frac{n}{r+1} \right\rceil$. This implies $n \leq 2r + 2$. But $\chi = 2$ and hence G is a bipartite graph with bipartition (X, Y). Since G is r - regular, $|X| = |Y|$ and thus n = 2r or 2r + 2. If n = 2r, then clearly $G \cong K_{r,r}$. If n = 2r + 2, then $|X| = |Y| = r+1$ and so each vertex in X is not adjacent to exactly one vertex in Y and vice versa. Thus $G \cong K_{r+1,r+1}$ - F. The converse part is trivial. ■

2. 2 - REGULAR AND 3 - REGULAR GRAPHS

Let G be a connected 2 - regular graph. Then clearly G is a cycle. Moreover, for even cycles $\chi = 2$ where as for odd cycles, $\chi = 3$. Hence for cycles if $\gamma_{cc} = \chi$ then $\gamma_{cc} = 2$ or 3. But we have $\gamma_{cc}(C_n) = n - 2$, for $n \geq 3$. If $\gamma_{cc}(C_n) = 2$, then n = 4 and so $G \cong C_4$ and if $\gamma_{cc}(C_n) = 3$, then $G \cong C_5$. Thus, $G \cong C_4$ or C_5. Therefore, we can conclude that C_4 and C_5 are the only connected 2 - regular graphs for which $\gamma_{cc} = \chi$. Next, we characterize the 3 - regular graphs for which $\gamma_{cc} = \chi = 3$. To attain this we need Lemma 1.

Lemma 1 Let G be an r - regular graph on 2r + 2 vertices. If $\gamma_{cc}(G) = 2$, then there exist two non adjacent vertices u and v such that $N(u) \cap N(v) = \phi$.

Proof Let G be an r - regular graph on 2r+2 vertices. Suppose $\gamma_{cc}(G) = 2$. Let S = {u, v} be a γ_{cc} - set. Then $V(G) = N[u] \cup N[v]$. If u and v are adjacent, then $|N[u] \cup N[v]| \leq 2r$, which is a contradiction. Thus u and v are not adjacent. If $N(u) \cap N(v) \neq \phi$, then we get $|N[u] \cup N[v]| \leq 2r+1$, a contradiction. Hence, $N(u) \cap N(v) = \phi$. Thus there are two non adjacent vertices u and v such that $N(u) \cap N(v) = \phi$.
Hence the theorem. ■

Lemma 2 There does not exist a 3 - regular graph on 6 vertices with $\gamma_{cc}(G) = \chi(G) = 3$.

Proof Let G be a 3 - regular graph on 6 vertices with $\gamma_{cc} = \chi = 3$. Then the girth of G is 3 or 4. If it is 3, then $G \cong C_3 \times P_2$. Otherwise, $G \cong K_{3,3}$. But $\gamma_{cc}(C_3 \times P_2) = \gamma_{cc}(K_{3,3}) = 2$, which is a contradiction. ■

For further discussion, we need a list of graphs shown in Figure 5.

Lemma 3 Let G be a 3 - regular graph on 8 vertices. Then $\gamma_{cc}(G) = \chi(G) = 3$ if and only if $G \cong G_1$ or G_2.

Proof Let G be a 3 - regular graph on 8 vertices with $\gamma_{cc} = \chi = 3$. Then G has 12 edges. Let $v_1, v_2, \ldots, v_8$ be the vertices of G and let $S = \{v_6, v_7, v_8\}$ be a γ_{cc} - set. Take V_s = V- S. Since S is a γ_{cc} - set, $< V_s >$ is connected. Also $\Delta(V_s) = 2$ and hence $< V_s >$ is either P_5 or C_5.

Case (i) If $< V_s >$ is isomorphic to C_5, then $|E(V_S, S)| = 5$. This implies that < S > has 2 edges and 3 vertices. Therefore, $< S > = P_3 = v_6v_7v_8$. Without loss of generality, we can assume that v_7 is adjacent to v_1. If $N(v_6) = \{v_4, v_5, v_7\}$, then $N(v_8) = \{v_2, v_3, v_7\}$ and the resultant graph has $\gamma_{cc} = 2$ with $S = \{v_5, v_8\}$, a contradiction. When the neighbours of v_6 are $\{v_3, v_5, v_7\}$ (or $\{v_2, v_5, v_7\}$), we get the graph G_2 (or G_1).

Case (ii) $< V_s > \cong P_5$. In this case $|E(V_S, S)| = 7$. Thus, < S > has only one edge. That is, $< S > = P_2 \cup K_1$. Without loss of generality, we can assume that v_8 is the isolated vertex in < S >. Then it is enough to deal the case in which $N(v_6)$ and $N(v_8)$ have a common neighbour or the case in which $N(v_7)$ and $N(v_8)$ have a common neighbour. In the remaining cases, by Lemma 1, we get $\gamma_{cc} = 2$, which is impossible.

Suppose the neighbours of v_8 are $\{v_1, v_2, v_5\}$. When $N(v_7) = \{v_1, v_3, v_6\}$, the given graph has $\gamma_{cc} = 2$ with $S = \{v_2, v_6\}$ which is impossible. And the graph G_1 is obtained when the neighbours of v_7 are $\{v_1, v_4, v_6\}$. On the other side, let $\{v_1, v_3, v_5\}$ be the neighbours of v_8. If $N(v_7) = \{v_1, v_2, v_6\}$, then $N(v_6) = \{v_4, v_5, v_7\}$. Here $\{v_1, v_4\}$ is a ccd set, a contradiction. If $N(v_7) = \{v_1, v_4, v_6\}$, then the resultant graph is G_2.

Conversely, since for i = 1, 2, $rad(G_i) = diam(G_i) = 2$, any two non adjacent vertices have a common neighbour and thus $\gamma_{cc} \neq 2$. Also it is easy to see that $\gamma_{cc}(G_1) = \gamma_{cc}(G_2) = 3$. In addition, it is clear that $\chi(G_1) = \chi(G_2) = 3$.
This completes the lemma.

■

Lemma 4 Let G be a 3 - regular graph on 10 vertices. Then $\gamma_{cc}(G) = \chi(G) = 3$ if and only if $G \cong G_i$, $3 \leq i \leq 17$.

Proof Let G be a 3-regular graph on 10 vertices for which $\gamma_{cc}(G) = \chi(G) = 3$. Then G contains 15 edges. Let v_i, $1 \leq i \leq 10$ be the vertices of G and let $S = \{v_8, v_9, v_{10}\}$ be a γ_{cc}- set. Since G is 3-regular and by the definition of a γ_{cc} - set, every vertex in $< V_S >$ has degree 1 or 2. Hence $< V_S >$ is isomorphic to C_7 or P_7.

Case (i) $< V_s > \cong C_7$.

Since G is 3-regular, $|E(V_S, S)| = 7$. Then $< S >$ contains only one edge as G has 15 edges. Thus $<S> = P_2 \cup K_1$ in which v_{10} is the isolated vertex.

Let the three consecutive vertices of C_7, say, v_1, v_2, v_3 be the neighbours of v_{10}. Now, the vertex v_4 belongs to either v_9 or v_8. Without loss of generality, take $v_4 \in N(v_9)$. If $N(v_9) = \{v_4, v_5, v_8\}$, then $N(v_8) = \{v_6, v_7, v_9\}$ and the resultant graph is G_9. When the neighbours of v_9 are $\{v_4, v_6, v_8\}$ (or $\{v_4, v_7, v_8\}$), we get the graph G_{16} (or G_6). In a similar way, we can deal the case when $v_4 \in N(v_{10})$.

Let $N(v_{10}) = \{v_1, v_2, v_4\}$ and let $v_3 \in N(v_9)$. If we take the neighbours of v_9 as $\{v_3, v_5, v_8\}$, $\{v_3, v_6, v_8\}$ or $\{v_3, v_7, v_8\}$, then we get the graphs G_3, G_5 and G_7 respectively.

Let $N(v_{10}) = \{v_1, v_2, v_5\}$. Without loss of generality, let $v_3 \in N(v_9)$. Then the neighbours of v_9 are $\{v_3, v_4, v_8\}$, $\{v_3, v_6, v_8\}$ or $\{v_3, v_7, v_8\}$ and so we get the graphs respectively G_{12}, G_5 and G_4.

Let $N(v_{10}) = \{v_1, v_3, v_5\}$. Take $v_3 \in N(v_9)$. If $N(v_9)$ are $\{v_2, v_4, v_8\}$, $\{v_2, v_6, v_8\}$ or $\{v_4, v_6, v_9\}$, then we get the graphs G_4, G_{10} and G_{14} respectively.

Case (ii) $< V_s > \cong P_7$.

Since G is 3-regular, $|E(V_S, S)| = 9$. Thus $< S >$ has no edge, that is $< S > \cong K_3^c$.

Let $N(v_{10}) = \{v_1, v_2, v_3\}$. Then $N(v_7) = \{v_6, v_8, v_9\}$. Without loss of generality, we can take $v_1 \in N(v_9)$. One can easily check that if the neighbours of v_9 are $\{v_1, v_4, v_7\}$, $\{v_1, v_5, v_7\}$ or $\{v_4, v_6, v_7\}$, the corresponding graphs are isomorphic to G_8, G_6 and G_9 respectively.

Now, we consider the three consecutive internal vertices in P_7 are adjacent to a vertex, say v_{10} in $< S >$. That is $N(v_{10}) = \{v_2, v_3, v_4\}$. This implies that $N(v_1) = \{v_2, v_8, v_9\}$ and $N(v_8) = \{v_1, v_6, v_7\}$. And the only possibility is that $N(v_9) = \{v_1, v_5, v_7\}$. In this case, we get the graph isomorphic to G_6.

Take $N(v_{10}) = \{v_1, v_2, v_4\}$. If the neighbours of v_9 are $\{v_1, v_5, v_7\}$ (or $\{v_1, v_6, v_7\}$), then we get the graph isomorphic to G_7 (or G_3). The remaining possibilities are already discussed in the earlier cases.

Now, consider $N(v_{10}) = \{v_2, v_3, v_5\}$. This forces that $N(v_1) = \{v_2, v_8, v_9\}$ and $N(v_7) = \{v_6, v_8, v_9\}$. The only possibility is that $N(v_9) = \{v_1, v_4, v_7\}$, and the resultant graph is isomorphic to G_7.

Let $N(v_{10}) = \{v_3, v_4, v_6\}$. This implies that $N(v_1) = \{v_2, v_8, v_9\}$ and $N(v_7) = \{v_4, v_8, v_9\}$. Then $N(v_9) = \{v_1, v_2, v_7\}$ and the resultant graph is isomorphic to G_3.

Let $N(v_{10}) = \{v_4, v_5, v_7\}$. By leaving the repeated cases, we get the graph G_7 when the neighbours of v_9 are $\{v_1, v_2, v_6\}$ or $\{v_1, v_2, v_7\}$.

Suppose, $N(v_{10}) = \{v_1, v_2, v_5\}$. This implies that $N(v_1) = \{v_6, v_8, v_9\}$. When the neighbours of v_9 are $\{v_1, v_4, v_7\}$ or $\{v_1, v_6, v_7\}$, we get the graph isomorphic to G_{13} or G_{12}. The remaining cases have been dealt already.

Let $N(v_{10}) = \{v_2, v_3, v_6\}$. This forces that $N(v_1) = \{v_2, v_8, v_9\}$ and $N(v_7) = \{v_6, v_8, v_9\}$. The only possibility is $N(v_9) = \{v_1, v_4, v_7\}$and so we get the resultant graph isomorphic to G_5.

Let $N(v_{10}) = \{v_3, v_4, v_7\}$. This gives $N(v_1) = \{v_2, v_8, v_9\}$. By omitting the repeated cases, we get the graphs isomorphic to G_{12} and G_{17}, when the neighbours of v_9 are $\{v_1, v_2, v_5\}$(or $\{v_1, v_2, v_7\}$) and $\{v_1, v_2, v_6\}$ respectively.

Let $N(v_{10}) = \{v_1, v_2, v_6\}$ which implies that $N(v_7) = \{v_6, v_8, v_9\}$. The only case is that if $N(v_9) = \{v_1, v_4, v_7\}$, then we get the graph isomorphic to G_5.

Let $N(v_{10}) = \{v_2, v_3, v_7\}$. This forces that $N(v_1) = \{v_2, v_8, v_9\}$. If $N(v_9) = \{v_1, v_4, v_6\}$(or $\{v_1, v_4, v_7\}$), then the corresponding resultant graph is isomorphic to G_5(or G_{13}).

Let $N(v_{10}) = \{v_1, v_2, v_7\}$. The only possibility is that $N(v_9) = \{v_1, v_4, v_6\}$, and so the resultant graph is isomorphic to G_4.

Suppose $N(v_{10}) = \{v_2, v_4, v_6\}$. This implies that $N(v_1) = \{v_2, v_8, v_9\}$ and $N(v_7) = \{v_6, v_8, v_9\}$. The only case left is that $N(v_9) = \{v_1, v_3, v_7\}$ and $N(v_8) = \{v_1, v_5, v_7\}$. In this case, we get $\chi = 2$, a contradiction.

Let the neighbours of v_{10} be $\{v_1, v_3, v_6\}$. This forces that $N(v_7) = \{v_6, v_8, v_9\}$. By leaving the repeated possibilities, we get the isomorphic graphs G_{11} or G_{10}, when $N(v_9) = \{v_1, v_4, v_7\}$or $\{v_1, v_5, v_7\}$ respectively.

Let $N(v_{10}) = \{v_2, v_4, v_7\}$. Then $N(v_7) = \{v_2, v_8, v_9\}$ and hence $N(v_9) = \{v_1, v_3, v_6\}$. Then we get the graph isomorphic to G_{10}.

Conversely, suppose $\gamma_{cc} = 2$ and let $\{v_r, v_s\}$ be a γ_{cc} - set. Then, $|N(v_r) \cup N(v_s)| \leq 8$ a contradiction, since G has 10 vertices. Thus $\gamma_{cc} \neq 2$. Also it is clear that $\gamma_{cc}(G_i) = 3$, for all $3 \leq i \leq 17$. Moreover, one can easily verify that $\chi(G_i) = 3$,for all i, $3 \leq i \leq 17$.

Hence the lemma ■

Lemma 5 Let G be a 3- regular graph on 12 vertices. Then $\gamma_{cc}(G) = \chi(G) = 3$ if and only if $G \cong G_i$, $18 \le i \le 42$.

Proof Let G be a 3-regular graph on 12 vertices for which $\gamma_{cc}(G) = \chi(G) = 3$. Then G contains 18 edges. Let $\{v_i / i = 1, 2,\dots,12\}$ be the vertex set of G and let $S = \{v_{10}, v_{11}, v_{12}\}$ be a γ_{cc}-set. Since G is 3 - regular and since S is a dominating set, every vertex in $< V_S >$ is of degree 1 or 2. Therefore $< V_S >$ is isomorphic to P_9 or C_9. If $< V_s > \cong P_9$, then $|E(V_S, S)| = 11$, which is impossible as G contains 18 edges. This forces that $< V_s > \cong C_9$. In this case, $|E(V_S, S)| = 9$ which means that $< S >$ contains no edge. That is, $< S >$ is isomorphic to K_3^c.

Case (i) let the neighbours of v_{12} be three consecutive vertices of C_9 say, v_1, v_2, v_3. Now, the vertex v_4 belongs to either $N(v_{10})$ or $N(v_{11})$. Without loss of generality, we can assume that $v_4 \in N(v_{11})$. If $N(v_{11}) = \{v_4, v_5, v_6\}$, then $N(v_{10}) = \{v_7, v_8, v_9\}$ which gives the graph isomorphic to G_{38}. In a similar way, one can easily check that if the neighbours of v_{11} are $\{v_4, v_5, v_7\}$, $\{v_4, v_5, v_8\}$ (or $\{v_4, v_6, v_7\}$), $\{v_4, v_5, v_9\}$(or $\{v_4, v_8, v_9\}$), $\{v_4, v_6, v_8\}$, $\{v_4, v_6, v_9\}$(or $\{v_4, v_7, v_9\}$) and $\{v_4, v_7, v_8\}$ the corresponding graphs are isomorphic to G_{26}, G_{35}, G_{25}, G_{29}, G_{30} and G_{39} respectively.

Case(ii) Let $\{v_1, v_2, v_4\}$ be the neighbours of v_{12} and let $v_3 \in N(v_{11})$. Then we get the graphs isomorphic to G_{27}, G_{22}, G_{23}, G_{28}, G_{19} and G_{36} when we take the neighbours of v_{11} as $\{v_3, v_5, v_7\}$, $\{v_3, v_5, v_8\}$, $\{v_3, v_6, v_7\}$, $\{v_3, v_6, v_8\}$, $\{v_3, v_6, v_9\}$ and $\{v_3, v_7, v_8\}$ respectively. The case when $v_3 \in N(v_{10})$ follows in a similar manner. By case(i), we can omit the remaining cases in which one of the vertices of v_{11} and v_{10} have the three consecutive neighbours.

Case(iii) Let $N(v_{12}) = \{v_1, v_2, v_5\}$. Without loss of generality, assume that $v_3 \in N(v_{11})$. Then the neighbours of v_{11} can be $\{v_3, v_6, v_7\}$, $\{v_3, v_6, v_8\}$, $\{v_3, v_6, v_9\}$, $\{v_3, v_7, v_8\}$ or $\{v_3, v_7, v_9\}$. The corresponding graphs are isomorphic to G_{31}, G_{24}, G_{20}, G_{33} and G_{32} respectively.

Case(iv) Let $N(v_{12}) = \{v_1, v_2, v_6\}$. Let $N(v_{11})$ contain v_3. Then the neighbours of v_{11} are $\{v_3, v_4, v_8\}$, $\{v_3, v_5, v_8\}$or $\{v_3, v_7, v_8\}$. Thus the graphs are G_{34}, G_{41} or G_{40} respectively.

Case(v) Suppose $N(v_{12}) =\{v_1, v_3, v_5\}$. Without loss of generality, let v_2 belong to $N(v_{11})$. Then we get the isomorphic graphs G_{37} and G_{42} respective to the neighbours of v_{11} which are $\{v_2, v_6, v_8\}$ and $\{v_2, v_7, v_9\}$.

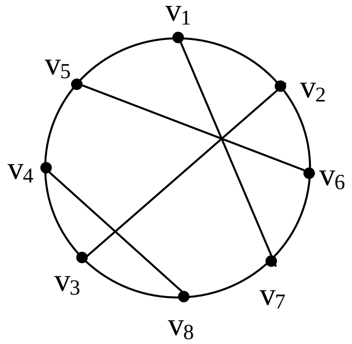
Case(vi) Let $N(v_{12}) =\{v_1, v_3, v_6\}$. Then the neighbours of v_{11} are $\{v_2, v_5, v_8\}$ and so we get the graph isomorphic to G_{21}. The remaining possibilities have been dealt in the earlier cases.

Case(vii) Let the neighbours of v_{12} be $\{v_1, v_4, v_7\}$. By leaving the repeated possibilities, we get the only case that $N(v_{11}) = \{v_2, v_5, v_8\}$. Then the resultant graph is isomorphic to G_{18}.

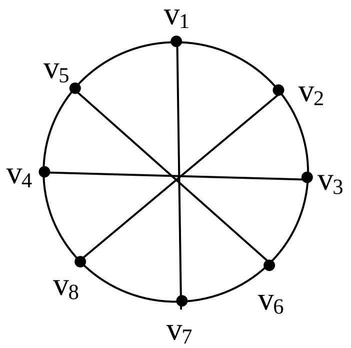
Conversely, it is easy to verify that each G_i has a ccd-set with three elements and so $\gamma_{cc}(G_i) \le 3$. In addition, each G_i is a 3 - regular graph on 12 vertices. Therefore $\gamma_{cc}(G_i) \ge \left\lceil \frac{n}{\Delta+1} \right\rceil = 3$. This means that $\gamma_{cc}(G_i) = 3$, for all i, $18 \le i \le 42$. Also, one can easily check that $\chi(G_i) = 3$, for all i, $18 \le i \le 42$. Hence $\gamma_{cc}(G_i) = \chi(G_i) = 3$, for all i, $18 \le i \le 42$. Thus the converse follows. ■

Theorem 3 Let G be a 3 - regular graph. Then $\gamma_{cc} = \chi = 3$ if and only if $G \cong G_i$, for $1 \le i \le 42$.

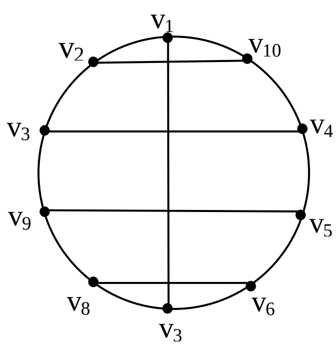
Proof Let G be any one of the above 42 graphs. Then it can be easily verified that $\gamma_{cc} = \chi = 3$. Conversely, we know that $3 = \gamma_{cc} \ge \left\lceil \frac{n}{\Delta+1} \right\rceil = \left\lceil \frac{n}{4} \right\rceil$. This implies that $n \le 12$. Since G is 3 - regular, n = 6, 8, 10 and 12. Now the result follows from the Lemmas 2, 3, 4 and 5.



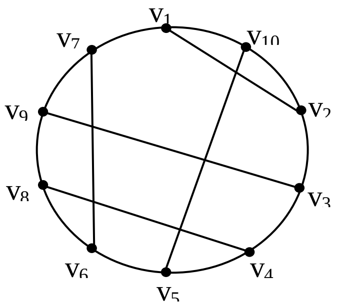
G_1



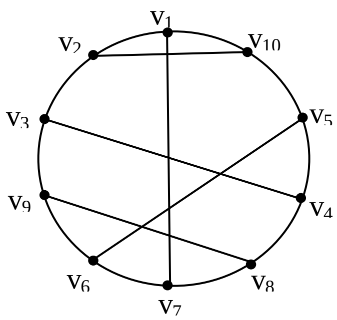
G_2



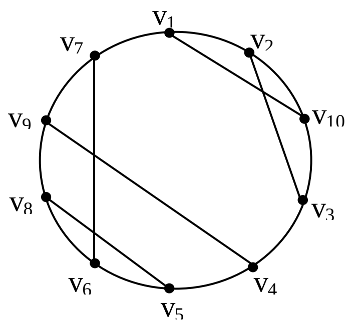
G_3



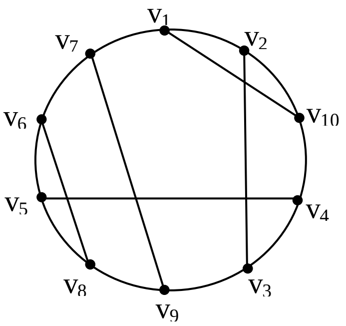
G_4



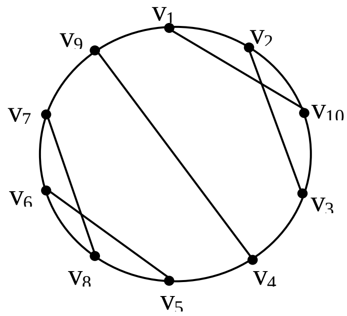
G_5



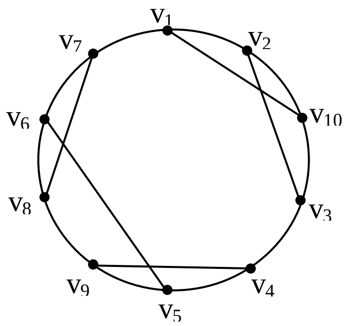
G_6



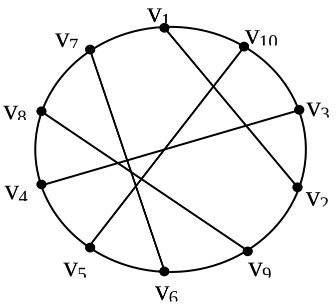
G_7



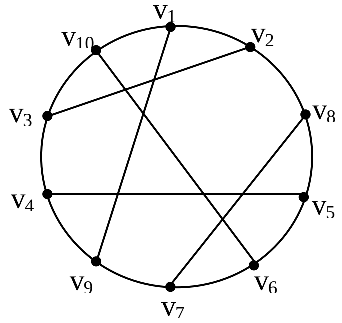
G_8



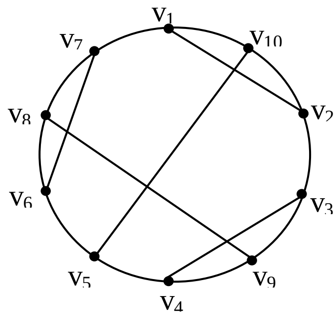
G_9



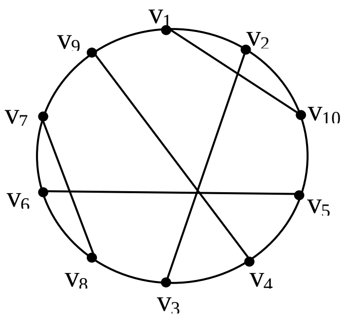
G_{10}



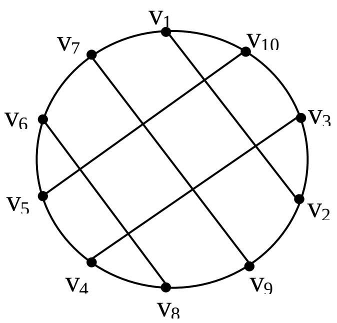
G_{11}



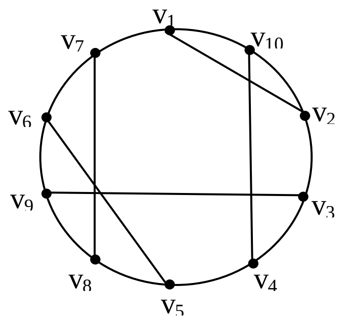
G_{12}



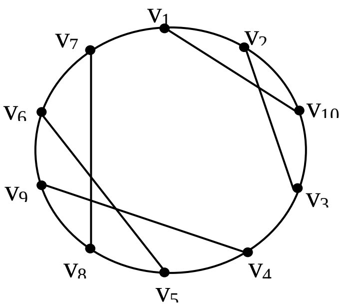
G_{13}



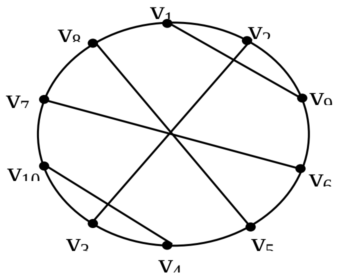
G_{14}



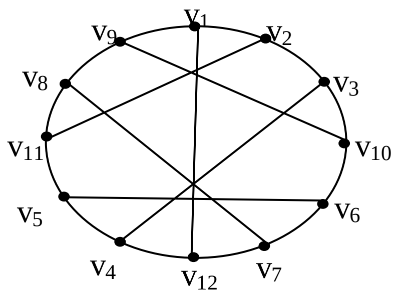
G_{15}



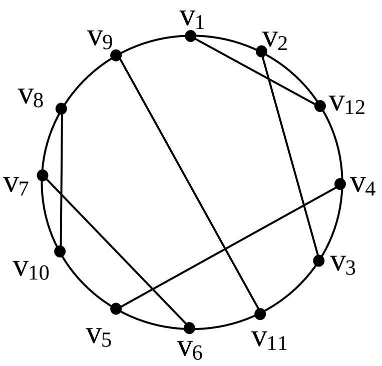
G_{16}



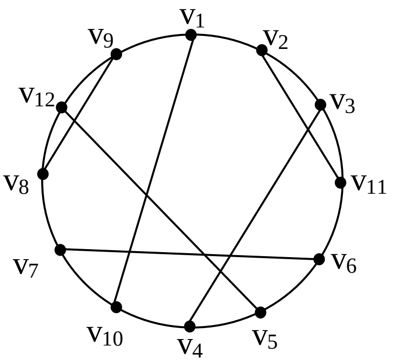
G_{17}



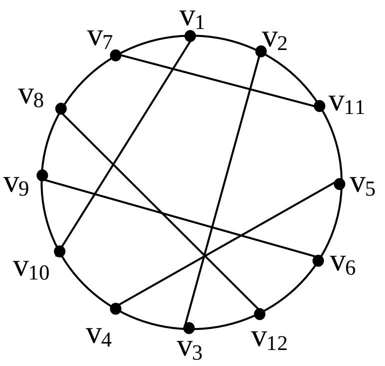
G_{18}



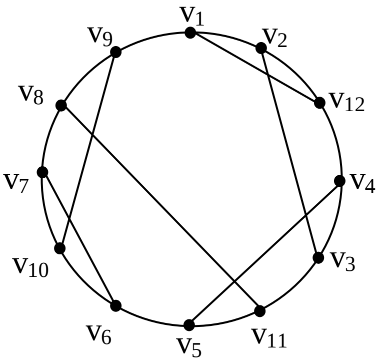
G_{19}



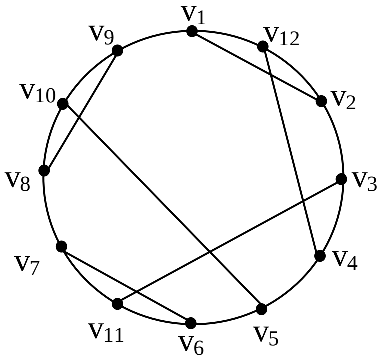
G_{20}



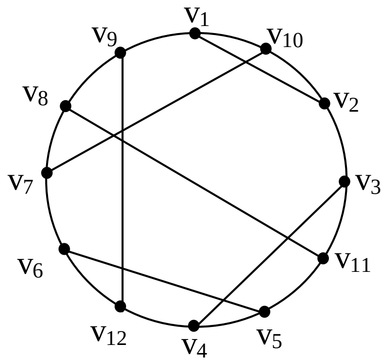
G_{21}



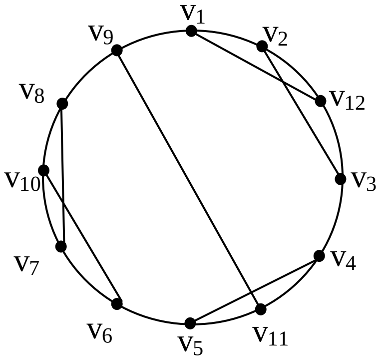
G_{22}



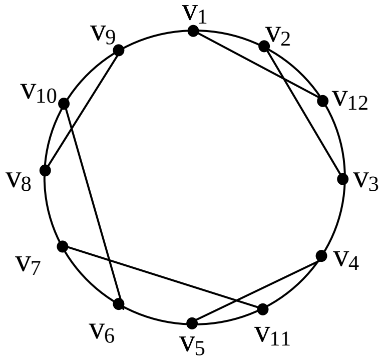
G_{23}



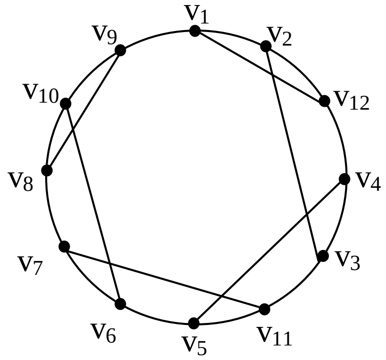
G_{24}



G_{25}



G_{26}



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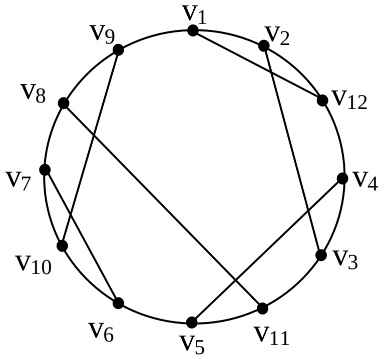
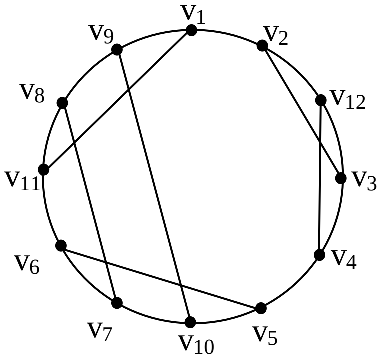
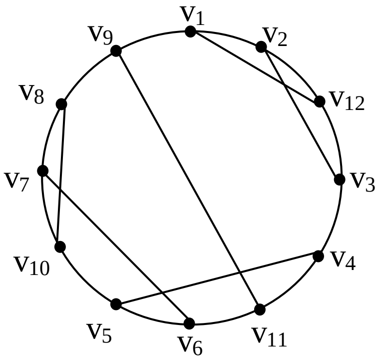


G_{28}



G_{29}



G_{30}

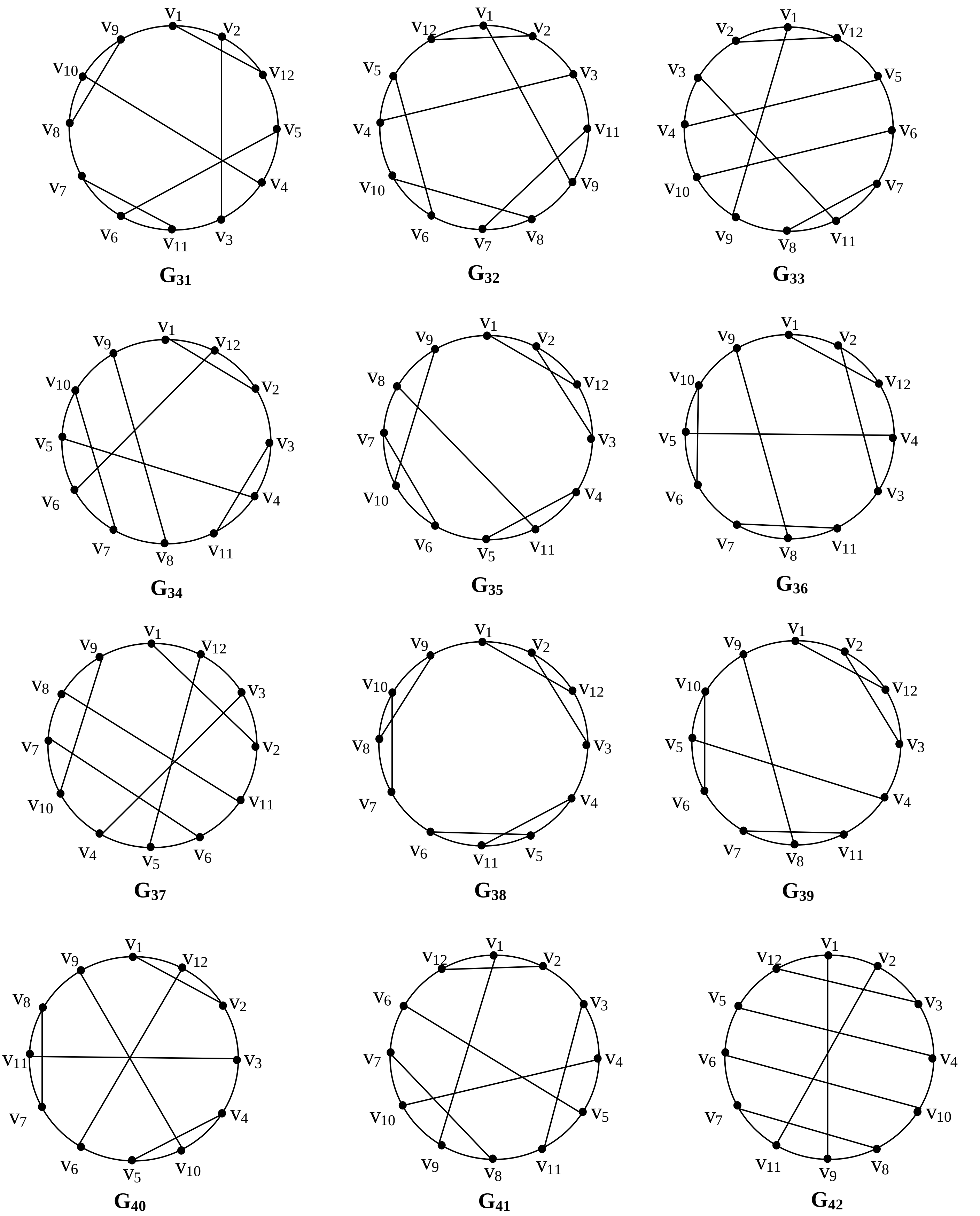


Figure 5

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