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An Application of 2d Rigid Link Theory in Flexible Link Manipulator: Kinematics

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ABSTRACT: *This paper presents kinematic and dynamic modeling techniques for flexible robots. . The main emphasis is to discretize whole flexible link in very small parts each considering as a rigid link. This approach is based on a "discretization method". In kinematics position or deflection is solved by deflection of cantilever beam theory while orientation is solved by forward kinematics. Dynamics is solved by applying Lagrange-Euler formulation.*

Keywords: *Discretization, Dynamics, Flexible link manipulator (FLM), Kinematics, Rigid link manipulator*

I. INTRODUCTION

1.1 Kinematics of rigid link

Kinematics is the branch of physics which involves the description of motion, without considering the forces which produce the motion (dynamics or kinetics, on the other hand, involves an examination of both a description of motion and the forces which produce it). A subset of kinematics is that of rigid body kinematics concerns the motions of one or more rigid bodies. A rigid body experiences zero deformation. In other words, all points lying on a rigid body experience no motion relative to each other [1].

There are seven methods to solve kinematics as:

1. Forward Kinematics
2. Inverse Kinematics
3. Algebraic method
4. Geometric method
5. Symbolic elimination method
6. Continuation method
7. Iterative method

1.2 Kinematics of flexible link

In rigid robot manipulator kinematics can be described by employing Denavit-Hartenberg representation. The main idea is to use 4×4 transformation matrices which can be determine uniquely as a function of only 4 parameters. However this procedure cannot be used directly to describe the kinematics of a FLM due to link deformation. In order to overcome this drawback, the procedure has been modified by including some transformation matrices which take link elasticity in account. A description of Denavit- Hartenberg representation of rigid body is assumed to be known.

In general, the homogeneous transformation of frame i with respect to the base frame can be characterized through the following composition of consecutive transformation:

$$^{0}T_i=T_i=A_1E_1A_2E_2\ldots A_{i-1}E_{i-1}=T_{i-1}A_i \qquad (1)$$

or $$T_1=A_1 \qquad (2)$$

Where A_i is the standard homogeneous transformation matrix for joint i due to rigid motion and E_i is the homogeneous transformation matrix due to link i length and deflection. Notice that, even though the superscript is not explicitly indicated, each transformation matrix is referred to the frame determine by the preceding transformation.

The transformation matrix A_i can be computed just like in the case of the rigid body. On the other hand, the transformation matrix E_i deserves special attention.

Assuming small link deformation, E_i can be expressed by:

$$Ei = \begin{matrix} 1 & \cos\left(\frac{\pi}{2}+\theta_{zi}\right) & \cos\left(\frac{\pi}{2}-\theta_{yi}\right) & l_i+\delta_{xi} \\ \cos\left(\frac{\pi}{2}-\theta_{zi}\right) & 1 & \cos\left(\frac{\pi}{2}+\theta_{xi}\right) & \delta_{yi} \\ \cos\left(\frac{\pi}{2}+\theta_{yi}\right) & \cos\left(\frac{\pi}{2}-\theta_{xi}\right) & 1 & \delta_{zi} \\ 0 & 0 & 0 & 1 \end{matrix}$$

(3)[2].

Where θ_{xi}, θ_{yi}, θ_{zi} are the angles of rotation, and δ_{Xi}, δ_{Yi}, δ_{Zi} represent link i deformation along x, y, z, respectively, being l_i the length of link without deformation. By taking into account the fact cos(π/2+ α) =-sin(α) and assuming small angles, so that sin(α)= α is valid, the matrix E_i can be approximated as

$$Ei = \begin{matrix} 1 & -\theta_{zi} & \theta_{yi} & l_i+\delta_{xi} \\ \theta_{zi} & 1 & -\theta_{xi} & \delta_{yi} \\ -\theta_{yi} & -\theta_{xi} & 1 & \delta_{zi} \\ 0 & 0 & 0 & 1 \end{matrix} \qquad (4)$$

1.3 Dynamics of rigid-link

Manipulator dynamics is concerned with the equations of motion, the way in which the manipulator moves in response to torques applied by the actuators, or external forces. The history and mathematics of the dynamics of serial-link manipulators is well covered by Paul and Hollerbach[2].

There are methods by which we can solve the dynamics of the rigid manipulator as:

1. Newton-Euler formulation
2. Langrange-Euler formulation
3. Generalized d'Alembert equation of motion

There are two problems related to manipulator dynamics that are important to solve:

- Inverse dynamics in which the manipulator's equations of motion are solved for given motion to determine the generalized forces and
- Direct dynamics in which the equations of motion are integrated to determine the generalized coordinate response to applied generalized forces.

To derive the dynamic equations of motion of manipulators types of methods can be followed for rigid link.

1.3.1 Lagrange-Euler Formulation

The general motion equation of a manipulator can conveniently be expressed through the direct application of the Lagrange-Euler formulation to non-conservative system. Many investigators utilize the Denavit-Hartenberg matrix representation to describe the spatial displacement between the neighboring link coordinate frame to obtain link kinematics information and they employ the L-E equation to derive dynamic equation of manipulator.

The derivation of the dynamic equation of an n degree of freedom manipulator is based on the understanding of:

- The 4X4 homogeneous coordinate transformation matrix, ${}^{i-1}A_i$, which describes the spatial relationship between the ith and (i-1)th link coordinate frame. It relates the point fixed in link i expressed in homogeneous coordinates with respect to i^{th} coordinate system to the $(i-1)^{th}$ coordinates system.

- The Lagrange-Euler equation

$$\frac{d}{dt}\left[\frac{\partial L}{\partial \dot{q}i}\right]-\frac{\partial L}{\partial qi}=\tau i \quad i=1,2,\ldots,n \qquad (5)$$

Where

L= Lagrangian function= kinetic energy K – potential energy P

K= total kinetic energy of the robot arm

P= total potential energy of robot arm

q_i=generalized coordinates of the robot arm

$\dot{q}_i$=first time derivative of the generalized coordinates, q_i

τi= generalized force or (torque) applied to the system at joint i to drive link i

From the above lagrangian equation one is required to properly choose a set of generalized coordinates to describe the system. Generalized coordinates are used as a convenient set of a coordinates which completely describe the location of a system with respect to reference coordinate frame.

1.4 Dynamics of flexible link

There are methods by which we can solve the dynamics of the flexible manipulator as:

1. Newton-Euler formulation
2. Langrange-Euler formulation
3. Generalized d'Alembert equation of motion
4. Recursive Gibbs-Appell formulation
5. Finite dimensional approximation
6. Hamilton's principle and FE approach
7. Assume mode method and Langrange approach

1.4.1 Dynamics using Lagrange-Euler approach

In order to obtain a set of differential equations of motion to adequately describe the dynamics of a flexible link manipulator, the Lagrange-Euler approach can be used. A system with n generalized coordinates q_i must satisfy n differential equations of the form

$$\frac{d}{dt}\left[\frac{\partial L}{\partial \dot{q}i}\right]-\frac{\partial L}{\partial qi}+\frac{\partial D}{\partial \dot{q}i}=\tau i \qquad i=1,2,\ldots,n \qquad (6)$$

Where L is the so called Lagrangian which is given by L=K-P; K represents the kinetic energy of the system and P the potential energy. Also, D is the Rayleigh's dissipation function which allows dissipative effects to be included, and τ_i is the generalized force acting on q_i.

II. MATHEMATICAL ANALYSIS

2.1 Problem statement

A flexible robotic arm having length of 0.500 m, width of 0.08 m, thickness of 0.001 is having point load at B as shown in figure of 0.200 kg. Modulus of elasticity of link material is 20 GPa consider as an example.

Kinematics and Dynamics of the flexible link using rigid link theories (deflection of cantilever beam, Forward kinematics and Lagrange-Euler approach) with discretization approach are to be solved.

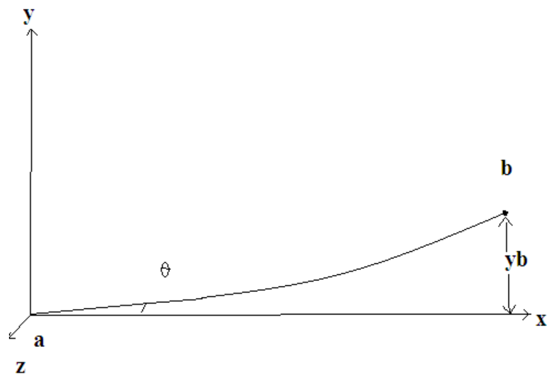

Fig.1 Deflection of flexible link

SOLUTION

L=500 mm E=2×10^{5}N/mm^2

w=80 mm

t=1 mm W=20 KN/m

Let,

- Moment of inertia of link is I, given as

$$I=\frac{wt^3}{12} \qquad (7)$$

$$I=\frac{80\times 1^3}{12} \qquad (8)$$

I=6.66mm^4

Flexure rigidity F=EI=13.32× 10^{6}N-mm^2

Deflection of link is given as

$$y_b=\frac{WL^3}{3EI} \qquad (9)$$

$$y_b=\frac{20\times(500)^3}{3\times 13.32\times 10^6}$$

y_b=62.56 mm

Now to use cantilever beam deflection theory for flexible link, discretize whole link in 50 parts each having length of 10 mm and find deflection of point B.

So deflection for single link,

$$y_1=\frac{Wl_1{}^3}{3EI}$$

$$y_1 = \frac{20 \times (10)^3}{3 \times 13.32 \times 10^6}$$

y_1=0.0005mm

Deflection of all the links (1 to 50) calculated as above and angle of each from horizontal axis.

TABLEDeflection and angle of each link

Link no. (n)	Deflection from fixed point A (y_{n})	Deflection (Yn)= y_n-y_{n-1}	θ from each point θn=$\tan^{-1}$Yn
1	0.0005	0.0005	0.0028
2	0.0040	0.0035	0.0200
3	1.0130	0.0090	0.0515
4	0.0320	0.0190	0.1088
5	0.0625	0.0305	0.1747
6	0.1081	0.0456	0.2612
7	0.1716	0.0635	0.3638
8	0.262	0.0846	0.4847
9	0.3648	0.1086	0.6222
10	0.5005	0.1357	0.7774
11	0.6661	0.1656	0.9487
12	0.8648	0.1987	1.1383
13	1.0995	0.2347	1.3444
14	1.3733	0.2738	1.5683
15	1.6891	0.3158	1.8087
16	2.0500	0.3609	2.0669
17	2.4589	0.40879	2.3415
18	2.9181	0.4600	2.6337
19	3.4329	0.5140	2.9424
20	4.0040	0.5711	3.2686
21	4.6351	0.6311	3.6111
22	5.3293	0.6942	3.9711
23	6.0895	0.7602	3.9711
24	6.9189	0.8294	4.7412
25	7.8203	0.9014	5.1507
26	8.7967	0.9764	5.5596
27	9.8513	1.0546	6.0201
28	10.9869	1.1356	6.4787
29	12.2067	1.2198	6.9545
30	13.5135	1.3068	7.4452
31	14.9104	1.3969	7.9521
32	16.4004	1.4900	8.4747
33	17.9864	1.5860	9.0120
34	19.6716	1.6852	9.5656
35	21.4589	1.7873	10.1334
36	23.3513	1.8924	10.7159
37	25.3518	2.0005	11.3126
38	27.4634	2.1116	11.9234
39	29.6891	2.2257	12.5477
40	32.0320	2.3429	13.1859
41	34.4949	2.4629	13.8360
42	37.0810	2.5861	14.4995
43	39.7932	2.7122	15.1747
44	42.6346	2.8414	15.8619
45	45.6081	2.9735	16.5598
46	48.7167	3.1086	17.2683
47	51.9634	3.2467	17.9870
48	55.3513	3.3879	18.7158
49	58.8833	3.5320	19.4532
50	62.5625	3.6795	20.2010

For whole link,

$$y_{total} = y_b = \sum_{1}^{50} y_n - y_{n-1} \quad (10)$$

$$y_{total} = y_b = 62.5628\ mm$$

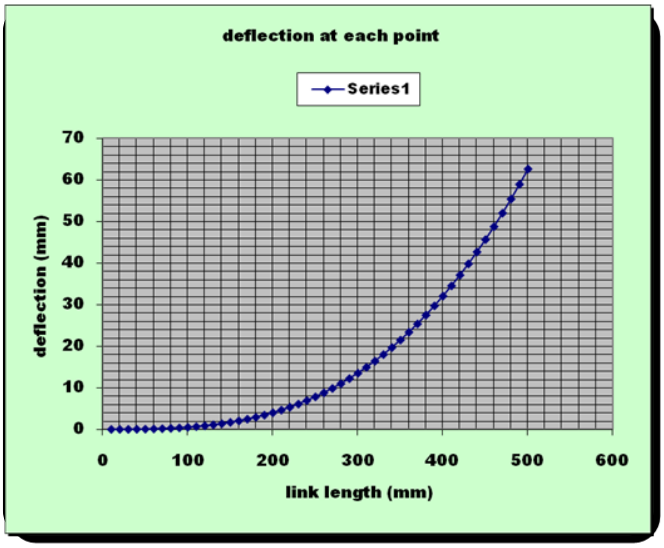

Fig.2 Deflection at each point

The deflection of link is 62.5628 mm which has been manually founded. The same result has been founded using ANSYS is 62.563 mm with two digit precision.

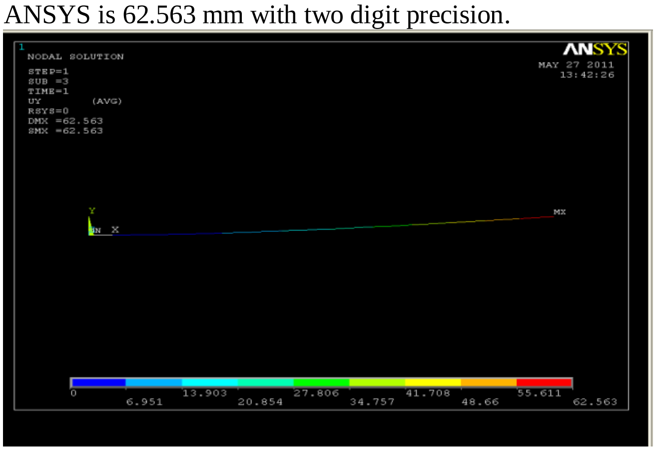

Fig.3Nodal solution Y-component displacement

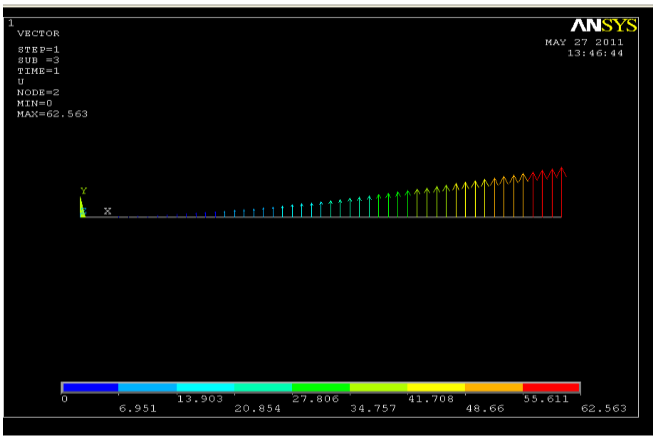

Fig.4Predefine vector plot on ANSYS

Same as to find rotation of link discretize whole link in 50 equal parts and find each link rotation matrix and use forward transformation for n rigid link will give rotation matrix of flexible link.

Rotation of θ angle with respect to Z- axis[1].

$$R_{z.\theta} = \begin{matrix} cos\theta & -sin\theta & 0 \\ sin\theta & cos\theta & 0 \\ 0 & 0 & 1 \end{matrix} \quad (11)$$

For link-1-50, rotation matrices are

So $\mathbf{{}^{0}R_{50}={}^{0}R_{1}{}^{1}R_{2}{}^{2}R_{3}\,....{}^{49}R_{50}}$

$$ {}^{0}R_{10} = \begin{matrix} 0.337 & -0.899 & 0 \\ 0.899 & 0.337 & 0 \\ 0 & 0 & 1 \end{matrix} $$

III. CONCLUSION

From above given approach of discretizing the flexible link in number of rigid links one can solve the kinematics and dynamics of the flexible link. Usually this approach is very much applicable in hyper-redundant or serpentine type of robot.

Future scope:Solve the dynamics by discretizing approach. By applying Lagrange-Euler theory of motion equation of rigid link for number of links and considering that the flexible link is made of from number of small rigid links one can solve dynamics for flexible link.

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