International Journal of Modern Engineering Research (IJMER)
www.ijmer.com Vol.2, Issue.5, Sep.-Oct. 2012 pp-3123-3127 ISSN: 2249-6645

Unsteady MHD free convective flow past a semi-infinite vertical porous plate

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Abstract : *In this article, we studied the effects of variable viscosity and thermal conducti Dept. of vity on an unsteady two-dimensional laminar flow of a viscous incompressible electrically conducting fluid past a semi-infinite vertical plate taking into account the mass transfer. The fluid viscosity is assumed to vary as a linear function of temperature. The governing equations for the flow are transformed into a system of non-linear ordinary differential equations are solved by a perturbation technique. The effects of the various parameters on the velocity, temperature, concentration and skin-friction profiles are presented graphically and discussed qualitatively.*

Keywords: *Radiation parameter, Thermal conductivity, MHD, porous medium and viscosity.*

I. Introduction

Natural convection flow over vertical surfaces immersed in porous media has paramount importance because of its potential applications in soil physics, geohydrology, and filtration of solids from liquids, chemical engineering and biological systems. Study of fluid flow in porous medium is based upon the empirically determined Darcy's law. Such flows are considered to be useful in diminishing the free convection, which would otherwise occur intensely on a vertical heated surface. In addition, recent developments in modern technology have intensified more interest of many researchers in studies of heat and mass transfer in fluids due to its wide applications in geothermal and oil reservoir engineering as well as other geo-physical and astrophysical studies.

Cramer, K. R. and Pai, S. I. [1] taken transverse applied magnetic field and magnetic Reynolds number are assumed to be very small, so that the induced magnetic field is negligible Muthucumaraswamy et al. [2] have studied the effect of homogenous chemical reaction of first order and free convection on the oscillating infinite vertical plate with variable temperature and mass diffusion. Sharma [3] investigate the effect of periodic heat and mass transfer on the unsteady free convection flow past a vertical flat plate in slipflow regime when suction velocity oscillates in time. Chaudhary and Jha [4] studied the effects of chemical reactions on MHD micropolar fluid flow past a vertical plate in slip-flow regime. Anjalidevi et al. [5] have examined the effect of chemical reaction on the flow in the presence of heat transfer and magnetic field. Muthucumaraswamy et al. [6] have investigated the effect of thermal radiation effects on flow past an impulsively started infinite isothermal vertical plate in the presence of first order chemical reaction. Moreover, Al-Odat and Al-Azab [7] studied the influence of magnetic field on unsteady free convective heat and mass transfer flow along an impulsively started semi-infinite vertical plate taking into account a homogeneous chemical reaction of first order.. The effect of radiation on the heat and fluid flow over an unsteady stretching surface has been analyzed by El-Aziz [8]. Singh et. al. [9] studied the heat transfer over stretching surface in porous media with transverse magnetic field. Singh et. al. [10] and [11] also investigated MHD oblique stagnation-point flow towards a stretching sheet with heat transfer for steady and unsteady cases. Elbashbeshy et. al. [12] investigated the effects of thermal radiation and magnetic field on unsteady boundary layer mixed convection flow and heat transfer problem from a vertical porous stretching surface. Ahmed Sahin studied influence of chemical reaction on transient MHD free Convective flow over a vertical plate. Recently, the chemical reaction, heat and mass transfer on MHD flow over a vertical stretching surface with heat source and thermal stratification have been presented by Kandasamy et al.[13]. The opposing buoyancy effects on simultaneous heat and mass transfer by natural convection in a fluid saturated porous medium investigated by Angirasa et al.[14]. Ahmed [15] investigates the effects of unsteady free convective MHD flow through a porous medium bounded by an infinite vertical porous plate. Ahmed Sahin [16] studied the Magneto hydrodynamic and chemical reaction effects on unsteady flow, heat and mass transfer characteristics in a viscous, incompressible and electrically conduction fluid over a semi-infinite vertical porous plate in a slip-flow regime.

The objective of the present study is to investigate the effect of various parameters like chemical Reaction parameter, thermal Grashof number, mass Grashof number, magnetic field parameter, radiation parameter on convective heat transfer along an semi-infinite vertical plate in porous medium. The governing non-linear partial differential equations are first transformed into a dimensionless form and thus resulting non-similar set of equations has been solved using the perturbation technique. Results are presented graphically and discussed quantitatively for parameter values of practical interest from physical point of view.

II. Mathematical Formulation

In a situation of two dimensional unsteady laminar natural convection flows of a viscous, incompressible, electrically conducting, radiating fluid past an impulsively started semi-infinite vertical plate in the presence of transverse magnetic field with viscous dissipation is considered. The fluid is assumed to be gray, absorbingemitting but non-scattering. The x - axis is taken along the plate in the upward direction and the y - axis is taken normal to it. The fluid is assumed to be slightly conducting and hence the magnetic Reynolds number is much less than unity and the induced magnetic field is negligible in comparison with the transverse applied magnetic field. Initially, it is assumed that the plate and the fluid are at the same temperature T'_∞ and concentration level C'_∞ everywhere in the fluid. At time $t' > 0$, the plate starts moving impulsively in the vertical direction with constant velocity u_0 against the gravitational field. Also, the temperature of the plate and the concentration level near the plate are raised to T'_w and C'_w, respectively and are maintained constantly thereafter. It is assumed that the concentration C' of the diffusing species in the binary mixture is very less in the comparison to the other chemical species, which are present and hence the Soret and Dufour effects are negligible. It is also assumed that there is no chemical reaction between the diffusing species and the fluid. Then, under the above assumptions, in the absence of an input electric field, the governing boundary layer equations with Boussinesq's approximation are

Continuous equation

$$\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}=0 \qquad (1)$$

Momentum conservation

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=g\beta(T-T_\infty)+ g\beta^*(C-C_\infty)+\nu\frac{\partial^2 u}{\partial y^2}-\frac{\sigma B_0^2}{\rho}u-\frac{\nu}{K'}u \qquad (2)$$

Energy conservation

$$\frac{\partial T}{\partial t}+u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}-\frac{1}{\rho c_p}\frac{\partial q_r}{\partial y}+\frac{\nu}{c_p}\left(\frac{\partial u}{\partial y}\right)^2 \qquad (3)$$

Species conservation

$$\frac{\partial C}{\partial t}+u\frac{\partial C}{\partial x}+v\frac{\partial C}{\partial y}=D\frac{\partial^2 C}{\partial y^2}-K'_r(C-C_\infty) \qquad (4)$$

The initial and boundary conditions are as follows:

$$\left.\begin{array}{l} t\le 0, u=0,\ \ v=0,\ T=T_\infty,\ C=C_\infty\ \forall\ y \\ t>0, u=u_0,\ v=0, T=T_w,\ \ C=C_w\ \text{ at }\ y=0 \\ u\to 0,\ T\to T_\infty,\ C'\to C_\infty \quad \text{as}\ \ y\to\infty \end{array}\right\} \qquad (5)$$

Thermal radiation is assumed to be present in the form of a unidirectional flux in the y - direction i.e., q_r (transverse to the vertical surface). By using the Rosseland approximation, the radiative heat flux q_r is given by

$$q_r=-\frac{4\sigma_s}{3k_c}\frac{\partial T'^4}{\partial y} \qquad (6)$$

where σ_s is the Stefan-Boltzmann constant and K_c - the mean absorption coefficient. In the Rosseland approximation, the present analysis is limited to optically thick fluids. If temperature differences within the flow are sufficiently small, then equation (6) can be linearized by expanding T'^4 into the Taylor series about T'_∞, which after neglecting higher order terms takes the form:

$$T'^4\cong 4T'^3_\infty T'-3T'^4_\infty \qquad (7)$$

In view of equations (6) and (7), equation (3) reduces to

$$\frac{\partial T'}{\partial t'}+u\frac{\partial T'}{\partial x}+v\frac{\partial T'}{\partial y}=\alpha\frac{\partial^2 T'}{\partial y^2}+\frac{16\sigma_s T'^3_\infty}{3k_c\sigma c_p}\frac{\partial^2 T'}{\partial y^2}+\frac{\nu}{c_p}\left(\frac{\partial u}{\partial y}\right)^2 \qquad (8)$$

The non-dimensionless quantities introduced in these equations are defined as

$$\left.\begin{array}{l} X=\dfrac{xu_0}{\nu},\ Y=\dfrac{yu^0}{\nu},t=\dfrac{t'u_0^2}{\nu},K_r=\dfrac{K'_r\nu}{u_0^2},U=\dfrac{u}{u_0}, \\ V=\dfrac{v}{u_0},r=\dfrac{\nu g\beta(T'_w-T'_\infty)}{u_0^3},\ Gm=\dfrac{\nu g\beta^*(C'_w-C'_\infty)}{u_0^3}, \\ N=\dfrac{k_c k}{4\sigma_s T'^3_\infty},M=\dfrac{\sigma\beta_0^2\nu}{u_0^2},T=\dfrac{T'-T'_\infty}{T'_w-T'_\infty},\ C=\dfrac{C'-C'_\infty}{C'_w-C'_\infty}, \\ K=\dfrac{K'u_0^2}{\nu^2}.\ \Pr=\dfrac{\nu}{\alpha},\ Sc=\dfrac{\nu}{D}, \end{array}\right\} \qquad (9)$$

In a situation where only one dimensional flow is considered, the above set of equations (1), (2), (8) and (4) are reduced to the following non-dimensional form:

$$\frac{\partial V}{\partial Y}=0\Rightarrow V=-V_0\ \text{(where } V_0=1) \qquad (10)$$

$$\frac{\partial U}{\partial t}-\frac{\partial U}{\partial Y}=GrT+GmC+\frac{\partial^2 U}{\partial Y^2}-\left(M+\frac{1}{K}\right)U \qquad (11)$$

$$\frac{\partial T}{\partial t}-\frac{\partial T}{\partial Y}=\frac{1}{\Pr}\left(1+\frac{4}{3N}\right)\frac{\partial^2 T}{\partial Y^2} \qquad (12)$$

$$\frac{\partial C}{\partial t}-\frac{\partial C}{\partial Y}=\frac{1}{Sc}\frac{\partial^2 C}{\partial Y^2}-K_r C \qquad (13)$$

The corresponding initial and boundary conditions are as follows

$$\left.\begin{array}{l} t\le 0:\ U=0,\ T=0 \quad C=0\ \ \forall\ y \\ t>0:\ U=1,\ T=1, \quad C=1 \quad \text{at }\ Y=0 \\ \qquad U\to 0,\ T\to 0, \quad C\to 0 \quad \text{as }\ Y\to\infty \end{array}\right\} \qquad (14)$$

III. Solution Of The Problem

In order to solve equations (11) - (13) with respect to the boundary conditions (14) for the flow, let us take

$$U(y,t)=U_0(y)+U_1(y)e^{\omega t} \qquad (15)$$

$$T(y,t)=T_0(y)+T_1(y)e^{\omega t} \qquad (16)$$

$$C(y,t)=C_0(y)+C_1(y)e^{\omega t} \qquad (17)$$

Substituting the Equations (15) - (17) in Equations (11) – (13), we obtain:

$$u_0''+u_0'-(M+1/K)u_0=-[GrT_0+GmC_0] \qquad (18)$$

$$u_1''+u_1'-(M+1/K+\omega)u_1=-[GrT_1+GmC_1] \qquad (19)$$

$$T_0''+K_1^2T_0'=0 \qquad (20)$$

$$T_1''+K_1^2T_1'-\omega K_1^2T_1=0 \qquad (21)$$

$$C_0''+ScC_0'-ScK_rC_0=0 \qquad (22)$$

$$C_1''+ScC_1'-Sc(K_r+\omega)C_1=0 \qquad (23)$$

where prime denotes ordinary differentiation with respect to y.

The corresponding boundary conditions can be written as

$$u_0 = 0,\ T_0 = 1, T_0 = 1,\ C_0 = 1, C_1 = 0 \quad \text{at} \quad y = 0 \qquad (24)$$
$$u_0 = 0,\ \ T_0 = T_1 = 0,\ C_0 = C_1 = 0 \qquad \text{as} \quad y \to \infty$$

Solving equations (19) – (23) under the boundary conditions (24), we obtain the velocity, temperature and concentration distribution in the boundary layer as:

$$U(y,t) = (1 + A_3)e^{-m_3 y} - A_1 e^{-m_2 y} - A_2 e^{-m_1 y}$$
$$T(y,t) = e^{-m_2 y}$$
$$C(y,t) = e^{-m_1 y}$$

where

$$m_1 = 0.5\left[Sc + \sqrt{Sc^2 + 4K_r Sc}\right],$$
$$n_1 = 1 + 4/3N,\ m_2 = \sqrt{\Pr/n_1};$$
$$A_1 = Gr/\left[m_2^2 - m_2 - (M + 1/K)\right],$$
$$A_2 = Gm/\left[m_1^2 - m_1 - (M + 1/K)\right]$$
$$A_3 = A_1 + A_2; K_1 = \frac{\Pr}{1 + 4/3N}$$

Skin-friction: The dimensionless shearing stress on the surface of a body, due to the fluid motion, is known as skin-friction and is defined by the Newton's law of viscosity.

The skin-friction is

$$\tau = \left(\frac{\partial U}{\partial y}\right)_{y=0} = -m_3(1 + A_3) + A_1 m_2 + A_2 m_1$$

IV. Results And Discussion

In order to get a physical insight in to the problem the effects of various governing parameters on the physical quantities are computed and represented in Figures 1-13 and discussed in detail.The formulation of the effects of chemical reaction and heat absorption on MHD convective flow and mass transfer of an incompressible, viscous fluid along a semi infinite vertical porous moving plate in a porous medium has been performed in the preceding sections. This enables us to carry out the numerical calculations for the distribution of the velocity, temperature and concentration across the boundary layer for various values of the parameters.

The influence of Magnetic field on the velocity profiles has been studied in Fig .1. It is seen that the increase in the applied magnetic intensity contributes to the decrease in the velocity. Further, it is seen that the magnetic influence does not contribute significantly as we move away from the bounding surface. The influence of the porosity of the boundary on the velocity of the fluid medium has been shown in Fig 2. It is seen that as the porosity of the fluid bed increases, the velocity also increases which is in tune with the realistic situation. Further, the porosity of the boundary does not influence of the fluid motion as we move far away from the bounding surface. The contribution of radiation parameter on the velocity profiles is noticed in Fig.3. It is observed that the radiation parameter increases, the velocity field is an increasing. Further, it is noticed that the velocity decreases as we move away from the plate which is found to be independent of radiation parameter. The effect of Prandtl number on the velocity profiles has been illustrated in Fig.4. It is observed that as the Prandtl number increases, the velocity decreases in general. The dispersion in the velocity profiles is found to be more significant for smaller values of Pr and not that significant at higher values of Prandtl number.

The influence of Schmidt number *Sc* on velocity profiles has been illustrated in Fig.5. It is observed that, while all other participating parameters are held constant and *Sc* is increased, it is seen that the velocity decreases in general. Further, it is noticed that as we move far away from the plate, the fluid velocity goes down. The effect of Grashof number on the velocity profiles as shown in Fig. 6. Increase in *Gr* contributes to an increase in velocity when all other parameters that appear in the velocity field are held constant. Also it is noticed that as we move away from the plate the influence of *Gr* is not that significant. The effect of modified Grashof number *Gc* on the velocity profiles is observed in Fig.7. Increase in *Gc* is found to influence the velocity to increase. Also, it is seen that as we move far away from the plate it is seen that the effect of *Gc* is found to be not that significant. Fig.8 shows that the effect of increasing the chemical reaction parameter on velocity profiles. It is noticed that velocity of flow field are decreasing, as the values of chemical reaction are increasing. The Effect of Prandtl number on the temperature field has been illustrated in Fig.9. It is observed that as the Prandtl number increases, the temperature in the fluid medium decreases. Also, as we move away from the boundary, the Prandtl number has not much of significant influence on the temperature. The dispersion is not found to be significant. Fig.10 illustrates the influence of the radiation parameter on the temperature profiles in the boundary layer. As radiation parameter increases, temperature distributions increase when the other physical parameters are fixed.

The influence of Schmidt number on the concentration is illustrated in Fig. 11. It is observed that increase in Sc contributes to decrease of concentration of the fluid medium. Further, it is seen that Sc does not contributes much to the concentration field as we move far away from the bounding surface. Fig.12 shows that the effect of increasing the chemical reaction parameter on concentration profiles. It is noticed that species concentration are decreasing, as the values of chemical reaction are increasing. The effect of chemical reaction on velocity and temperature is less dominant in comparison to concentration. Skin friction for various values of magnetic field strength is portrayed through Fig 13. It is seen that skin friction decreases, as magnetic parameter increases, whereas it is increasing when radiation parameter is increasing.

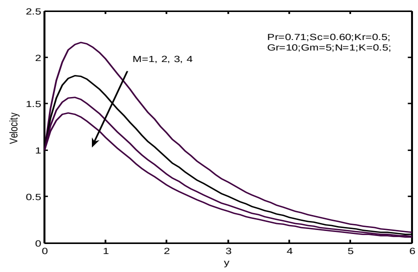

Fig.1. Effects of magnetic parameter on velocity profiles.

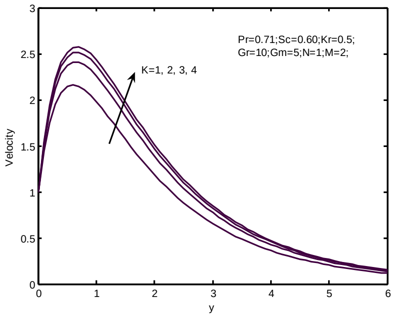

Fig.2. Effects of permeability parameter on velocity profiles.

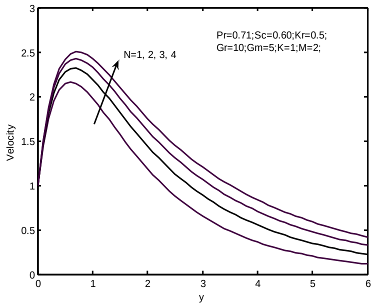

Fig.3. Effects of radiation parameter on velocity profiles.

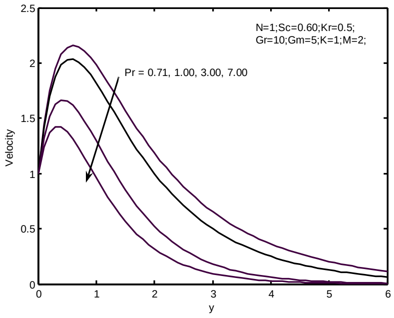

Fig.4. Effects of Prandtl number on velocity profiles.

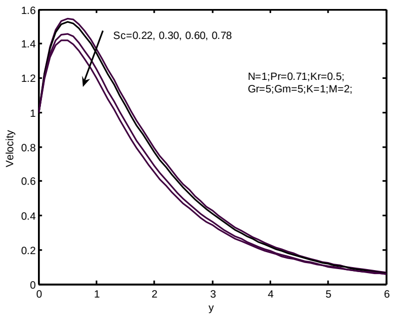

Fig.5. Effects of Schmidt number on velocity profiles.

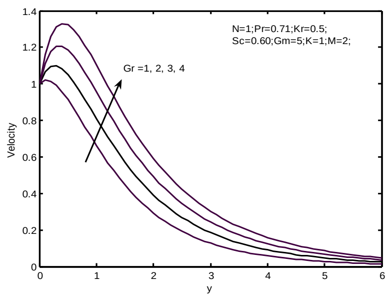

Fig.6. Effects of Grashof number on velocity profiles.

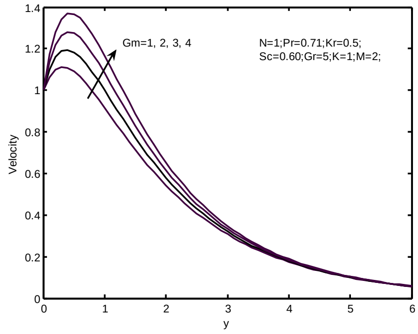

Fig.7. Effects of modified Grashof number on velocity profiles.

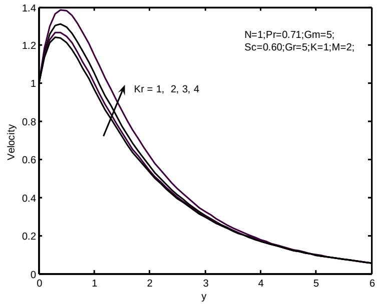

Fig.8. Effects of magnetic parameter on velocity profiles.

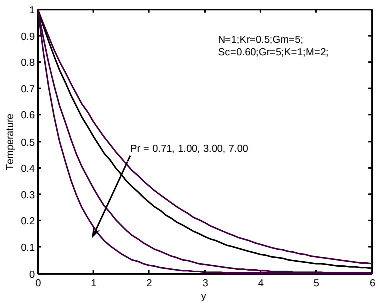

Fig.9. Effects of prandtl number on temperature profiles.

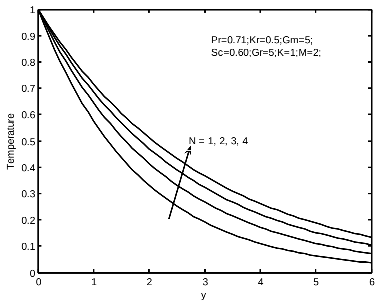

Fig.10. Effects of radiation parameter on temperature profiles.

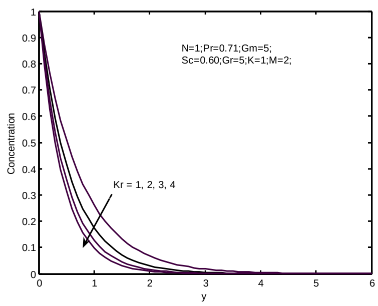

Fig.11. Effects of chemical reaction parameter on concentration profiles

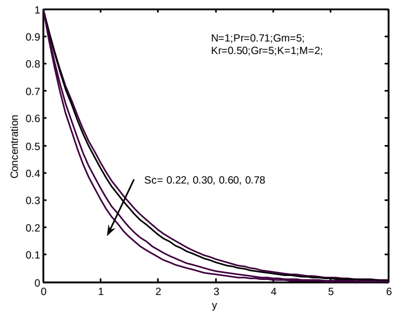

Fig.12. Effects of Schmidt number on concentration profiles.

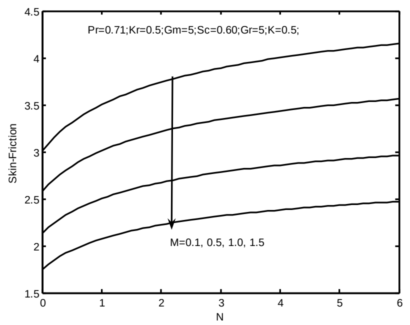

Fig.13. Effects of magnetic parameter on skin-friction.

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