

A Study of Net Section Failure between Two Equal Cracks in an Infinite Plate

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ABSTRACT: Multi site damage is one of the important aspects to be studied to ensure the safety of the aircraft structure. The rivet hole locations are one of the stress concentration regions. The current study includes a panel which represents the fuselage splice joint. The fuselage splice joint is a location where it experiences the uniform stress field at many rivet locations in a row. This study has relevance in the structural integrity evaluation of aging transport aircraft due to multisite damage. Fatigue cracks will emanate from the rivet holes simultaneously as they experience identical stresses due to internal pressure. In service, the cracks in the fuselage will grow due to internal pressurization load cycling. The objective is to investigate the first failure mechanism out of two competing mechanisms of failure; Failure due to fracture or Failure due to plastic collapse at the net section between two advancing crack tips.

Key words: Fuselage of an aircraft, Fatigue, Net section, Stress intensity factor, finite element analysis

I. INTRODUCTION

The primary objective of the aircraft structure is to carry the required flight loads with as little weight as possible. Today's airplanes use the most advanced lightweight materials and the most advanced structural design and analysis tools to produce the most efficient structures possible.

An ideal aircraft structure would be designed so that every part fails at exactly the same limit load and fatigues at exactly the same number of cycles and these failure conditions are selected, so that they just cannot happen under normal operating conditions. The ideal structure also would have no margin above these conditions because that just means extra weight.

The airframe consists of the fuselage, which is the main component of the airplane. The simultaneous presence of cracks in the same structural element is usually referred to as multi-site fatigue damage. Aging aircraft may develop multiple site fatigue damage that can reduce the structural integrity of fuselage structures.

The recent concept of damage tolerance supposes an aircraft structure to be redundant so that a catastrophic failure should not occur after fatigue failure of a structural element. As a consequence the concept also admits the existence of cracks in the aircraft structure. The probability of crack existence is particularly enhanced in ageing aircraft. In these structures multiple-site damage [1] [2] (MSD) is also more likely to occur. Some examples are frequently referenced, such as the Aloha accident of the Boeing 737, the C5A wing and JAL accident.

The MSD [3] [8] problem is generally associated with a large number of small neighboring cracks located in one line. As a more or less uniform stress field is required for MSD, the cracks mostly originate at the edges of several adjacent and collinear fastener holes in longitudinal skin splices [4] of a pressurized fuselage structure. They occur at the same time, grow and can suddenly coalesce to form a single critical crack, which can lead to a catastrophic failure of the structure. The critical size of the individual cracks can be relatively small, even less than the length easily detected during visual in-service inspections. With MSD the fatigue crack growth and fracture characteristics are significantly different from the characteristics of the isolated cracks. The fatigue lifetime becomes shorter than that of a single-site crack having the same length.

The existence of small cracks emanating from adjacent rivet holes in a fuselage [5] lap splice joint is of major concern. Small collinear cracks greatly reduce the residual strength of a panel with a lead crack.

Thus there is a need to conduct detailed fracture analysis of the crack link up phenomenon in butt-splice joints and Z-stiffener with rivet-loaded fasteners.

Multiple Site fatigue Damage (MSD) – as in the Aloha Airlines Boeing 737 – where fatigue cracks occur at many locations in the same structural element, such that fatigue cracks may coalesce to form one large crack. Failure of specimen [1] due to MSD is shown in the fig. 1

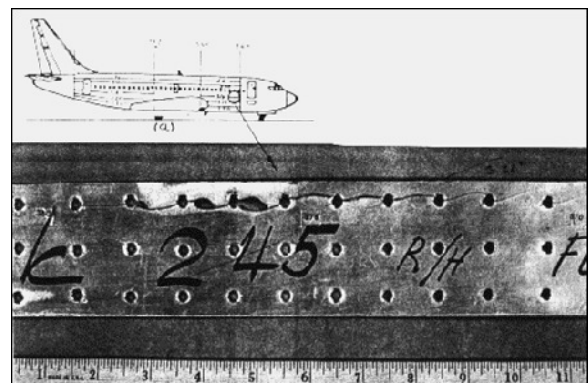


Figure 1: Multi-site damage (MSD) at a B-737 fuselage lap joint [1].

II. MATERIAL USED

Material used for the analysis of fuselage splice joint is Aluminum 2024 –T3 and its composition and properties are given in Table 1 and Table 2.

Table 1: Material compositions

COMPONENT	WEIGHT PERCENTAGE
Aluminum	90.7-94.7
Chromium	Max 0.1
Copper	3.8-4.9
Ferrous	Max 0.5
Magnesium	1.2-1.8
Manganese	0.3-0.9
Titanium	Max 0.15
Zinc	Max 0.25
Other, Total	Max 0.15

Table 2: Material Properties in Al 2024-T3

Properties	Material Aluminum 2024-T3
Density	27.27 N/mm ³
Ultimate Tensile Strength	483 N/mm ²
Tensile Yield Strength	362 N/mm ²
Modulus of Elasticity	72000 N/mm ²
Poisson's Ratio	0.33
Fracture Toughness	98.90 MPa√m

III. GEOMETRICAL CONFIGURATION

The first step is to understand the complex loading conditions in a fuselage structure. The pressurization of the fuselage causes the structure to expand outward like simple balloon. This expansion creates the hoop stress in the circumferential and an axial stress in the longitudinal direction. Due to this complexity in structure, loading conditions and test set-up simplification to simpler test specimen is required.

With full pressurization, the skin and underlying structure will move outward. It is not too difficult to see that a frame or stiffener will not move the same distance as the skin would due to higher local stiffness, thus creating differences in outward movements and higher hoop stresses in the skin between the frames.

Setting up a test as large as a full-scale aircraft structure requires an enormous amount of time and money. Reducing full scale-test to a more simple, easier to understand test specimen such as barrel or fuselage panel including stiffeners and frames reduces the size of the test. Elimination of the stiffeners, frames and curvature reduces the structure to flat sheet longitudinal splice and circumferential butt joints.

The global analysis of the structure is carried out to find the stress distribution. Riveted connection is the common feature in the built up airframe structure. The fatigue crack will initiate from the locations of the maximum tensile stress. The rivet hole locations are one of the stress concentration regions. Therefore rivet hole locations are the most probable location for the fatigue crack initiation.

The fuselage splice joint with Z-Stiffener is the location where it experiences the uniform stress field at many rivet locations in a row. Therefore fine meshing is done at the splice joint with Z-stiffener location to achieve the exactness of the stress and the riveting is being done by 1D element for global model of the fuselage.

Local panel is the sectional cut out of the fuselage to do stress analysis by validating local panel with hoop stress obtaining from local panel is equal to the hoop stress obtained from the global model, so by this way we can reduce the time consumed for analysis without compromise on the result variation.

Local panel with rivet holes are next step after the local panel with 1D rivet element, for our problem rivet holes should be there for crack initiation and crack propagation. Here also the hoop stress is validated with global model by applying same uniformly distributed tension load for local panel with rivet holes. So at the rivet holes in the direction of load transfer, the multiple points of semicircle are constrained to transfer the load to obtain the uniform load distribution on the semicircle of the rivet holes in direction of load, which is obtained in practical rivets on the rivet holes. So, it confirms that except at rivet holes, the remaining part of the panel is having the same hoop stress as obtained for global model of the fuselage.

IV. STRESS ANALYSIS

The global analysis of the structure is carried out to find the stress distribution. The fatigue crack will initiate from the locations of the maximum tensile stress. The rivet hole locations are one of the stress concentration regions. Therefore rivet hole locations are the most probable location for the fatigue crack initiation.

4.1. LOAD CALCULATIONS AND GLOBAL MODEL DIMENSIONS FOR THE FUSELAGE

- Length of the fuselage(L) = 2500 mm
- Radius of the fuselage(R) = 1600 mm
- Width of the splice plate(b) = 70 mm
- Length of the splice plate(L) = 2500 mm
- Thickness of the fuselage skin(T) = 1.8 mm
- Thickness of the splice plate(t) = 2 mm
- Diameter of the Rivet Hole(d) = 4.8 mm
- Internal pressurization(P_r) = 0.06695 N/mm²

Both the ends of fuselage are constrained for rotation and translation and internal pressurization is applied.

Hoop stress = (P_r × D) / (2 × T) ----- (1)

Hoop stress (σ_h) = (0.06695 × 3200) / (2 × 1.8)

Hoop stress (σ_h) = 59.51 N/mm²

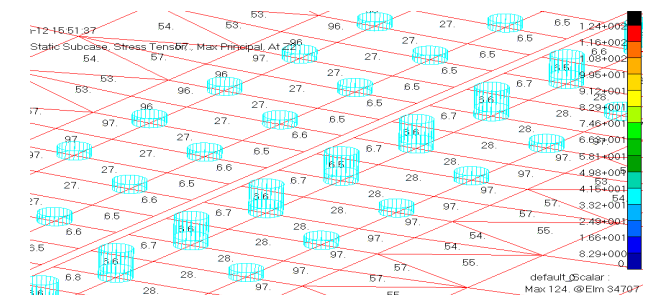


Figure 2: stresses are maximum at rivet holes in global model of the fuselage for 1D rivet element

It can be observed from the Figure 2 that the stress is distributed uniformly and the maximum stress is experienced in the riveted splice joint region for global model for 1D rivet element.

4.2. STRESS ANALYSIS OF THE LOCAL PANEL

A local analysis was done which represents the fuselage splice joint panel. Loading and boundary conditions for the local analysis of the panel are

Length of the panel =1200mm
 Width of the panel =500mm

The total tension load [9] acts on the fuselage structure from the global analysis is found as 53559.47 N. This total load is converted into uniformly distributed load (UDL) and applied at the top side of the panel. Uniformly distributed load of 107.12 N/mm was applied at top end of the plate and other end is fixed. A two dimensional linear static stress analysis is carried out using finite element analysis software tool. Mesh independent stress magnitudes are obtained through iterative mesh refinement process. Aluminum 2024-T3 is well-known aluminum alloy is used for the panel analysis.

It can be observed that the same Hoop stress value is experienced in both global and local analysis by applying same boundary conditions. Even the rivet loading are similar in both the cases. This is the indication to proceed further.

Large structures are usually assemblies of smaller parts are joined together by the variety of production techniques. There are two important joining methods namely; adhesive bonding and mechanically fastening. Mechanically fastened joints are an interesting subject to investigate. The present investigation focuses on solid rivets installed in aluminum plate. The expansion of solid rivet in the rivet hole is important with respect to the fatigue properties of joints. In reality rivet holes will be present. So the stress analysis of panel with rivet holes was carried out with the same applied boundary conditions by adapting multipoint constraint (MPC) at rivet holes as shown in Figure 3.

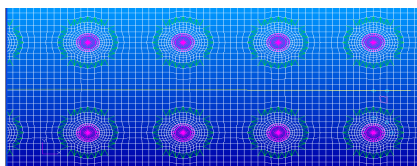


Figure 3: Meshed panel with riveted loading with MPC

V. CRACK ANALYSIS AT THE RIVETED HOLE LOCATION

CALCULATION OF SIF FOR 20 mm PITCH RIVET HOLE

Thus the periodic increase of crack lengths has applied for the panel. By applying same boundary conditions as the stress analysis has been carried out. Near the crack tip the stress intensity factor values are calculated for all the elemental crack length of 20mm pitch rivet hole.

Around the riveted hole section fine meshing has been done with quad4 a 2D elements will results the accurate stress values.

Stress Intensity Factor (SIF), $K = \sigma_R \sqrt{\pi a}$

Where, σ_R = Remote stress in N/mm² and 2a = Crack length in mm

By using FEA and Modified Virtual Crack Closure Integral (MVCCI) [6-7] method:

$$K = \sqrt{G \times E}$$

Where, G = strain energy release rate and E = Young's modulus of elasticity in N/mm²

$$G = \frac{(\Delta v \times f)}{2 \Delta a \times t}$$

Where, Δa =Element edge length at the crack tip in mm, Δv =Crack opening displacement in mm, f = Force at the crack tip in N and t =Skin thickness in mm

Considering the following analysis for 3rd iteration of 7.5mm crack length of 20 mm pitch rivet hole, where the SIF (k) for mode I can be calculated as

$$K = \sqrt{G \times E}$$

$$K = \sqrt{(0.015 \times 438 \times 72000) / (2 \times 0.45 \times 1.8)}$$

$$K = 540.37 \text{ MPa}\sqrt{\text{mm}}$$

$$K = 17.1 \text{ MPa}\sqrt{\text{m}}$$

MVCCI procedure is used for calculating the stress intensity factor. Similarly for all crack lengths and for different pitch holes, the stress intensity factor values has been calculated and compare those values with the fracture toughness of the material, where the fracture toughness of the material is 98.90 MPa√m. once the stress intensity factor value reaches the fracture toughness of the material then it leads to failure through fracture. This is one mode of failure. The other mode of failure is the structure with stress concentration may fails by net section yielding due to local yield at the crack tip. The average stress value between the two advancing crack tips will be compared with the yield strength of the material, where the yield strength of the material is 362 N/mm².

VI. STUDY OF NET SECTION FAILURE

The net section is the region or cross sectional area available between two rivet holes to carry whatever the load the component has to transfer. Due to rivet holes the stress concentration will be more around the rivet holes and also fatigue load will be acting for fuselage skin due to pressure variation, which tends to initiate the crack at the rivet hole edges perpendicular to the direction of the load acting.

Since the pressure variation occurs inside the fuselage according to altitude at which aircraft flies, there will be an uniform load acting on the component due to pressure variation, which in turn causes the growth of cracks in all the rivet holes simultaneously called as MSD.

So, as and when crack grows simultaneously in all the rivet holes, the net section available between the two crack tips will be reducing to carry the required load and at certain crack length if the stress intensity factor of the crack reaches the fracture toughness value of the component, then the component will fail due to fracture leading to bigger crack. But the failure may also occur due to plastic collapse [9] as the crack grows, at the tip of the crack the stress concentration will be more and before propagation there

will be local yield around the crack tip, when the plastic deformation at the two advancing crack tips towards each other coalesce, then the catastrophic failure occurs due to plastic deformation leading to bigger crack.

VII. CRACK TIP PLASTICITY

Under linear elastic fracture mechanics assumptions, the stress at the crack tip is theoretically infinite. Clearly, all materials have a finite strength, thus there will always be a small plastified zone around the crack tip.

If this zone is small compared to the crack size, then our linear elastic assumptions are correct, if not, LEFM is not applicable (thus it would be incorrect to use a *K* or *G* criterion) and a nonlinear model must be used. This damaged zone is referred to as a plastic zone for metals.

The appearance of the plastic zone at the tip does not allow its material to bear high stresses predicted by the elastic analysis. Also the material is soft in front of the crack tip and therefore the effective crack length is longer than the actual.

In fact, owing to the presence of the plastic zone, the stiffness of the component decreases. Consequently, the crack is equivalent to a length that is longer than actual length. The size of the plastic zone in front of the crack tip determines the effective crack length.

Therefore, considerable efforts have been made by many investigators such as Irwin plastic zone approach and Dugdale plastic zone approach to determine the plastic zone size and effective crack length.

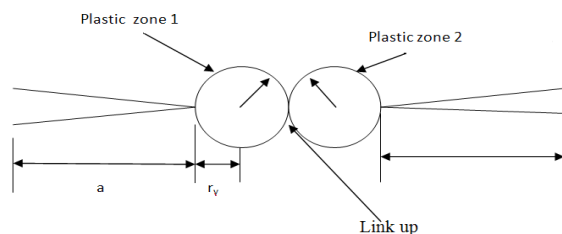


Figure 4: plastic zone around the crack tip

The net section plastic collapse is applied considering the crack sizes. As at each crack tips, the effective crack length is more than the actual crack, there will be plastic zone around the crack tip as shown in Figure 4. As these plastic zone approach each other as crack grows and at certain crack length these two crack length will link up to form the plastic collapse [10] and becomes a bigger crack. The Net section yielding stresses for the presence of multiple cracks are calculated based on the asymptotic formula for stresses near a crack tip, based upon the Irwin and Dugdale formulas [12-14].

Irwin formula

$$r_y = \left(\frac{1}{2 \times 3.14}\right) \times (K/\sigma_y)^2 \text{ ----- (2)}$$

Dugdale Formula

$$r_y = \left(\frac{3.14}{8}\right) \times (K/\sigma_y)^2 \text{ ----- (3)}$$

where, σ_y is the yield stress and r_y is the plastic zone radius.

7.1. CALCULATION OF NET SECTION YEILDING

The local panel taken for the net section failure calculation will have the same dimensions as given for the SIF calculation with same loads and boundary condition. For the different crack lengths the stress intensity factor was calculated, also for the same crack length, net section yielding will be calculated between two advancing crack tips by taking an average value of the elemental stresses obtained between the two crack tips and it is compared with the Irwin formula for validating the average elemental length for plastic zone and its stresses.

Since, the elemental stress at the crack tip will be higher and gradually decreases as moved away from the crack tip and it attains least value at centre in-between the crack tips. The average values of all the elemental stresses are then compared with the yield strength of the material Al 2024-T3 is 362 N/mm².

7.2. NET SECTION YIELDING FOR THE 20 MM PITCH RIVET HOLE

The net section yielding for the 20 mm pitch rivet hole is calculated by taking average of elemental stresses between two crack tips for each iteration in increasing order of crack lengths.

For the crack length of 12 mm, the average of elemental stress is $\sigma_{avg} = 312 \text{ N/mm}^2$

therefore, $\sigma_{avg} \leq \sigma_y$

From Irwin plastic zone formula, we have

$$K_I = 425.351183 \text{ MPa}\sqrt{\text{mm}}$$

$$\sigma_y = 362 \text{ N/mm}^2$$

From equation (1), one can obtain

$$r_y = (1/(2 \times 3.14)) \times (425.351183/362)^2$$

$$r_y = 0.219845 \text{ mm}$$

Since, problem requires only the length of the plastic zone. So, one can add actual crack length and plastic zone length Therefore,

$$a_{effective} = a + 2r_y$$

$$a_{effective} = 2.85 + (2 \times 0.219845) = 3.28969 \text{ mm}$$

but, actual crack length,

$$a = 2.85 \text{ mm}$$

therefore,

$$a_{effective} \geq a$$

The above values indicate that the plastic zone is increasing as crack grows, which is an indication of the plastic deformation.

Similarly, effective plastic length and net section yielding for other crack lengths for different pitch rivet hole distances such as 25mm and 30mm are tabulated below in the Table 3, Table 4 and Table 5.

Table 3: Net section average stress and effective crack length for plastic collapse for 20 mm pitch rivet hole

Crack Length, 2a in mm	Half Crack length, a in mm	K_{FEA} in $MPa\sqrt{mm}$	Net Section average stress, σ in MPa	Yield Strength σ_y in N/mm^2	Irwin formula for plastic zone length, r_y in mm	Effective Crack Length, a_{eff} in mm	Plastic zone length between two cracks tips with a_{eff} in mm
5.7	2.85	425.351	195.77	362	0.21985	3.28969	6.57938
6.6	3.30	493.004	205.05	362	0.29534	3.89068	7.78137
7.5	3.75	541.243	217.98	362	0.35596	4.46193	8.92386
8.4	4.20	581.195	231.98	362	0.41046	5.02091	10.0418
9.3	4.65	619.314	248.50	362	0.46606	5.58213	11.1643
10.2	5.10	658.098	267.53	362	0.52627	6.15253	12.3051
11.1	5.55	699.134	289.03	362	0.59394	6.73789	13.4758
12.0	6.00	757.788	312.00	362	0.69778	7.39556	14.7911
13.0	6.50	799.069	348.14	362	0.77588	8.05175	16.1035
14.0	7.00	862.914	396.00	362	0.90481	8.80962	17.6192

Table 4: Net section average stress and effective crack length for plastic collapse for 25 mm pitch rivet hole

Crack Length, 2a in mm	Half Crack length, a in mm	K_{FEA} in $MPa\sqrt{mm}$	Net Section average stress, σ in MPa	Yield Strength σ_y in N/mm^2	Irwin formula for plastic zone length, r_y in mm	Effective Crack Length, a_{eff} in mm	Plastic zone length between two cracks tips with a_{eff} in mm
6.05	3.025	487.341	201.00	362	0.28860	3.60219	7.20438
7.30	3.650	552.821	211.70	362	0.37136	4.39271	8.78543
8.55	4.275	603.350	222.33	362	0.44235	5.15969	10.3194
9.80	4.900	702.678	228.19	362	0.59998	6.09996	12.1999
11.70	5.850	716.993	250.50	362	0.62467	7.09935	14.1987
13.60	6.800	786.998	282.42	362	0.75261	8.30522	16.6104
15.50	7.750	863.658	324.50	362	0.90637	9.56275	19.1255
17.40	8.700	955.394	383.69	362	1.10915	10.9183	21.8366

Table 5: Net section average stress and effective crack length for plastic collapse for 30 mm pitch rivet hole

Crack Length, 2a in mm	Half Crack length, a in mm	K_{FEA} in $MPa\sqrt{mm}$	Net Section average stress, σ in MPa	Yield Strength σ_y in N/mm^2	Irwin formula for plastic zone length, r_y in mm	Effective Crack Length, a_{eff} in mm	Plastic zone length between two cracks tips with a_{eff} in mm
5.93	2.965	466.198	185.74	362	0.26410	3.49320	6.98639
7.06	3.530	530.951	191.31	362	0.34256	4.21511	8.43023
8.20	4.100	578.362	201.78	362	0.40647	4.91293	9.82586
9.33	4.665	617.945	210.00	362	0.46401	5.59301	11.18600
10.50	5.250	654.840	217.75	362	0.52107	6.29213	12.58430
11.60	5.800	730.374	222.50	362	0.64821	7.09642	14.19280
13.13	6.565	741.519	236.52	362	0.66814	7.90128	15.80260
14.67	7.335	790.809	254.70	362	0.75992	8.85483	17.70970
16.20	8.100	842.602	276.40	362	0.86272	9.82543	19.65090
17.73	8.865	897.031	302.72	362	0.97777	10.8205	21.64110
19.30	9.650	955.827	335.43	362	1.11015	11.8703	23.74060
20.80	10.400	1021.550	377.04	362	1.26806	12.9361	25.87220

VIII. RESULTS AND DISCUSSIONS

8.1. RESULTS AND DISCUSSIONS OF SKIN FOR 20MM PITCH RIVET HOLE

The stress intensity factor value is calculated for periodic increase of crack length. For each crack length, the stress intensity factor value is compared with the fracture

toughness of the material. Table 3 shows the results of stress intensity factor values for periodic increase of crack lengths. The graph in the Figure 5 shows the crack analysis result which is obtained for crack length versus stress intensity factor value. The distance between two rivet hole edges are 15.2mm, from the result, it is found that at the half crack length of 6.5 mm the stress intensity

factor value does not reaches the fracture toughness value of the material, where the material does not leads to failure through fracture. In figure 5. the graphical view is shown clearly.

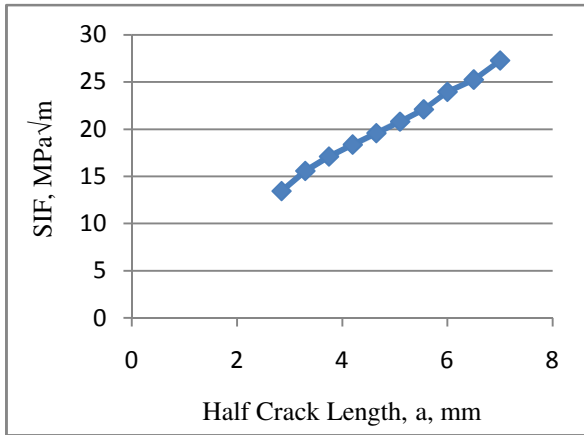


Figure 5: SIF graph for 20mm pitch rivet hole

Similarly the net section yielding calculations are done by means of taking the average stress value between the two advancing crack tips and it is compared with the yield strength of the material. Table 3 shows the results of net section yielding for periodic increase of crack lengths. From the result, it is found that at the crack length of 14 mm the has material crossed the yield strength value of material, where it leads to material yielding failure. The following Figure 6 shows the graph of net section yielding result which is plotted for crack length versus average yield stress of the material for different crack lengths.

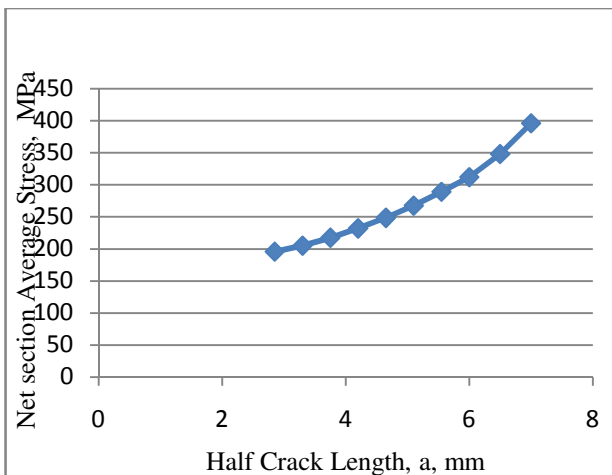


Figure 6: Net section yielding graph for 20mm pitch rivet hole

So from the analysis it is came to know that the structure with stress concentration will fail by net section yielding. Similarly, the results for 25 mm and 30 mm pitch rivet hole, the graphical representation values are given in the Figure 7, Figure 8, Figure 9 and Figure 10.

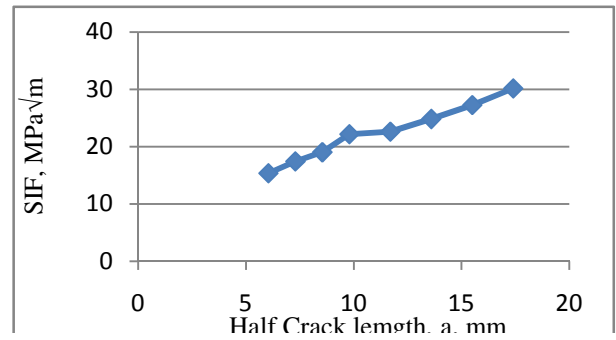


Figure 7: SIF graph for 25mm pitch rivet hole

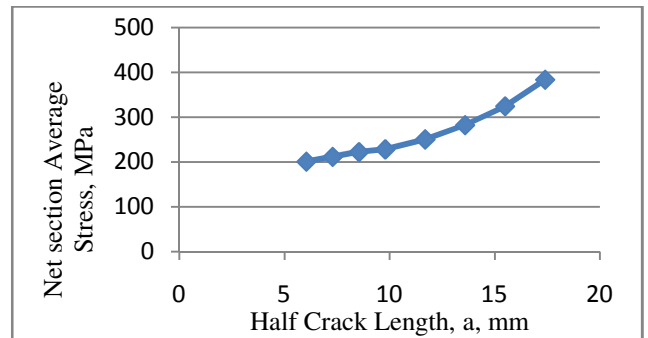


Figure 8: Net section yielding graph for 25mm pitch rivet hole

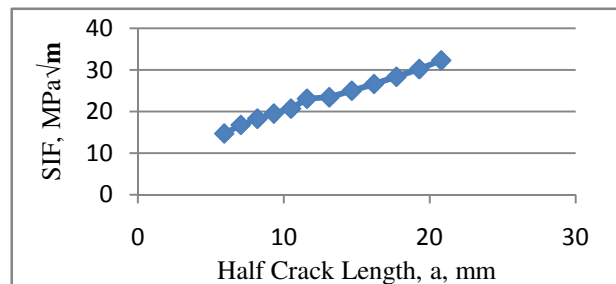


Figure 9: SIF graph for 30mm pitch rivet hole

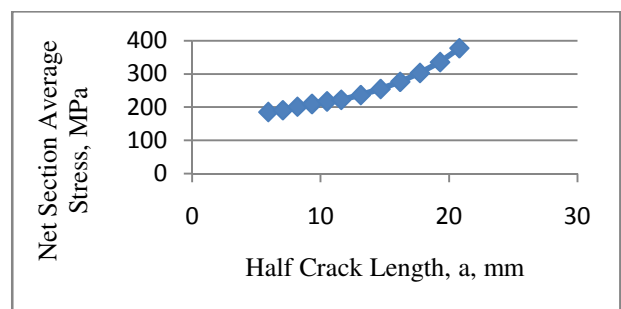


Figure 10: Net section yielding graph for 30mm pitch

IX. DISCUSSIONS

The results of the Net section failure have been discussed in the previous section; from that one can understand that, if the component has a crack, it does not mean that it should fail by fracture. From results of current study, one can understand that the component will fail by plastic collapse even if it has crack.

Also, one can observe that if the distance between rivet holes are increasing net section failure will also occur for bigger cracks. Due to fatigue loading, the net section

yielding is more susceptible to failure than the fracture failure. So, one should not concentrate only on fracture during design and inspection of a component at what crack length the component will fail, one should concentrate also on plastic deformation which occurs due to fracture.

X. CONCLUSIONS

The static stress analysis of the fuselage of a transport aircraft has been performed in the presented work. This study has relevance in the structural integrity evaluation of aging transport aircraft due to multisite damage. Here the MSD analysis was carried out for the aged aircraft.

In the present work only the fuselage with splice through butt joint has been analyzed. Fatigue loads due to internal pressurization acting on the fuselage, stress concentration will be high at rivet holes locations of the fuselage joint, which causes the initiation of cracks on all rivet hole edges due to uniform stress acting on the fuselage due to internal pressurization.

Usually the fuselage of the aircraft structure is subjected to different kinds of loads that include aerodynamic loads, landing loads, taxing loads, pressurization and reaction loads. The present study deals only with MSD, so the loads considered are only internal pressurization.

Taking all the above points into consideration modeling and finite element analysis of fuselage and its local segment was carried out and from that work, some of the information's are concluded as follows

- The Fuselage model was created using a 3D modeling software tool according to the dimensions and the 3D modeled component was imported to analysis tool for pre-processing. The fuselage component was meshed with 2D elements such as quad 4 and Tria 3 elements and loads and boundary conditions were applied.
- The material used was Al 2024-T3, which is widely used in aircraft industry for its good fatigue strength and corrosion resistance.
- Stress analysis of the global model of the fuselage has been carried out to observe the hoop stress on skin is equal to the analytical value of the hoop stress 59.5 N/mm^2 .

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