The Comparison of Moment Invariants and to Remove Blur By Using Zernike Moments

T. Rajesh Kumar
M.Tech Student Scholar, DECS,
Dept of Electronics and Communication Engineering,
Nalanda Institute of Engineering and technology,
Sattenapalli (M); Guntur (Dt); A.P, India.

Bhookya Nageswararao Naik
M. Tech , Asst professor
Dept of Electronics and Communication Engineering,
Nalanda Institute of Engineering and technology,
Sattenapalli (M); Guntur (Dt); A.P, India.

Abstract: *In this paper, we propose various types of moments like Legendre moments, complex moments and Zernike Moments invariants. In the past decades has been extensively investigated about moment invariants. In Zernike moments we construct to derive set of invariants which is simultaneously invariant to similarity transformation and to convolution with spread spectrum(PSF).In this mainly two contributions are provided, The frame work for theoretical is derived from Zernike moments of a blurred image and the way to construct the one of invariant moment combined geometric blur invariants. To evaluate the performance of the invariants with various PSFs and similarity Transformations. Experimental results show that the propose dinvarients perform better to compared with other invariants like Legendre moments and complex Moments.*

Keywords: *Moment Invariants, Legendre Moments, Complex moments, Zernike Moments, Pattern Recognisation*

I. INTRODUCTION

Analysis and interpretation of an image which was acquired by a real (i.e. non-ideal) imaging system is the key problem in many application areas such as remote sensing, astronomy and medicine, among others. Since real imaging systems as well as imaging conditions are usually imperfect, the observed image represents only a degraded version of the original scene. Various kinds of degradations (geometric as well as radiometric) are introduced into the image during the acquisition by such factors as imaging geometry, lens aberration, wrong focus, motion of the scene, systematic and random sensor errors, etc.

Size and orientations are important concern in recognition objects. The history about invariants includes so many techniques [1]–[5],Fourier descriptors [6] and point set invariants [7]–[10] have been Proposed . Among them, moment invariants have been extensively used for image description in object recognition [11], [12], image classification [13] and scene matching . However, much less attention has been paid to invariants with respect to changes of the image intensity function (known as radiometric invariants) as to joint radiometric- geometric invariants. The principle behind doing the blur is not too hard, although it seems like magic. What we do is take the image and the kernel, and perform the Fourier transform on them both. We then multiply the two together and inverse transform back.[14]

This is exactly the same as performing the long convolution. In image terms this means that each pixel in the source image gets spread over and mixed into surrounding pixels. Another way to look at this is that each pixel in the destination image is made up out of a mixture of surrounding pixels from the source image. The operation we need for this is called convolution. A quick diversion here to discuss a problem which often crops up: Imagine you want to blur a shape which is on a transparent background. You've got an empty image, and you draw a shape on it, then blur the image. blur to sharpen an image as well as blur it using a technique called un-sharp masking.

Orthogonal functions have been around for a very long time. The best known are the sine and cosine. Two functions or vectors are orthogonal if their inner product (defined as the sum of the product of their corresponding elements) is zero. An important class of orthogonal functions is orthogonal polynomials , which are orthogonal over various intervals of the real axis. Important orthogonal polynomials include Legendre, Hermite, Chebyshev, etc. Legendre polynomials, which are orthogonal over [-1, 1], can be taken as a product $P(x)P(y)$, and the result is an orthogonal set of polynomials over a square. Zernike working in optics in the 1930's derived a set of polynomials that are orthogonal over a unit disk, i.e., $r \leq 1$.[15]

Orthogonal moments are computed similar to regular moments, except that the set of orthogonal polynomials replaces the x^p or $x^p y^q$ monomial in all the above equations. That is, where $h_{pq}(x, y)$ is the pq-th orthogonal polynomial, and R is the region over which the polynomials are defined. for Legendre and Zernike polynomials, and where we have subtracted the centers, which can be determined from the centroid or some other means, so that the resulting moments are location invariant.

Zernike polynomials are orthogonal over the unit disk and are specified in polar coordinates in terms of a real valued radial component $R_{nl}(r)$ which is a polynomial of order n, and a complex exponential component: There are two ways of viewing moments, one based on statistics and one based on arbitrary functions such as $f(x)$ or $f(x, y)$. As a result moments can be defined in more than one way.[16]

Moments are the statistical expectation of certain power functions of a random variable. The most common moment is the mean which is just the expected value of a random variable: where $f(x)$ is the probability density function of continuous random variable X. More generally, moments of order p = 0, 1, 2, … can be calculated as $m_p = E[X^p]$. These are sometimes referred to as the raw moments.

There are other kinds of moments that are often useful. One of these is the central moments $\mu_p = E[(X-\mu)^p]$.

However, moments are easy to estimate from a set of measurements, The *p*-th moment is estimated as and the *p*-th central moment is estimated as where is the average of the measurements, which is the usual estimate of the mean.

II. PROPOSED CONCEPT

A. Legendre Moments:

The Legendre of order $(m+n)$ are defined as:

$$\lambda_{mn} = \frac{(2m+1)(2n+1)}{4}\int_{-1}^{1}\int_{-1}^{1} P_m(x)P_n(y)f(x,y)dx\,dy \quad (1)$$

where

$$m,n = 0,1,2,3,\ldots\ldots$$

Pm and *Pn* are the Legendre polynomials and *f(x,y)* is the continuous image function. The Legendre polynomials are a complete orthogonal basis set defined over the interval *[1,-1]* For orthogonality to exist in the moments, the image function $f(x,y)$ is defined over the same interval as the basis set, where the n^{th} order Legendre polynomial is defined as:

$$P_n(x) = \sum_{j=0}^{n} a_{nj}\, x^j \quad (2)$$

and a_{nj} are the Legendre coefficients given by:

$$a_{nj} = (-1)^{(n-j)/2}\frac{1}{2^n}\frac{(n+j)!}{\left(\frac{(n-j)}{2}\right)!\left(\frac{(n+j)}{2}\right)!\,j!} \quad where\ n-j = even \quad (3)$$

So, for a discrete image with P_{xy} current pixel

$$\lambda_{mn} = \frac{(2m+1)(2n+1)}{4}\sum_x\sum_y P_m(x)P_n(y)P_{xy} \quad (4)$$

and x, y are defined over the interval [-1,1].

B. Complex Zernike Moments:

The Zernike polynomials were first proposed in 1934 by Zernike. Their moment formulation appears to be one of the most popular, outperforming the alternatives (in terms of noise resilience, information redundancy and reconstruction capability). The pseudo-Zernike formulation proposed by Bhatia and Wolf further improved these characteristics. However, here we study the original formulation of these orthogonal invariant moments.

Complex Zernike moments are constructed using a set of complex polynomials which form a complete orthogonal basis set defined on the unit disc $(x^2+y^2) \geq 1$ .They are expressed as A_{pq} Two dimensional Zernike moment.

$$A_{mn} = \frac{m+1}{\pi}\int_x\int_y f(x,y)[V_{mn}(x,y)]^* dxdy \quad where\ x^2+y^2 \leq 1 \quad (5)$$

Where $m = 0,1,2,3,\ldots\ldots\ldots\ldots\ldots.\infty$

And defines the order $f(x,y)$ is the function being described and * denotes the complex conjugate. While n is an integer (that can be positive or negative) depicting the angular dependence, or rotation, subject to the conditions:

$$m - |n| = even, |n| \leq m \quad (6)$$

And $A^*_{mn} = A_{m,-n}$ is true . The Zernike polynomials *Vmn(x,y)* Zernike polynomials expressed in polar coordinates

$$V_{mn}(r,\theta) = R_{mn}(r)\exp(jn\theta) \quad (7)$$

Where (r,θ) are defined over the unit disc

$$j = \sqrt{-1}\text{an } R_{mn}(r)\text{d}$$

is the orthogonal radial polynomial, defined as *Rmn (r)* orthogonal radial polynomial

$$R_{mn}(r) = \sum_{s=0}^{\frac{m-|n|}{2}} (-1)^s F(m,n,s,r) \quad (8)$$

where:

$$F(m,n,s,r) = \frac{(m-s)!}{s!\left(\frac{m+|n|}{2}-s\right)!\left(\frac{m-|n|}{2}-s\right)!} r^{m-2s} \quad (9)$$

C. Zernike Moments of the Blurred Image :

In this subsection, we establish the relationship between the Zernike moments of the blurred image and those of the original image and the PSF. To that end, we first consider the radial moments.

Applying (1) to blurred image g(x,y), we have a

$$
\begin{aligned}
D^{(g)}_{q+2k,q} &= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (x-\hat{j}y)^{q+k}(x+\hat{j}y)^{k} g(x,y)\,dxdy \\
&= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (x-\hat{j}y)^{q+k}(x+\hat{j}y)^{k} \\
&\qquad \times \left[\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} h(a,b) f(x-a,y-b)\,dadb\right] \\
&\qquad \times dxdy \\
&= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} h(a,b) \\
&\qquad \times \left[\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} \left((x-\hat{j}y)+(a-\hat{j}b)\right)^{q+k}\right. \\
&\qquad\qquad \times \left((x+\hat{j}y)+(a+\hat{j}b)\right)^{k} \\
&\qquad\qquad \left.\times f(x,y)\,dxdy\right] dadb \\
&= \sum_{m=0}^{q+k}\sum_{n=0}^{k} \binom{q+k}{m}\binom{k}{n} \\
&\qquad \times \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} (x-\hat{j}y)^{m}(x+\hat{j}y)^{n} f(x,y)\,dxdy \\
&\qquad \times \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} (a-\hat{j}b)^{q+k-m}(a+\hat{j}b)^{k-n} h(a,b)\,dadb \\
&= \sum_{m=0}^{q+k}\sum_{n=0}^{k} \binom{q+k}{m}\binom{k}{n} D^{(f)}_{m+n,m-n} \\
&\qquad \times D^{(h)}_{q+2k-m-n,q+n-m}.
\end{aligned}
\tag{10}
$$

Applying blurred image g(x,y)=g(r,ɵ) and using above equation , we obtain

$$
Z^{(g)}_{q+2l,q} = \sum_{k=0}^{l}\sum_{m=0}^{q+k}\sum_{n=0}^{k} \binom{q+k}{m}\binom{k}{n} \times c^{q}_{l,k} D^{(f)}_{m+n,m-n} D^{(h)}_{q+2k-m-n,q+n-m}. \tag{11}
$$

the radial moments can also be expressed as a series of Zernike moments

$$
D^{(f)}_{q+2l,q} = \sum_{k=0}^{l} d^{q}_{l,k} Z^{(f)}_{q+2k,q} \tag{12}
$$

$$
d^{q}_{i,j} = \frac{i!(q+i)!\pi}{(i-j)!(q+i+j+1)!}, \qquad 0 \le j \le i \le l. \tag{13}
$$

We have

$$
D^{(f)}_{m+n,m-n} = \sum_{i=0}^{n} d^{m-n}_{n,i} Z^{(f)}_{m-n+2i,m-n} \tag{14}
$$

$$
D^{(h)}_{q+2k-m-n,q+n-m} = \sum_{j=0}^{k-n} d^{q+n-m}_{k-n,j} Z^{(h)}_{q+n-m+2j,q+n-m}. \tag{15}
$$

By using above two eqn's we obtain

$$
Z^{(g)}_{q+2l,q} = \sum_{k=0}^{l}\sum_{m=0}^{q+k}\sum_{n=0}^{k}\sum_{i=0}^{n}\sum_{j=0}^{k-n} \binom{q+k}{m}\binom{k}{n} c^{q}_{l,k} d^{m-n}_{n,i} \times d^{q+n-m}_{k-n,j} Z^{(f)}_{m-n+2i,m-n} Z^{(h)}_{q+n-m+2j,q+n-m}. \tag{16}
$$

III. MATLAB/SIMULINK RESULTS

In this evaluation, we also use the images shown in Fig. 1.The testing set was generated by adding motion blur, averaging blur with zero-mean. This was followed by adding a white Gaussian noise with different standard deviations and salt-and-pepper noise with different noise densities. Because the actual size of the PSF is usually unknown in practical application, in order to evaluate the performance of the different methods under such a situation.

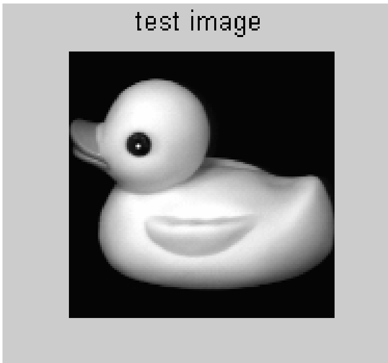

Figure 1-:Original Image

Figure 2: Combined invariants to similarity transform

We have computed the ZMIs, CMIs, and LMIs up to order M=1,3,5…., 17. The mean classification rates under different noise conditions for different values of M are shown in Fig. 3,4. It can be observed that the rate first increases, reaches the maximum value and then decreases for all three methods. In other words, there should exist an optimal order for each type of moment invariants. This behavior has also been observed and pointed out by Liao and Pawlak in image reconstruction due to the noise influence [17]. In this experiment, the optimal order for CMIs is M=7(the feature vector includes 17 invariants), for LMIs (the feature vector has 21 invariants), and M=9 for ZMIs (with 26 invariants).

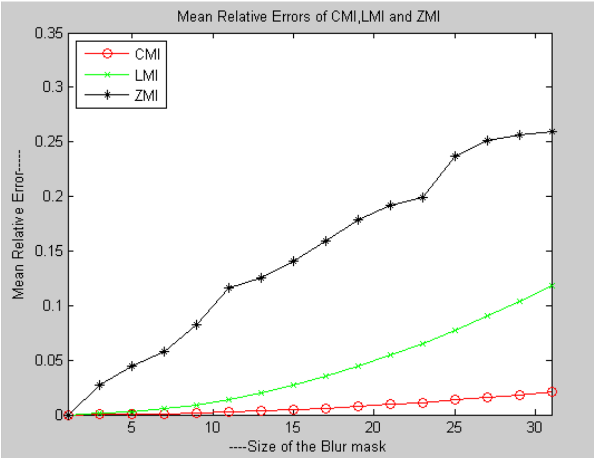

Figure 3: Mean relative errors of CMIs, LMIs, and ZMIs for motion blurred versions of the images

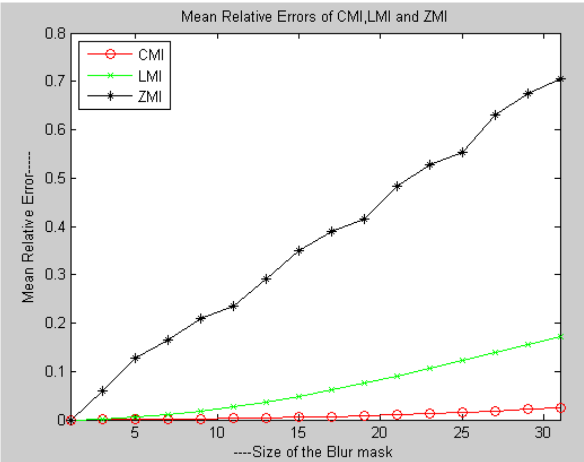

Figure 4: Mean relative errors of CMIs, LMIs, and ZMIs for average blurred versions of the image

It can be seen that the proposed descriptors ZMI is compared with CMIs And Legendre moments.

The figures 3, 4 are to calculate the mean relative error corresponding invariants. And to compare invariants are like Legendre and complex.

IV. VI CONCLUSION

In this paper, we have proposed a method to construct a set of combined geometric-blur invariants using the orthogonal Zernike moments. The relationship between the Zernike moments of the blurred image and those of the original image and the PSF has been established. Based upon this relationship, a set of invariants to convolution with circularly symmetric PSF has been derived. The advantages of the proposed method over the existing ones are the following: 1) The proposed descriptors are simultaneously invariant to similarity transformation and to convolution. Using these invariants, the image deblurring and geometric normalization process can be well avoided. 2) Like the method reported in [17], our method can also derive the even order invariants. The experiments conducted so far in very distinct situations demonstrated that the proposed descriptors

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