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Mathematical Model of ECG Signal for OFF Line Adaptive Signal Processing in MATLAB

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Abstract: *this paper is outlined basic model based on Fourier series is capable of generating realistic synthetic electrocardiogram (ECG) signals. The operator can specify the heart rate, the variation, position and amplitude of pulses. The beat-to-beat variation and timing of the human ECG, including QT and R-peak amplitude are shown in result. An adaptive LMS filter is implemented to remove the noise of ECG signal. This model may be employed to consider biomedical signal processing techniques that are used to calculate clinical statistics from the ECG signal. Model is derived and successfully simulated in MATLAB.*

Keywords: *ECG signal, Heart rate variation, realistic model, PQRST pulse, QRS analysis, Adaptive filtering, LMS algorithm.*

I. INTRODUCTION

ECG is vital in order to find out heart disease in early phase of its occurrence. The real time ECG signal is more multifaceted in nature and to represent ECG signal adequate mathematical analysis should prefer over the range and nature of signal. The various models have been proposed for clearing up ECG signal OFF line generation. A realistic synthetic ECG signal was modelling using dynamic model [1] and thus authors have taken the efforts to exact curve fitting of ECG signal. An intracellular potential based skin surface model [3] has been suggested and simulating for normal heart operation. In most of time percentage, root mean square difference [4] is used to reconstruct ECG signal. However, the voltage difference method [5] has recommended exact procedure of obtaining an ECG signal with enhanced features. The present paper discuses the natures of ECG signal and derivation of mathematical equation that describe resultant ECG pulse approximately close to realistic ECG signal. The ECG signal has modelled using Fourier series method [6].

ECG signal is influenced with many artefacts and thus noise can add into ECG signal. The de-noising of ECG signal is essential for detecting heart disease. An adaptive filter algorithm is used for de-nosing of ECG signal. The analog fitter is implemented followed by adaptive filter for better performance. The de-noising is essential especially for power line noise and base line wonder and should separate from ECG signal. An attempt is made in this present work to remove out ECG noise using adaptive filtering technique.

The ECG signal is the resultant function of heart activity and intracellular potential developed due to action of heart. SA node, atrial muscle, AV node, common bundle fibres, bundle Branches, Purkinje fibres, ventricular muscles are literal cause of resultant ECG signal, and hence any abnormality can be localised by observing ECG signal [7]. A normal form of ECG signal is presenting in figure 1.

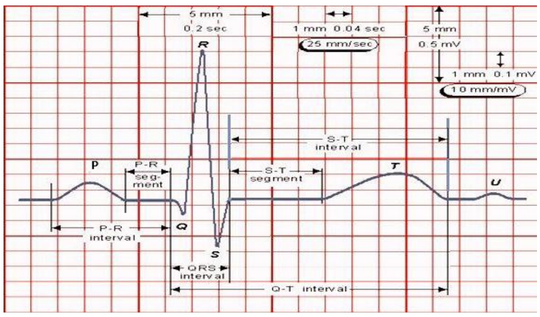

Figure 1: A normal form of ECG signal

II. MATHEMATICAL MODEL

An ECG signal is the combination of number of pulses resulting due to heart activity. if mathematical model of one pulse is derived then on similar basis other pulses can represented. Consider S(t) is the ECG signal in time domain. According to the geometry of one ECG pulse it represented as

$$S(t) = A_0(1 - e^{\frac{-t}{\tau}}) - A_0 e^{\frac{-t}{\tau}}$$

$$S(t) = A_0(1 - 2e^{\frac{-t}{\tau}})$$

ECG is the resultant signal obtaining after addition of minimum three to maximum n number of pulses.

$$S(t) = A_0(1 - 2e^{\frac{-t}{\tau_0}}) + A_1(1 - 2e^{\frac{-t}{\tau_1}}) + \dots\dots\dots + A_n(1 - 2e^{\frac{-t}{\tau_n}})$$

The pulses are periodic and continuous pulses and represented by using Fourier series method. The one pulse is given by using Fourier series method.

$$S(t) = a_0 + \sum_n a_n \cos(2\pi n f_0 t) + \sum_n b_n \sin(2\pi n f_0 t)$$

Where,

$$a_0 = \frac{1}{T}\int_{t_0}^{t_0+T} S(t)dt$$
$$a_n = \frac{2}{T}\int_{t_0}^{t_0+T} S(t)\cos(2\pi n f_0 t)dt$$
$$b_n = \frac{2}{T}\int_{t_0}^{t_0+T} S(t)\sin(2\pi n f_0 t)dt$$

Consider general one pulse segment

$$S(t) = A(1-2e^{\frac{-t}{\tau}})$$
$$a_0 = \frac{1}{T}\int_0^T A(1-2e^{\frac{-t}{\tau}})dt$$
$$a_n = \frac{2}{T}\int_0^T A(1-2e^{\frac{-t}{\tau}})\cos(2\pi n f_0 t)dt$$
$$b_n = \frac{2}{T}\int_0^T A(1-2e^{\frac{-t}{\tau}})\sin(2\pi n f_0 t)dt$$

The solution is

$$a_0 = A + \frac{2A\tau}{T}\left(e^{\frac{-T}{\tau}} - 1\right)$$
$$a_n = \frac{4/T}{\left(\frac{1}{\tau}\right)^2 + \left(2\pi n f_0\right)^2}\left(e^{\frac{-T}{\tau}} - \frac{1}{\tau}\right)$$
$$b_n = \frac{-A}{\pi n} - \frac{\left(4/T\right)\left(2\pi n f_0\right)}{\left(\frac{1}{\tau}\right)^2 + \left(2\pi n f_0\right)^2}\left(1 - e^{\frac{-T}{\tau}}\right)$$

One pulse of ECG signal is represented by using Fourier series as

$$S(t) = A + \frac{2A\tau}{T}\left(e^{\frac{-T}{\tau}} - 1\right) + \sum_{n=1,2,3,\ldots}\left(\frac{4/T}{\left(\frac{1}{\tau}\right)^2 + \left(2\pi n f_0\right)^2}\left(e^{\frac{-T}{\tau}} - \frac{1}{\tau}\right)\right)\cos(2\pi n f_0 t) + \sum_{n=1,2,3,\ldots}\left(\frac{-A}{\pi n} - \frac{\left(4/T\right)\left(2\pi n f_0\right)}{\left(\frac{1}{\tau}\right)^2 + \left(2\pi n f_0\right)^2}\left(1 - e^{\frac{-T}{\tau}}\right)\right)\sin(2\pi n f_0 t)$$

III. MATLAB MODEL

Mathematical model of ECG signal is discussed in previous section. The equation in terms of Fourier series is representing ECG signal. MATLAB model is constructed to implement the mathematical ECG signal. Figure 2 shows one pulse model of ECG signal. Thus one pulse is generated using MATLAB model. Minimum three pulses are needed to represent ECG signal. Seven pulses are sufficient to simulate complete ECG signal. Figure 3 shows a five pulse model of ECG signal and finally figure 4 is ECG toolbox created for OFF line ECG processing.

Removal of power line noise and base line wonder is vital problem in ECG signal processing. The various types of filters are useful for removing of noise. Here, an attempt is made using adaptive List Mean Square (LMS) algorithm based filters to remove the ECG noise. Figure 5 shows the matlab model containing OFF line ECG generator, noise adder, equalised LMS filter, analog filter, and scope. Analog filter is used after the adaptive LMS filter, the figure 6 shoes the results.

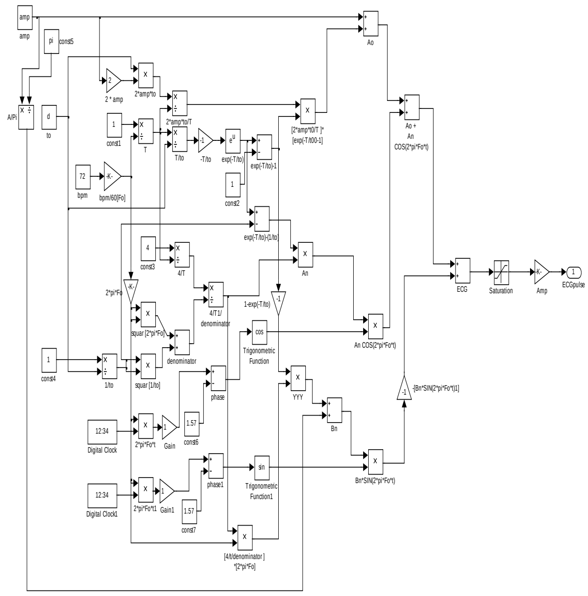

Figure 2 MATLAB equation model

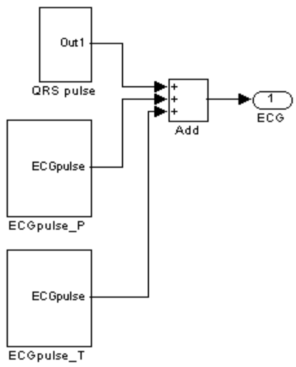

Figure 3 Five pulse ECG signal

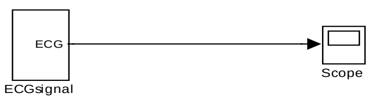

Figure 4 ECG tool box

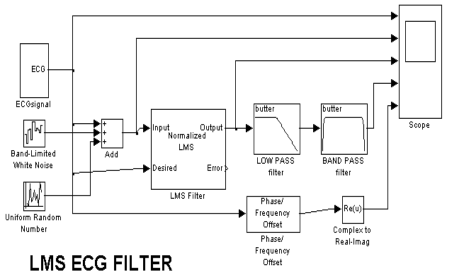

Figure 5 LMS filter

IV. RESULTS

Properties of ECG pulses are given in table 1.

Table 1 Properties of ECG pulses [1]

Index (i)	P	Q	R	S	T
Time (secs)	-0.2	-0.05	0	0.05	0.3
θ_i (radians)	$-\frac{1}{3}\pi$	$-\frac{1}{12}\pi$	0	$\frac{1}{12}\pi$	$\frac{1}{2}\pi$
a_i	1.2	-5.0	30.0	-7.5	0.75
b_i	0.25	0.1	0.1	0.1	0.4

Typical ECG is used to suggest suitable times, angles and values of the PQRST points. The times and angles are specified relative to the position of the R-peak as shown in Table I. Output result is shown in figure 6. The Ist wave is reference ECG signal generated using Fourier series method. The 2nd waveform is showing ECG signal with added noise. The 3rd waveform shows output of adaptive LMS filter. This output waveform contains high and mid band frequency noise. This noise may remain present due to adaptive iteration action. Finally analog low pass and band pass filter is used to remove out remaining noise present in ECG signal. The 4th wave form indicating final output after analog filtering.

Figure 6 Results

V. CONCLUSION

Mathematical model has been introduced which is capable of intrigues the significant features of the ECG signal. Realistic ECG has important relevance in medical signal processing techniques. As compared with noise ECG signal with adaptive filter output waveform, it is clear that LMS filtering technique removes most of the noise present in the ECG signal. When analog filter is followed by after adaptive filtering, it is clear from the waveform that noises have been removed completely. Hence, the combination of adaptive and analog filtering removes noises present in the signal.

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