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Multichannel Image Restoration Using Scattered Techniques

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Abstract: *consider the problem of joint enhancement of multichannel images with pixel based constraints on the mul-tichannel data. We formulate an optimization problem that jointly enhances complex-valued mul-tichannel images while preserving the cross-channel information, which we include as constraints tying the multichannel images together. We first reformulate it as an equivalent (un-constrained) dual problem and develop a numerically-efficient method for solving it. We develop the Dual Descent method, which has low complexity, for solving the joint optimization problem. The algorithm is applied to a synthetic multimodal medical image example.*

I. INTRODUCTION

The collection and subsequent processing of multiple-channel datasets to characterize and extract features present in the images. These systems either operate by sending wideband electromagnetic signals and processing the backscattered waves from the targets or by using the measurements at different frequency bands to capture the required features from the scene of interest. For these examples limited resolution and low signal to noise ratios for individual modalities can be addressed by joint reconstruction of the image or scene using multiple measurements or modalities. Regularization methods which incorporate the prior information about the point spread function, have been able to achieve higher resolution and reduced side lobe artifacts. Specifically, in the deconvolution problem is formulated the paper is organized as follows: Section I present the model we have used for the noisy, complex-valued multi-channel images and formulate the joint enhancement problem. Section II develops the Dual Descent method for solving the joint enhancement problem. Finally concludes this paper by discussing some of the contributions of this work and also a few future directions.

II. MULTICHANNEL CASE

In this section, we present a measurement model for multichannel noisy images using a linear operator and formulate the joint enhancement problem.

A. Modeling

Let us assume that there are n channels of datasets from which noisy real or complex-valued images (G_i, i=1…n) are formed. We formulate the more general complex-valued case below. Each of the noisy images G_i is assumed to be obtained (Independently) by a linear transformation Ti, which typically is a convolution operation with the PSF of the underlying Noiseless image, after which noise of variance σ2is added. The model is represented as follows:

$$\boldsymbol{g}_i = \boldsymbol{T}_i \boldsymbol{f}_i + \boldsymbol{w}_i \quad i = 1, \ldots, n \qquad (1)$$

As an optimization problem with a penalty term for inducing sparsity which forces the enhanced images to be sparse. These regularization methods can be used for enhancing, in general, any image which is known to be sparse in some domain. In this paper, we extend these regularization based methods for the multichannel case where processing each channel independent of the others lead to suboptimal results because of the underlying physical process which couples the datasets together. Many imaging technologies make use of data from multiple channels; these can be multiple elevations or multiple polarization data. In this paper we take a regularization-based approach for the deconvolution problem for the complex-valued multichannel case when the images are known to be sparse in some domain. We consider a linear model for the multichannel images and formulate it as a joint optimization problem with additional constraints to ensure that the enhancement process preserves the interchannel information. We develop Dual Descent method, which is similar to the hierarchical optimization technique, for solving the joint optimization problem. We find that the proposed method is approximately as fast as the independent processing of the multichannel. Where $g_{i,}f_i$ complex vectors obtained by stacking the columns of the observed and the actual (noiseless) images for the the channel respectively, $_I$ is the corresponding transformation matrix for the i_{th} channel, is the circularly symmetric complex additive white Gaussian noise with mean zero and variance and is the total number of pixels in each of the images. For many applications they are convolution matrices which act as low-pass filters, thereby making the inversion problem ill-posed in the absence of any further constraints on. We assume that the underlying (noiseless) images are sparse (with a few nonzero coefficients in some basis) or in general compressible meaning that it is approximated well by linear combination of a few basis vectors in some suitable form. The processing goal is to invert the linear transformation by using the sparsity knowledge about the underlying noiseless image. This kind of inverse problem has been extensively studied in the literature for the (real-valued) single channel case and regularization techniques have proven to be very effective especially for preserving edges. In this paper we consider a more general framework, where multiple channels are coupled through arbitrary con-strain functions. In addition, we consider the general case of complex-valued imagery applicable to microwave reflectivity functions underlying radar imagery.

B. Joint Enhancement Problem

We next formulate the problem of joint enhancement first as a (constrained) joint optimization problem and then reduce it to an equivalent (unconstrained) dual problem. The constrained joint optimization problem representing the multichannel problem is mathematically formulated as follows:

$$\min_{\boldsymbol{f}_1,\boldsymbol{f}_2,\ldots,\boldsymbol{f}_n} \quad L(\boldsymbol{f}_1,\boldsymbol{f}_2,\ldots,\boldsymbol{f}_n)$$

$$\text{s.t.} \quad \sum_{i=1}^{n} h_{ij}(\boldsymbol{f}_i)=0, \quad j=1,\ldots,N$$

where $L(\boldsymbol{f}_1,\boldsymbol{f}_2,\ldots,\boldsymbol{f}_n)$ is given by

$$L(\boldsymbol{f}_1,\boldsymbol{f}_2,\ldots,\boldsymbol{f}_n)=\sum_{i=1}^{n}\|\boldsymbol{g}_i-\boldsymbol{T}_i\boldsymbol{f}_i\|_2^2+\sum_{i=1}^{n}\lambda_{1i}^2\|\boldsymbol{f}_i\|_p^p+\sum_{i=1}^{n}\lambda_{2i}^2\|\boldsymbol{D}|\boldsymbol{f}_i|\|_p^p$$

$p \leq 1 \qquad \lambda_{1i}^2 \qquad \lambda_{2i}^2$

Weight to the data-fitting, the sparseness and the region-en-hancement terms in the optimization problem. The NXN*T_i matrices are the forward linear operators and Are vectors representing the observed and the enhanced images respectively for the channels, where the image pixels are stacked into a vector. The is the discrete approximation of the 2-D derivative operator. In order to avoid the difficulty of differentiating the terms near zero, we approximate it as in by where is an arbitrary small positive constant (e.g.,is the th component of the th channel. hence, the approximated cost function is given by,

$$\|\boldsymbol{f}_i\|_p^p \approx \sum_{j=1}^{N}(|(f_i)_j|^2+\epsilon)^{p/2} \quad i=1,\ldots,n$$

One method for solving this optimization problem would be to include the constraint directly in the joint optimization problem, i.e., is an invertible map. For example consider an equal-magnitude constraint, the reconstructed images. In such a case, we can directly include the constraint in the cost function and reduce the number of parameters to be optimized from real variables to real variables and then do a gradient descent. Even though the complexity of such an approach is lower, the gradient descent was found to take significantly more time to converge to the solution than decoupled solutions to. Second, such an approach is not general, in the sense that it cannot be applied for more general constraint functions. Convert the joint (constrained) optimization problem given into an equivalent (unconstrained) dual problem. Assume to be a local minimum, for matrix i_p-norm) and where is an arbitrary small positive constant (e.g., is the th component of the th channel. Hence, the approximated cost function is given by,

$$\hat{L}_i(\boldsymbol{f}_i)=\|\boldsymbol{g}_i-\boldsymbol{T}_i\boldsymbol{f}_i\|_2^2+\lambda_{1i}^2\left\{\sum_{j=1}^{N}(|(f_i)_j|^2+\epsilon)^{p/2}\right\}+\lambda_{2i}^2\left\{\sum_{j=1}^{M}(|(D|f_i|)_j|^2+\epsilon)^{p/2}\right\}$$

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$$\xi(\boldsymbol{\beta})=\min_{\boldsymbol{f}_1,\boldsymbol{f}_2,\ldots,\boldsymbol{f}_n}\underbrace{\left[\hat{L}(\boldsymbol{f}_1,\boldsymbol{f}_2,\ldots,\boldsymbol{f}_n)+\sum_{i=1}^{n}\boldsymbol{\beta}^T\boldsymbol{h}_i(\boldsymbol{f}_i)\right]}_{\tilde{\xi}(\boldsymbol{f}_1,\boldsymbol{f}_2,\ldots,\boldsymbol{f}_n,\boldsymbol{\beta})}$$

Where the minimum is taken locally. It is straightforward from the Local Duality Theorem [23] to prove the equivalence between and the following dual problem:

$$\max_{\boldsymbol{\beta}} \ \xi(\boldsymbol{\beta}).$$

We include the following results from [23], used in proving the Local Duality Theorem, that will be important in formulating a solution strategy. Their proofs are straightforward and have been omitted for brevity.

Lemma 1: **The dual function has gradient**

$$\nabla\xi(\boldsymbol{\beta}) = \sum_{i=1}^{n} \boldsymbol{h}_i(\boldsymbol{f}_i(\boldsymbol{\beta}))$$

Where for is the unique solution to (7).

This lemma is useful as it gives the gradient of the dual function with respect to the Lagrange multipliers and, thus, can be used to develop fast and efficient algorithms for solving the dual problem. The second result establishes Hesitation which can be used to formulate quadratic methods

$$\nabla^2\xi(\boldsymbol{\beta}) = -\sum_{i=1}^{n} \nabla\boldsymbol{h}_i(\boldsymbol{f}_i(\boldsymbol{\beta}))\boldsymbol{H}_i^{-1}(\boldsymbol{f}_i(\boldsymbol{\beta}))\nabla\boldsymbol{h}_i(\boldsymbol{f}_i(\boldsymbol{\beta}))^H$$

Where the matrix H_i is defined although, from the Local Duality Theorem, it appears as though the dual formulation is equivalent only locally, we found through simulations that it is robust to the various initializations we used. Second-order sufficiency is approx-near, we dew here over be-function Hessian of the dual function

$$\hat{L}(\boldsymbol{f}_1, \boldsymbol{f}_2, \ldots, \boldsymbol{f}_n) = \sum_{i=1} \|\boldsymbol{g}_i - \boldsymbol{T}_i\boldsymbol{f}_i$$

$$+\sum_{i=1}^{n} \lambda_{2i}^2 \left\{\sum_{j=1}^{M} \|_2^2 + \sum_{i=1}^{n} \lambda_{1i}^2 \left\{\sum_{j=1}^{N} (|(f_i)_j|^2 + \epsilon)^{p/2}\right\}\right.$$

$$\left.\sum^{n} \ \hat{} \qquad (|(D|f_i|)_j|^2 + \epsilon)^{p/2}\right\}$$

$$:= \sum_{i=1} L_i(\boldsymbol{f}_i)$$

III. DUAL DESCENT METHOD

A. Algorithm

In this section, we develop a fast numerical method for solving the dual problem. The equivalent unconstrained optimization problem we need to solve is given by

$$\max_{\boldsymbol{\beta}} \ \min_{\boldsymbol{f}_1, \boldsymbol{f}_2, \ldots, \boldsymbol{f}_n} \left[\hat{L}(\boldsymbol{f}_1, \boldsymbol{f}_2, \ldots, \boldsymbol{f}_n) + \sum_{i=1}^{n} \boldsymbol{\beta}^T \boldsymbol{h}_i(\boldsymbol{f}_i)\right].$$

The constraint has been included in the cost function and the are the Lagrange multipliers which will also be taken as pa to be optimized. We develop a method which avoids directly solving the complicated optimization problem in (11) by adopting a step-by-step approach. The Dual Descent method contains two nested iterations to solve (11). For each fixed

The inner loop solves (7) for. Note that each minimization of (11) decouples into smaller minimization lems. The updated vectors are passed on to the outer loop which updates the vector using the gradient of the dual function given by (9). The derivative of the cost function with respect to the real and imaginary parts of is compactly represented as complex-gradient vectors as follows:

$$\nabla \hat{L}_{\boldsymbol{f}_i} = \boldsymbol{H}_i(\boldsymbol{f}_i)\boldsymbol{f}_i - 2\boldsymbol{T}_1^H \boldsymbol{g}_i$$

Where

$$\begin{aligned}
H_i(f_i) &:= 2T_i^H T_i + p\lambda_{1i}^2 A_{1i} + p\lambda_{2i}^2 \Phi_i^H D^T A_{2i} D\Phi_i + F_i \\
A_{1i} &:= diag\left\{\frac{1}{(|(f_i)_j|^2 + \epsilon)^{1-p/2}}\right\}, \quad j = 1, \ldots, N \\
A_{2i} &:= diag\left\{\frac{1}{(|(D|f_i|)_j|^2 + \epsilon)^{1-p/2}}\right\}, \quad j = 1, \ldots, M \\
\Phi_i &:= diag\left\{e^{\{-j\angle\{(f_i)_j\}\}}\right\}, \quad j = 1, \ldots, N \\
F_i &:= diag\left\{\frac{(\nabla(\beta^T h_i(f_i)))_j}{(f_i)_j}\right\}, \quad j = 1, \ldots, N. \qquad (13)
\end{aligned}$$

Note that can be seen as an approximated Hessian matrix for the th-channel. The last term in factor out in the radiant expression (12) for the cases where is not a polynomial in. Thus, the first derivative with respect to each of the, for a fixed, can be independently used to find the updated vectors. The quasi-Newton iteration equation for each, given, is as follows:

$$H_i(f_i^l)f_i^{l+1} = \gamma_D\, 2T_i^H g_i.$$

The previously shown equation (for each) can be solved using the conjugate gradient method [24], typically with until the residual error is less than some predefined (e.g.). Each is updated until the relative difference in the updated vectors is less than some user-selected)

$$\frac{\| f_i^{l+1} - f_i^l \|}{\| f_i^{l+1} \|} < \delta_1 .$$

Then, all the updated vectors are used to update using a gradient-based approach

$$\beta^{k+1} = \beta^k + \alpha \nabla \xi(\beta^k)$$

PSEUDOCODE FOR THE DUAL DESCENT METHOD

Input: Measured images g_i, forward operators T_i, sparseness paramters p, λ_{1i}^2, λ_{2i}^2, stepsize α and convergence parameters δ_1 and δ_2.

Output: Jointly enhanced images f_i.

Initialize: $f_i^0 = g_i$, $\beta^0 = 0$ and $\nabla\xi(\beta^0) = \sum_{i=1}^{n} h_i(g_i)$.

The Algorithm

```
while e > δ2 do
    Update β^k using (16)
    while Relative change in all f_i^l > δ1 do
        Form diagonal matrices Λ_1i, Λ_2i using vectors f_i^l and equation
        (13)
        Solve for the vectors f_i^(l+1) in equation (14) using β^(k+1) computed
        above
        Compute relative change in f_i^l: e_i = ||f_i^(l+1) − f_i^l|| / ||f_i^l||
    end while
    f̂_i^(k+1) = f_i^(l+1)
    Compute ∇ξ(β^(k+1)) using (17)
    Relative change in β:
    e = |ξ̃(f̂_1^(k+1), f̂_2^(k+1), ..., f̂_n^(k+1), β^(k+1)) − ξ̃(f̂_1^k, f̂_2^k, ..., f̂_n^k, β^k)| / ξ̃(f̂_1^k, f̂_2^k, ..., f̂_n^k, β^k)
end while
f_i = f̂_i^(k+1)
```

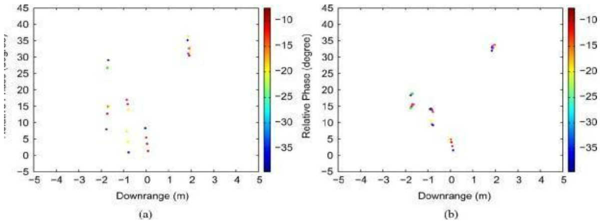
Where (e.g) is the user-selected step size and

$$\nabla\xi(\beta^k) = \sum_{i=1}^{n} h_i(\hat{f}_i^k)$$

Is the gradient of the dual function derived in Lemma 1. Al though a second-order descent is possible for Culating for all images and for each iteration is computationally demanding even for the cases where the convolution matrices. Hence, we elect to use a gradient approach here. The stopping criterion is chosen to be

$$\frac{|\tilde{\xi}(\hat{f}_1^{k+1}, \hat{f}_2^{k+1}, \ldots, \hat{f}_n^{k+1}, \beta^{k+1}) - \tilde{\xi}(\hat{f}_1^k, \hat{f}_2^k, \ldots, \hat{f}_n^k, \beta^k)|}{\delta \quad \tilde{\xi}(\hat{f}_1^k, \hat{f}_2^k, \ldots, \hat{f}_n^k, \beta^k)} < \delta_2.$$

The basic idea is to partially enhance the images indepen dently using slight modifications to the algorithm described in [25] and then use the updated vectors to estrus, two iterations, the inner iteration to find the enhanced images for each updated and the outer it employs a gradient-based method [22], to up refined estimates of the enhanced images. In this way, instead of simultaneously solving for all the vectors and, we fix and then minimize the objective function for that. Then we update imization, thereby finding the surface of over which must be maximized. We, thus, make an implicit assumption that remains unchanged throughout the inner iteration, thereby obviating the need to maximize the function simultaneously along with the minimization. This decoupled approach significantly reduces the size of any compound optimization problem. The pseudo code for the Dual Descent method cal images computationally are gradient-based independently se estimate. There are, it-eration, which update using the



We apply the proposed joint processing strategy to the 2images from the eight passes. The resulting images share the same support but differ in their phases. The sum magnitude over the eight passes for independent and joint enhancement techniques for. We observe that joint processing reduces clutter levels and provides better isolation of target features.

B. Medical Imaging Example

In many medical imaging applications, multichannel images are collected using different modalities, but the physical structure of the object being imaged remains the same across the channels. For example, even within MRI, T1weighted images provide complementary information, i.e., in T1 weighted images fat appears bright and fluids appear with Intermediate signal intensity, while in T2-weighted images fat appears dark and fluids are bright [33]. Typically these images are not sparse in the image domain but they contain a small number of homogeneous regions, so the gradient is sparse. Moreover each modality captures the regions of the underlying physical structure with different intensities because of which the need for joint enhancement becomes more relevant for reconstructing an accurate estimate of the physical structure. In such cases, the independent enhancement technique, with independently- and (b) jointly-enhanced plane images 2-D underlying T1- and T2-T1-foreover,

C. Computational Complexity

In this subsection, we briefly discuss the computational com-plexity of the joint enhancement method and compare it to the independent enhancement. For the medical examples considered the frequency domain representations of the Point Spread Function (for finding and for are stored. Then, the dominating terms for the computational complexity (number of multiplications per iteration of the algorithm) are, which are computed once for each iteration. So the computational complexity (for each iteration) of the Dual Descent method is on the order of , being the number of pixels in the image, as all other multiplications required for each iteration are of a lesser order. The total computational com plexity of the algorithm is determined by the number of iterations the method takes to converge and its dependent number of pixels, which is still an open research topic. Note that the computational complexity per iteration of the independent enhancement method, which involves two unconstrained optimization problems, is also. So we expect the time complexities of the Dual Descent and the independent enhance-ment to be on the same order of magnitude. In all our Simulations the ratio of computation time was always between 4 and 5 with the Dual Descent being the larger computational time method.

IV. CONCLUSION

In this paper we presented a joint enhancement problem for complex-valued coupled-multichannel-images/datasets. We formulated the problem as a joint optimization problem with constraints. We then reformulated it as an equivalent strained) dual problem and developed a fast numerical method, Dual Descent, for solving the optimization problem. We adopt a regularization approach to the enhancement problem by including -type terms in the cost function to counter the ill-posed nature of the inverse problem. Medical imaging examples that the independent processing of channels without considering the coupled-nature of the underlying datasets can result in degraded performance, whereas the joint approach, that includes additional constraints on the datasets, preserves the required features during the enhancement process. In this paper we focused on multichannel image enhancement, where the channels represent data from separate sensors. However, the techniques discussed in this paper are applicable for joint enhancement of images at multiple resolutions. in order to lower bound the minimum attainable value of the primal objective function, is not straightforward and is a topic of future research. Ana-lyzing the conditions on the objective and the constraint functions for which strong duality holds is another interesting topic to be explored.-,), used for Dethe complexity iterations dependence on the independent cities (unconstrained) true constraints enhancement, per Extensions of the joint enhancement technique for colour images and multiband hyper spectral images are some of our future directions.

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